125th birth anniversary of S. N. Bose

International Workshop on Bose-Einstein Condensation and related phenomena (IWBECRP)

SNBNCBS, Kolkata, 26-28 March 2018

Creating Atom-Atom Bound State in Continuum by Interference of Magneto-Optical Feshbach Resonances

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- **First introduced by von Neumann and Wigner in 1929**
  J von Neumann and E Wigner, Phys. Z. **30,** 465 (1929)
- Counter-intuitive concept
- Square-integrable discreet state in continuum
- It occurs due to destructive interference of out-going waves

- **First experimental realization:**
  - C. W. Hsu et al., Nature 499, 188 (2013)

An "unusual" trap for an electron: A. G. Smart, Phys.Today 66, 14 (2013)

Experimental realizations

Realization in photonic crystal

C. W. Hsu et al., Nature 499, 188 (2013) [MIT, J D Joannopoulos]

Light trapped in the radiation continuum

BIC of light by "destructive interference" of plane waves

Realization in wave guides by "symmetry incompatibility"

Y. Plotnik et al., PRL 107, 183901 (2011)

[Technion (Israel) and Friedrich-Schiller-Universität Jena (Germany)]

Using optical waveguide array: BIC is the guided mode

Realization of BIC laser: A. Kodigala et al, Nature, 12 Jan 2017

Using BIC cavity made of cylindrical nanoresonators

[University of California San Diego]

Neuman-Wigner BIC is a wave phenomenon: Should occur in all classical & quantum interference domains with appropriate tuning of the system parameters

Review: Hsu et al, Nat. Rev. Mater., 1, 16048 (2016)

Our proposal to realize two-atom (diatom) BIC in cold collisions

B Deb & G S Agarwal, PRA **90,** 063417 (2014)

Using quantum inference in photo- & magneto-associations

Purpose of this work:

- To demonstrate BIC of cold atoms by quantum interference
- To enhance the efficiency of formation of Feshbach molecules
- Coherent control of atomic and molecular processes with BIC

Early theoretical works (connections with resonances & reactions)

BIC of two particles in the continuum of scattering states

F H Stillinger & D R Herrick, PRA **11,** 446 (1975)

- Gave an exposition of the work by Neumann & Wigner
- Importance of two-electron BIC in atomic and molecular physics

BIC by interference resonances in multichannel scattering

F Friedrich & D Wintgen, PRA **32,** 3231 (1985)

Resonance reactions (three-channel meson-baryon scattering)

L Fonda & R G Newton, Annal of Physics 10, 490 (1960)

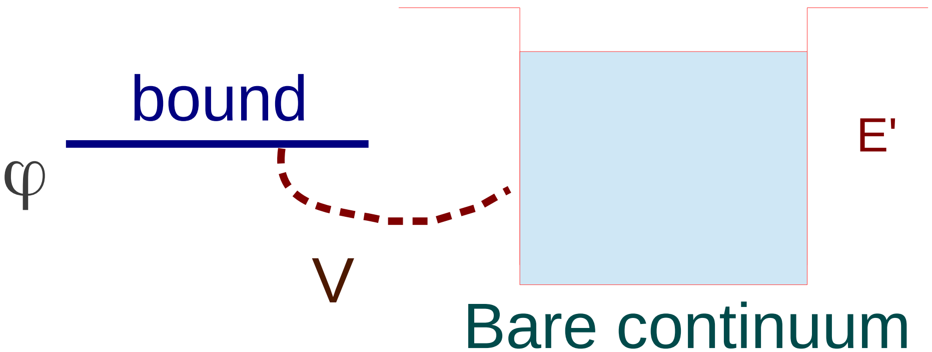
(Probably not aware of the work by Neumann & Wigner)

Plan of the Talk

- **Preliminaries:**
  - Fano resonance or Fano effect (U Fano, Phys. Rev. 1961)
  - Magneto-optical Feshbach resonance
  - Feshbach method (H Feshbach, 1958)
- **Model**
  - Two excited molecular bound states coupled to the continuum of two-atom scattering states with two PA lasers in the presence of a magnetic Feshbach resonance
  - Effective Hamiltonian using projection operator & resolvent methods
- **Solution**
  - BIC: Analytical and numerical results
  - Conditions for the occurrence of BIC
- **How to detect: Proposal for experimental realizations**
- **Applications**
  - Controlling FR and efficient formation of Feshbach molecules
  - Coherent spectroscopy using BIC

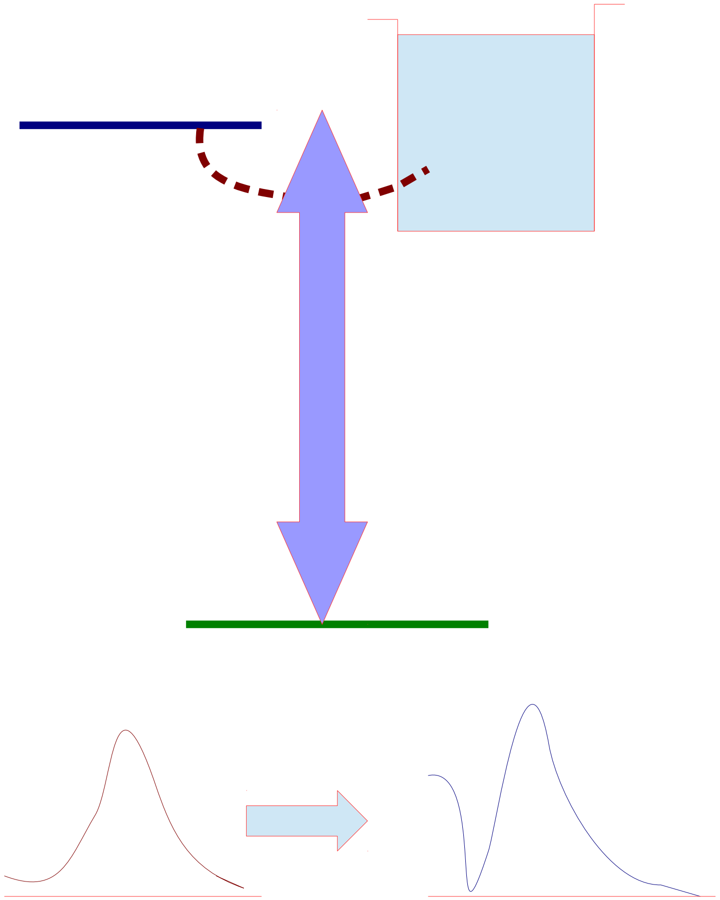
Fano effect: QI in continuum-bound coupled system

Continuum-bound interacting system



$$|E\rangle = a_E |\varphi\rangle + \int dE' \, C_{E'}(E) |E'\rangle$$

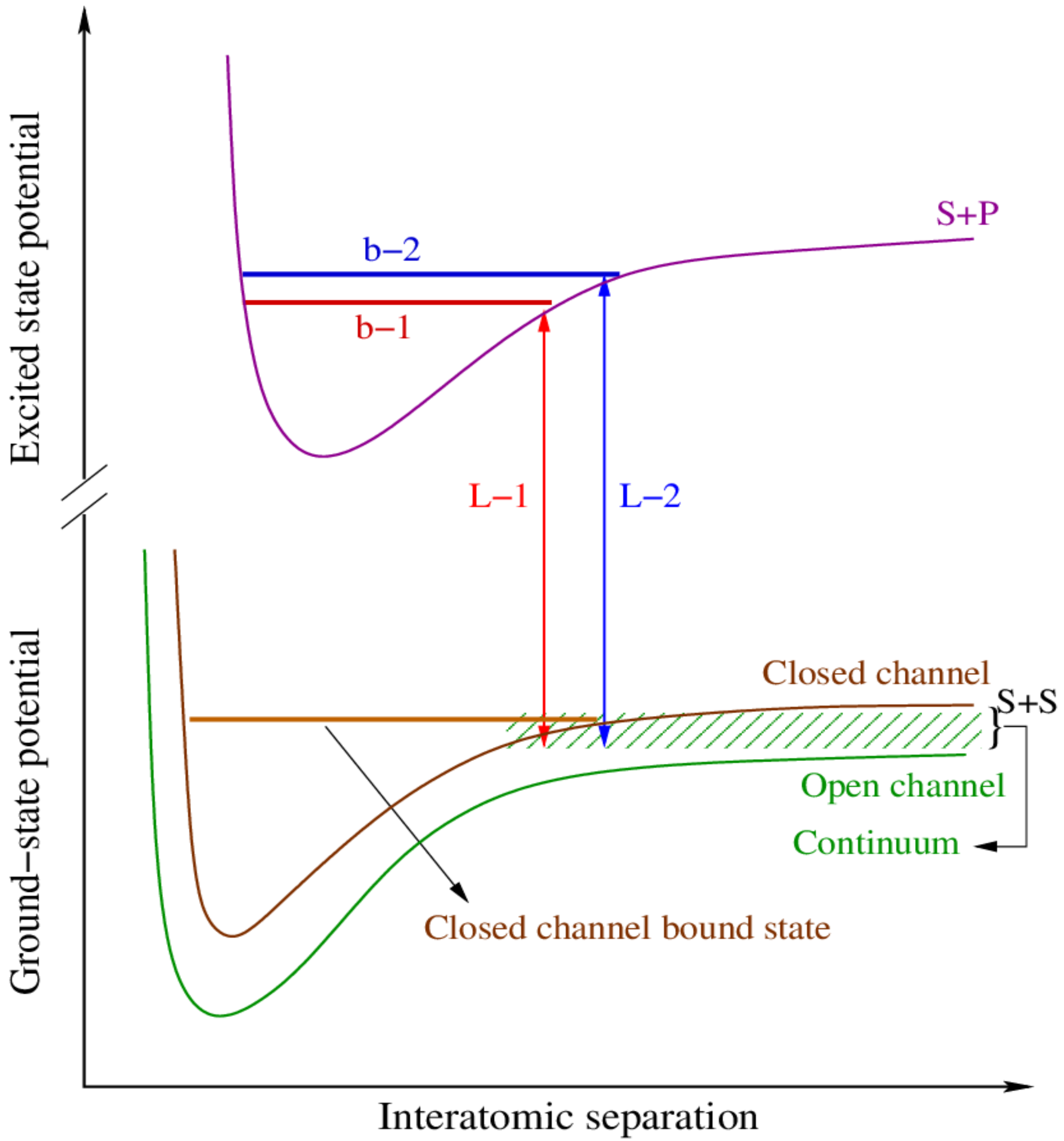
Dressed continuum



Asymmetry in absorption spectra results from quantum interference

Fano asymmetry parameter q

Model



MFR is described with a two-channel model (open and closed)
Quasi-bound state in CC
Continuum in open channel

Photoassociation (PA)
b-1 & b-2: excited bound states
Coupled to the continuum & the quasi-bound state with two **PA** lasers (L-1 & L-2)

Amplitudes of several transition pathways interfere

Two interfering
Fano resonances

Effective Hamiltonian

Use projection operators

$$P = \sum_{n=1}^{3} |b_n\rangle\langle b_n| \quad \text{and} \quad Q = \int dE\, |E\rangle\langle E| = 1 - P$$

The resolvent operator $G(z) = (z - H)^{-1}$ can be projected onto

$$PGP = (z - H_{eff})^{-1} \quad \text{where} \quad H_{eff} = \frac{\hbar \Gamma_f}{2} [A - iB]$$

$$A = \begin{bmatrix} \delta_1 & 0 & q_1\sqrt{g_1} \\ 0 & \delta_2 & q_2\sqrt{g_2} \\ q_1\sqrt{g_1} & q_2\sqrt{g2} & -(k a_s)^{-1} \end{bmatrix} \qquad B = \begin{bmatrix} g_1 & g_{12} & \sqrt{g_1} \\ g_{21} & g_2 & \sqrt{g_2} \\ \sqrt{g_1} & \sqrt{g_2} & 1 \end{bmatrix}$$

$$g_{12} = g_{21} = \sqrt{g_1 g_2}$$

H_{eff} is non-hermitian

Point to be noted:

B has two zero eigenvalues and one nonzero eigenvalue

Solution

The two real eigenvalues of effective Hamiltonian are

$$\lambda = \frac{1}{2}\left(\delta_2 - q_1\right) \ \pm\ \frac{1}{2}\left[\left(\delta_2 + q_1\right)^2 - 4 g_2 q_2 \left(q_1 - q_2\right)\right]^{1/2}$$

Subject to the condition

$$g_1 = \frac{\left(\delta_1 - \lambda\right)\left(\delta_2 - \lambda - g_2 q_2\right)}{q_1\left(\delta_2 - \lambda\right)}$$

Special case: When A and B commute, and assuming $q_1 \simeq q_2 = q$, the two real eigenvalues and eigenvectors are

$$E_A = \frac{\hbar \Gamma_f}{2} q\left(g_1 + g_2 - 1\right) \quad E_B = -\frac{\hbar \Gamma_f}{2} q$$

$$\boxed{|A>_{\mathrm{BIC}} = \frac{1}{\sqrt{g_1 + g_2}}\left[\sqrt{g_2}|b_1> - \sqrt{g_1}|b_2>\right]}$$

Dark state

$$|B>_{\mathrm{BIC}} = \frac{1}{\sqrt{\left(g_1 + g_2\right)\left(g_1 + g_2 + 1\right)}}\left[\sqrt{g_1}|b_1> + \sqrt{g_2}|b_2> + \left(g_1 + g_2\right)|b_c>\right]$$

- BIC will influence scattering and PA spectral properties. These can be calculated using Moller operators

$$\Omega_{\pm}(E) = 1 + G(E \pm i\epsilon)V \quad \Rightarrow \quad \boxed{|E>_{\text{dressed}} = \Omega_{+}|E>_{\text{bare}}}$$

- PA probability per unit collision energy

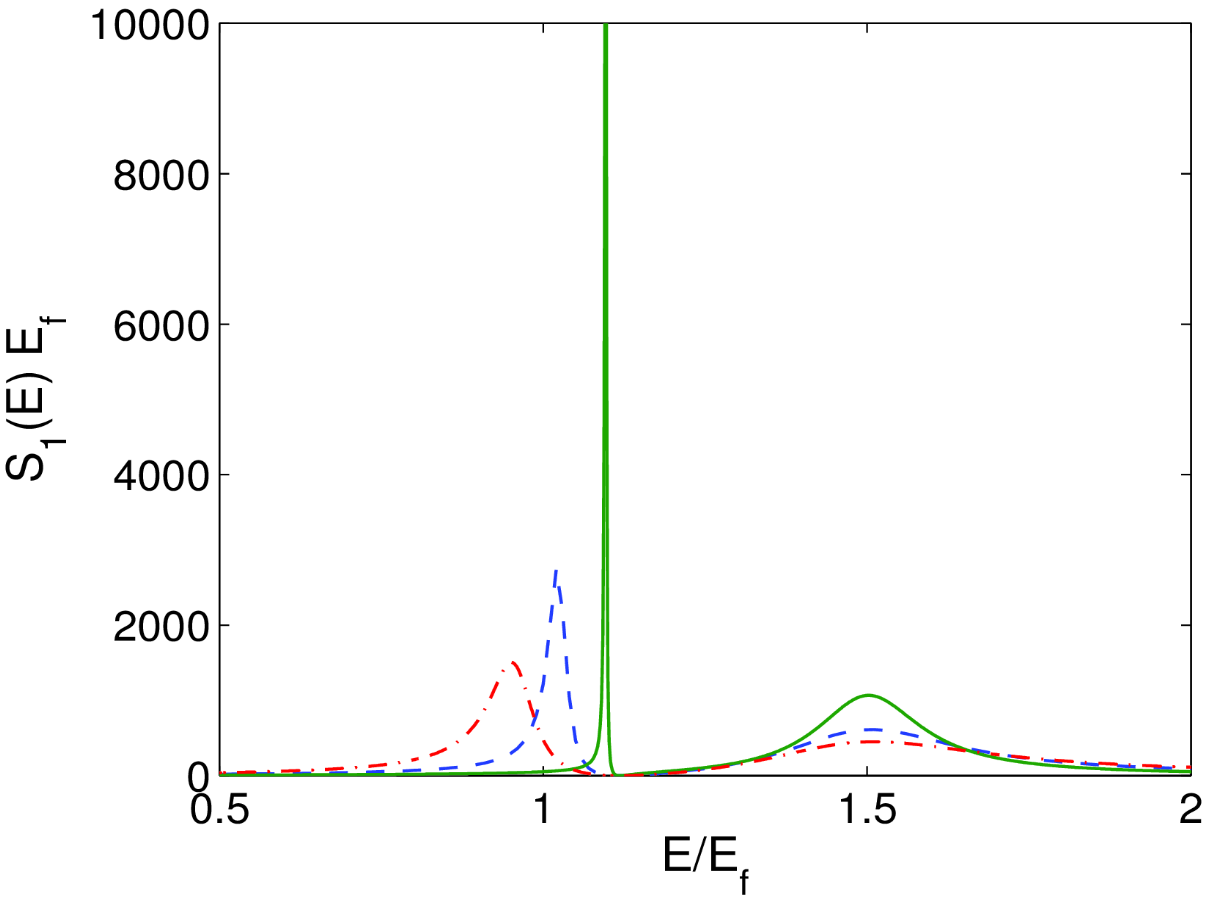
$$\boxed{S_n(E) = |<b_n|E>_{\text{dressed}}|^2 \propto \left(E - H_{eff}\right)^{-1}}$$

- Scattering T-matrix is

$$T = {}_{\text{bare}}<E|V|E>_{\text{dressed}} \propto \left(E - H_{eff}\right)^{-1}$$

Bound-bound Spectrum
No singularity

For a real eigenvalue, singularity appears in scattering cross section



g1 = 0.25 (solid),
g1 = 0.5 (dashed)
g1 = 0.75 (dashed dotted)

g2 = 2.0, δ1 = 1.5, δ2 = −0.5
q1 = − 0.5, q2 = −1.0,

E1= 1.096 − 0.001i
E2 = 1.499 − 0.099i
E3 = −1.595 − 3.150i

g1 = 0.1803 ⇨ E1=1.1180
⇨ BIC

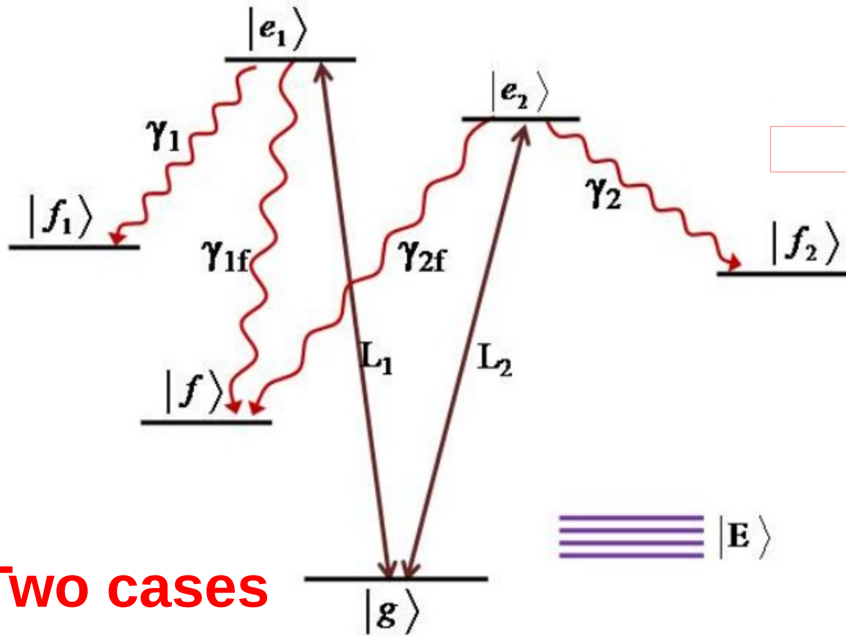
g1=0.5 E1=1.024 - 0.016i
g1=0.75 E1=0.958 - 0.042i

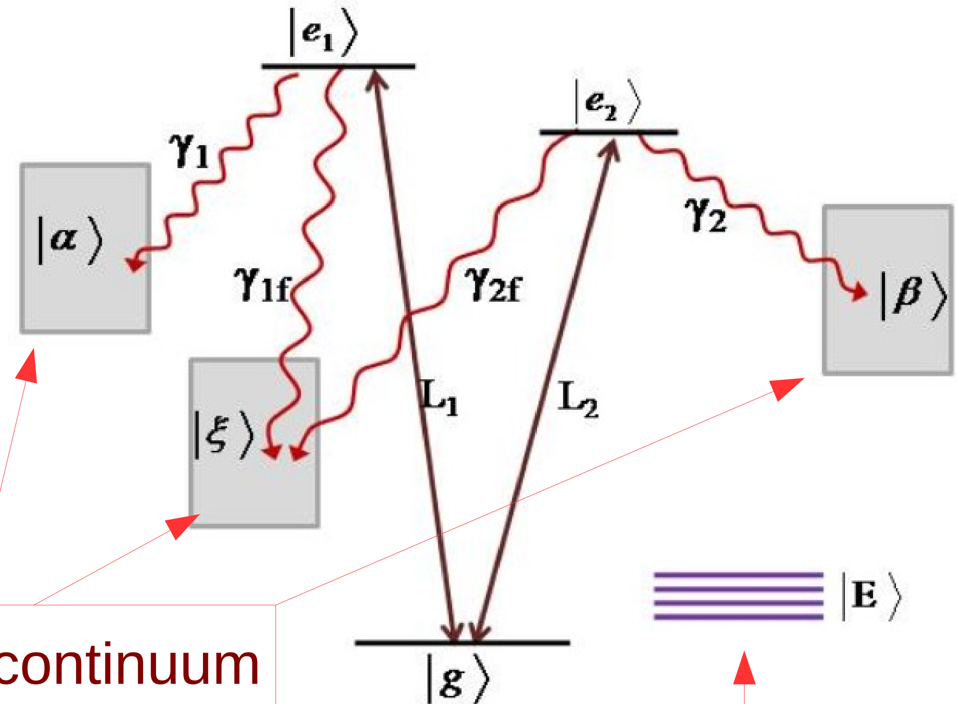
BIC implies scattering resonance with zero width

Close to a real eigenvalue, BIC will show up as an ultra-narrow resonance with asymmetric shape

B Deb & G. S. Agarwal, PRA **90**, 063417 (2014) [Editors' suggestion]

The effects of spontaneous emission on BIC





Two cases

Non-orthogonal Vs. Orthogonal EDM
VIC Vs. non-VIC

EM continuum (vacuum)

Atomic continuum (collision)

Eliminating all continuua, we have $H_{eff}=\frac{\hbar\Gamma_f}{2}[A-iB]$

$$A = \begin{matrix} \delta_1 & 0 & q_1\sqrt{g_1} \\ 0 & \delta_2 & q_2\sqrt{g_2} \\ q_1\sqrt{g_1} & q_2\sqrt{g2} & -(k a_s)^{-1} \end{matrix}$$

$$B = \begin{matrix} g_1+\gamma_1 & g_{12}+\gamma_{12} & \sqrt{g_1} \\ g_{21}+\gamma_{21} & g_2+\gamma_2 & \sqrt{g_2} \\ \sqrt{g_1} & \sqrt{g_2} & 1 \end{matrix}$$

γ_{12} → Vacuum-induced coherence

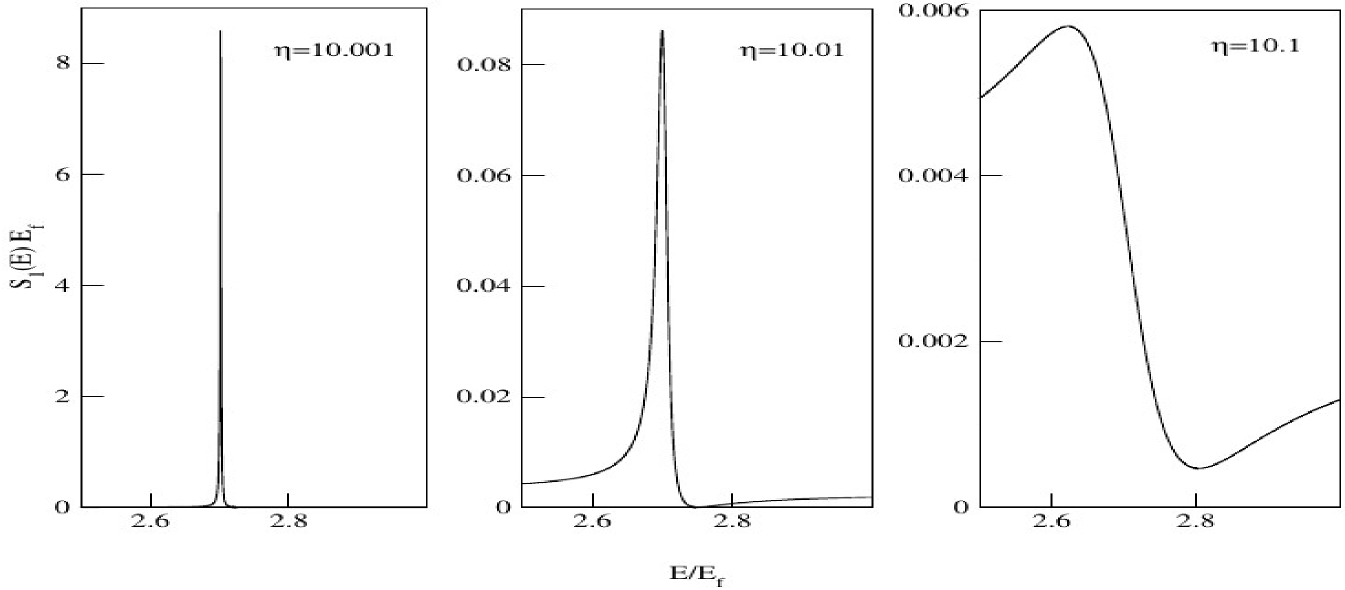
Case-I
Non-orthogonal EDM

B has one zero eigenvalue if

$$\gamma_{12} = \gamma_{21} = \sqrt{\gamma_1 \gamma_2} = \eta$$

$$\gamma_1 = \gamma_2 = \gamma = 10 \ \ \text{(say)}$$

$\eta = 10.001$ $\quad$ $\eta = 10.01$ $\quad$ $\eta = 10.1$



Case-II: Orthogonal EDM

$$\gamma_{12} = \gamma_{21} = \eta = 0$$

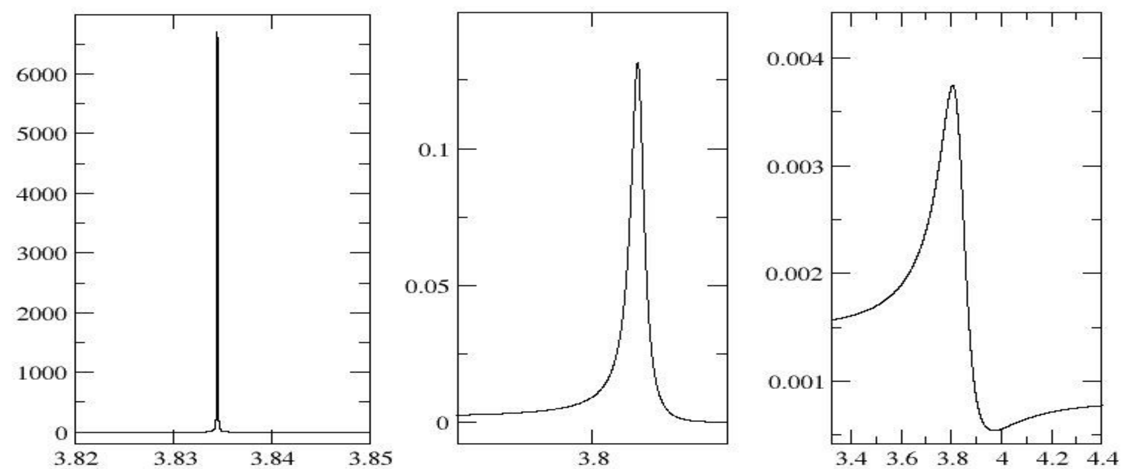
B has no zero eigenvalue if

$$\gamma_i \neq 0$$

There is no BIC in presence of spontaneous emission.

Laser phase may be another control parameter

$\gamma = 0.001$ $\quad$ $\gamma = 0.01$ $\quad$ $\gamma = 0.1$



Somnath Naskar, Dibyendu Sardar & B. Deb (To be submitted)

How to realize BIC of cold atoms? Some experimental aspects

- Magnetically tunable narrow Feshbach resonance are of advantage.

- Choose two excited molecular bound states that can be populated by two PA lasers. These bound states should have comparable or stronger PA coupling with the ground-state quasi-bound state, compared to free-bound couplings.

- Spontaneous emission from the excited bound states will be the biggest hindrance towards realization of the proposed BIC. This may be mitigated by choosing long-lived (such as PLR, meta-stable) states or vacuum-induced coherence [S Das, A Rakshit and B Deb, PRA (2012)].

- The system parameters should be tuned appropriately so that the BIC condition is nearly satisfied. The eigenvalue of a state near BIC should be much different from all other eigenvalues, so that BIC signal can be distinguished clearly. Otherwise, BIC signal can be missed out, instead an Autler-Townes like spectral structure will appear as in the experiment by Bauer et al, Nat. Phys. 5, 339 (2009).

Application-I

Efficient production of Feshbach molecules

- What is a Feshbach molecule ?
- Loss in production process (FR): Bosonic and Fermionic atoms

To mitigate loss and thereby to enhance the efficiency of production, one can make use of B-type BIC

Recall: B-type BIC

$$|B>_{\mathrm{BIC}}=\frac{1}{\sqrt{\left(g_1+g_2\right)\left(g_1+g_2+1\right)}}\left[\sqrt{g_1}|b_1>+\sqrt{g_2}|b_2>+\left(g_1+g_2\right)|b_c>\right]$$

For $g_1 \gg 1$ and $g_2 \gg 1$ we have $|B>_{BIC} \simeq |b_c>$

By using radio frequency pulse, this BIC can be converted into a Feshbach molecule.

Application-II

Controlling Feshbach resonance or cold collision

- Complex eigenvalue with small imaginary part and the real part being almost equal to the energy of BIC implies leakage of the probability amplitude into the continuum. Using Moller operator, the scattering T-matrix can be calculated.

- Our analytical and numerical results show that BIC leads to a Fano-type asymmetric resonant structure in scattering cross section Vs. energy plots. This is unlike usual scattering resonances. Thus BIC can be used to control the structure and the width of magneto-optical Feshbach resonances.

MOFR: B. Deb, J. Phys. B (2010): Importance of *q*-parameter

Shifts and width: B. Deb and A. Rakshit, JPB (2009)

Summary

- It is possible to generate BIC in cold collisions of atoms by magneto-optical Feshbach resonances
- BIC can be applied to make OFR more efficient by suppressing atom loss.
- BIC can be used for efficient production of Feshbach molecules.
- BIC will serve as an important tool for exploring coherent spectroscopy (such as EIT) and coherent control in a variety of physical systems involving a continuum of states.

Further studies

How to include the effects of spontaneous emission ?

- Inclusion of the continuum of vacuum EM modes

What is the relation of BIC with vacuum-induced coherence (VIC) ?

The role of quantum statistics on BIC in many-particle system ?

Thanks for your attention