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THE HEBREW UNIVERSITY OF JERUSALEM

Spin and mass superfluidity in spin-1 BEC of cold atoms

Edouard B. Sonin

International Workshop on BEC and related phenomena
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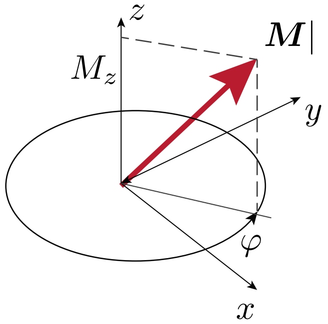
Content

- What is spin superfluidity?
- Metastability: Landau critical velocity, phased slip
- Experiment: B phase of superfluid ^3He, YIG magnetic films
- Ferromagnetic spin-1 BEC of cold atoms: coexistence of mass and spin superfluidity

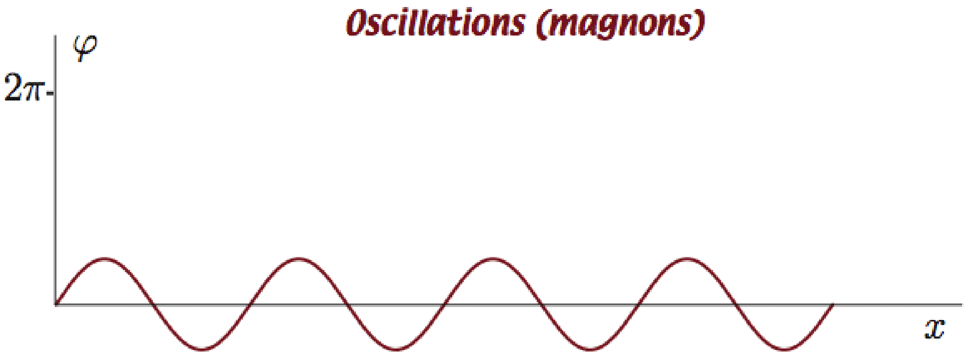
Analogy of superfluid hydrodynamics and magnetodynamics

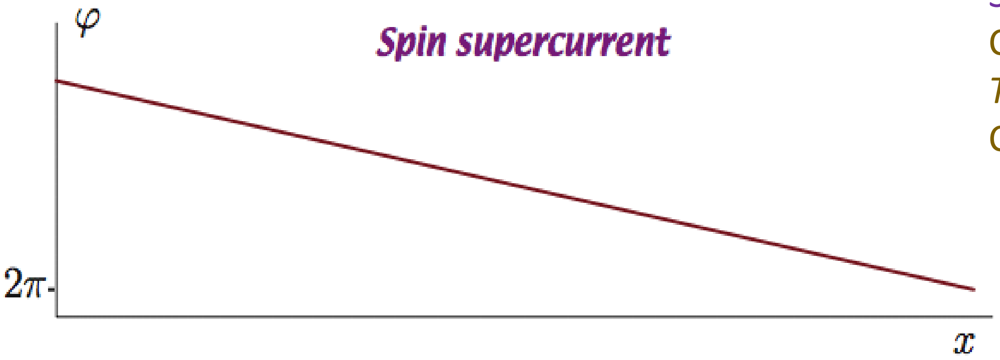
	Superfluid	Ferromagnet
Order parameter	$\psi = \psi_0 e^{i\varphi} = \sqrt{n} e^{i\varphi}$	$\boldsymbol{M}$
Pair of conjugated variables	Density n Phase φ	Magnetization M_z Rotation angle φ
Hamilton equations:	$\frac{d\varphi}{dt} = -\frac{\delta E}{\delta n},$ $\frac{dn}{dt} = \frac{\delta E}{\hbar \delta\varphi} = -\boldsymbol{\nabla} \cdot \boldsymbol{j}$	$\frac{d\varphi}{dt} = -\gamma \frac{M_z}{\chi},$ $-\frac{1}{\gamma}\frac{dM_z}{dt} + \boldsymbol{\nabla} \cdot \boldsymbol{J}^z = 0$

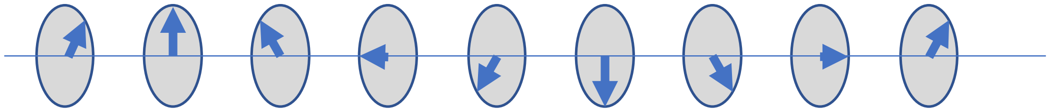
Current: $j \propto \nabla\varphi \qquad J^z \propto \nabla\varphi$



Halperin & Hohenberg, Phys. Rev. **188**, 898 (1969)
Hydrodynamic Theory of Spin Waves

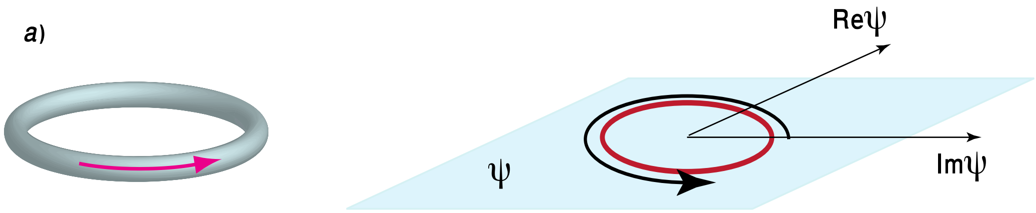






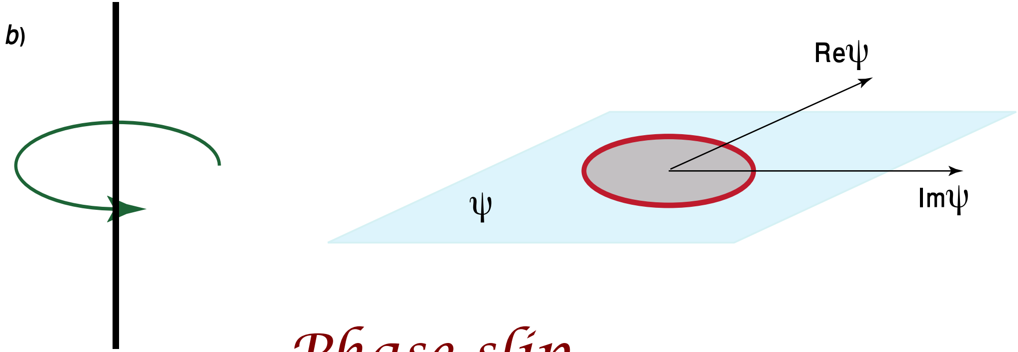
Sonin, JETP (1978), Adv. Phys. 59,181 (2010)
Chen & MacDonald, in: *Universal Themes of Bose-Einstein Condensation* Cambridge University Press, 2017

Topological stability of supercurrents (persistent currents)

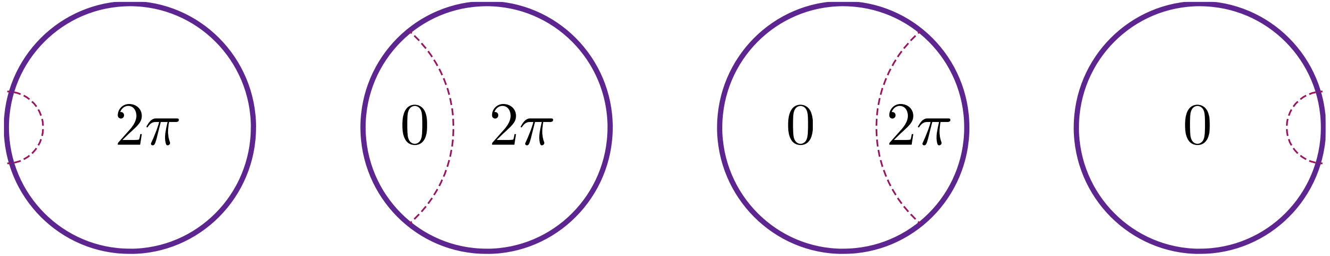


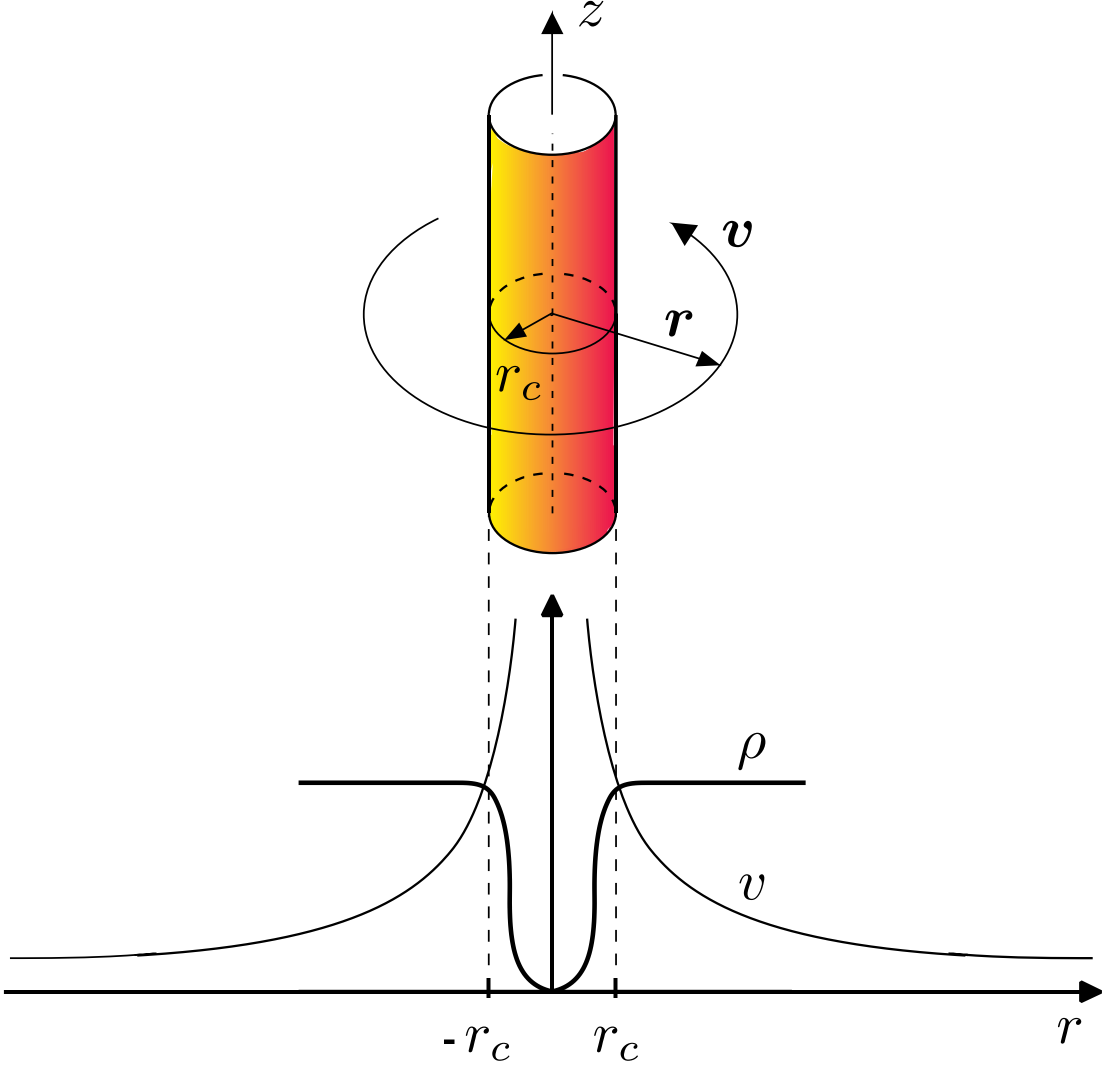
Phase variation around the ring: $\delta\varphi = 2\pi n$ $\quad n$ is a topological charge

Vortex:

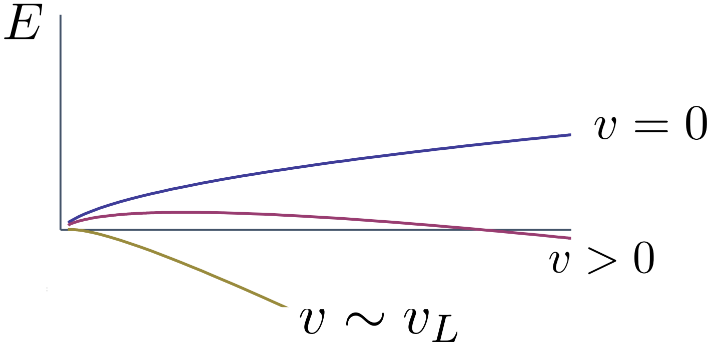


Phase slip





z
v
r
r_c
ρ
v
-r_c
r_c
r



Barrier height:

$$E_b = \frac{\rho\hbar^2}{4\pi m^2}\ln\frac{\hbar}{mvr_c}$$

Core radius: $r_c \sim \dfrac{\hbar}{mc_s}$

Barriers for vortex expansion vanish when the phase gradient becomes of the order of the inverse core radius:

$$\boldsymbol{v} = \frac{\hbar}{m}\boldsymbol{\nabla}\varphi, \quad \nabla\varphi \sim \frac{1}{r_c}$$

Landau criterion: any elementary excitation increases the energy of the current state

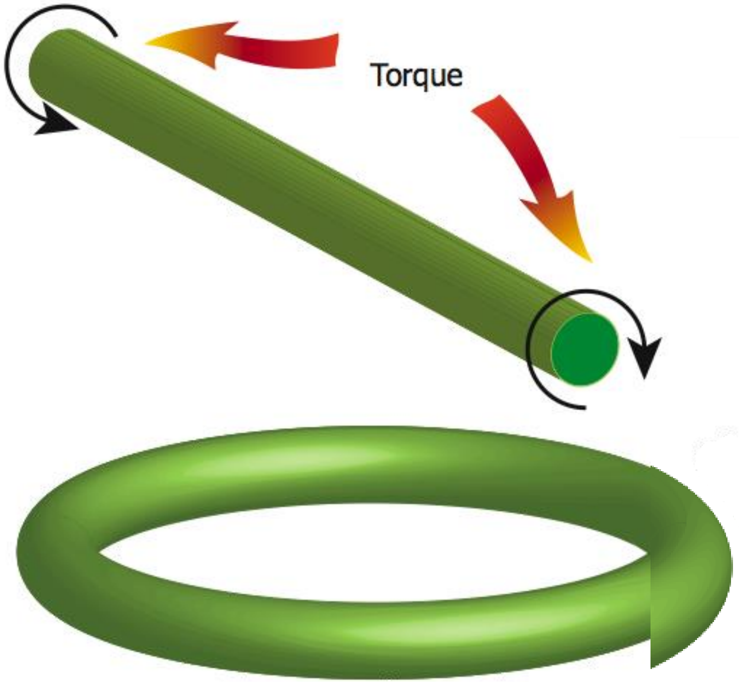
$$\varepsilon(\boldsymbol{p}) = \varepsilon_0(\boldsymbol{p}) - \boldsymbol{p}\cdot\boldsymbol{v}_s > 0$$

$$\omega(\boldsymbol{k}) = c_s k - \boldsymbol{v}_s\boldsymbol{k} > 0$$

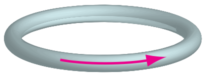
$$\boldsymbol{p} = \hbar\boldsymbol{k}, \quad \varepsilon(\boldsymbol{p}) = \hbar\omega(\boldsymbol{k})$$

Landau critical velocity: $v_L = c_s$

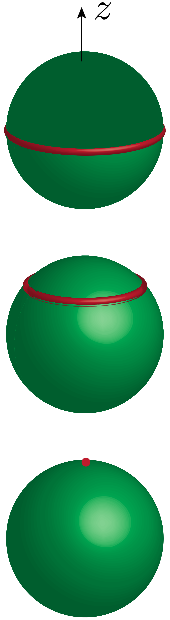
Mechanical analog of supercurrent (twisted elastic rod)



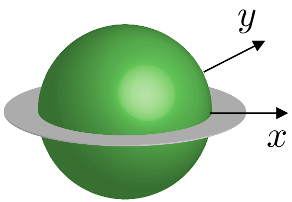
Topological stability of supercurrents in ferromagnets



Isotropic ferromagnet

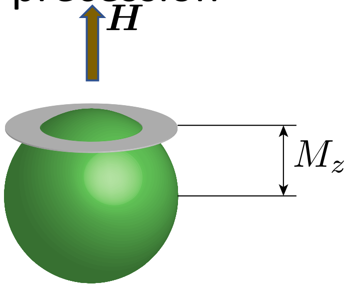


Easy-plane ferromagnet



Sonin, JETP (1978)

Pumping-supported precession

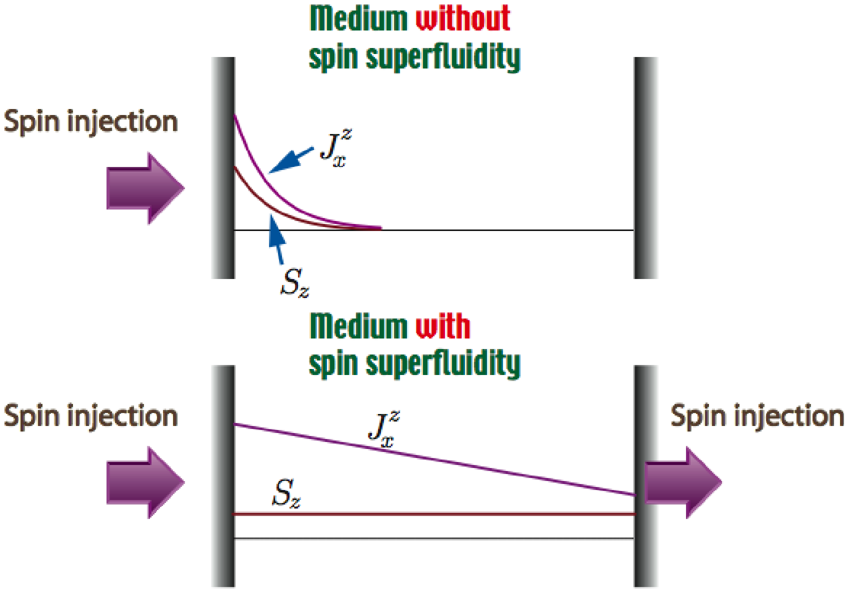


Magnetic vortex:

Barriers for vortex expansion vanish at the Landau critical velocity equal to spin-wave velocity

Observable consequences of spin supercurrents

Sonin, JETP (1978)



$$\frac{dS_z}{dt} + \boldsymbol{\nabla} \cdot \boldsymbol{J}_d^z = -\frac{S - z}{T_1}$$

$$\boldsymbol{J}_d^z = -D \boldsymbol{\nabla} S_z$$

$$\frac{d\varphi}{dt} = \frac{S_z}{\chi}$$

$$\frac{dS_z}{dt} + \boldsymbol{\nabla} \boldsymbol{J}^z = -\frac{S_z}{T_1}$$

Superfluid Spin Transport through Easy-Plane Ferromagnetic Insulators

So Takei and Yaroslav Tserkovnyak

Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA and Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California, 93106, USA

(Dated: November 5, 2013)

Superfluid spin transport — dissipationless transport of spin — is theoretically studied in a ferromagnetic insulator with easy-plane anisotropy. We consider an open geometry where spin current is injected into the ferromagnet from one side by a metallic reservoir with a nonequilibrium spin accumulation, and ejected into another metallic reservoir located downstream. Spin transport through the device is studied using a combination of magnetoelectric circuit theory, Landau-Lifshitz-Gilbert phenomenology, and microscopic linear-response theory. We discuss how spin superfluidity can be probed using a magnetically-mediated electron-drag experiment.

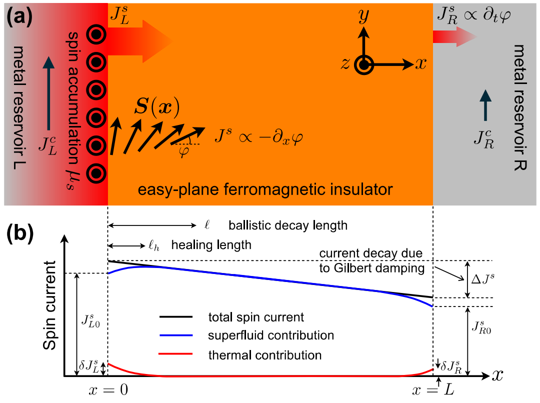
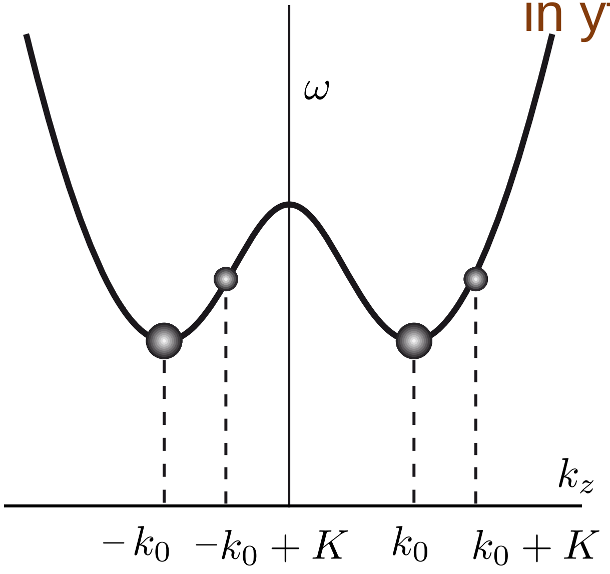


FIG. 1. (a) Schematic of the hybrid structure for realizing spin superfluidity. (b) A schematic plot showing the spatial distribution of the condensate and thermal contributions to the spin currents in the presence of Gilbert damping. See text for a detailed discussion.

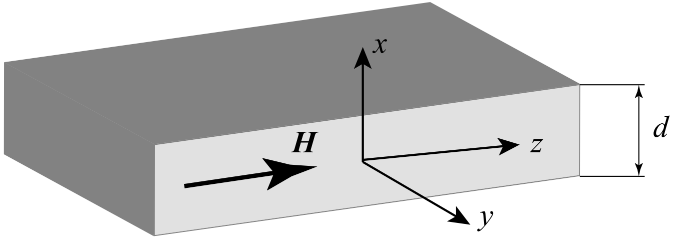
Coherent magnon condensate (magnon BEC) in yttrium-iron-garnet (YIG) films



Bose-Einstein condensation of quasi-equilibrium magnons at room temperature under pumping

Demokritov et al., Nature **443**. 430 (2006)

$$K = k_z - k_0$$



Spin superfluidity in YIG films

Sun, Nattermann, and Pokrovsky, PRL **116**, 257205 (2016)

Isotropic ferromagnet with magnetostatic interaction

$$\mathcal{H} = \int \left[-\boldsymbol{H} \cdot \boldsymbol{M} + D \frac{\nabla_i \boldsymbol{M} \cdot \nabla_i \boldsymbol{M}}{2} \right] d\boldsymbol{r} + \int \frac{\boldsymbol{\nabla} \cdot \boldsymbol{M}(\boldsymbol{r}) \boldsymbol{\nabla} \cdot \boldsymbol{M}(\boldsymbol{r}_1)}{2|\boldsymbol{r} - \boldsymbol{r}_1|} d\boldsymbol{r}\, d\boldsymbol{r}_1$$

Zeeman energy

inhomogeneous exchange energy

Dipolar energy

Is superfluidity possible?

Linear spin wave (boundary problem) → Nonlinear corrections → Landau criterion

Sonin, PRB **95**, 144432 (2017)

Landau criterion for superfluid transport:

Any perturbation of the current state increases its energy

Perturbation: $m_z = M_z - \langle M_z \rangle, \quad \nabla_z \varphi' = \nabla_z \varphi - K$

Critical phase gradient:

$M - \langle M_z \rangle \propto \text{magnon density}$

$$(\nabla_z \varphi)_{cr} = \sqrt{\frac{3\pi\gamma(M - \langle M_z \rangle)}{\frac{d^2\omega(k_0)}{dk_z^2}}} = \sqrt{\frac{3(M - \langle M_z \rangle)}{M}\frac{k_0^2 d}{4\pi}}$$

Critical magnon group velocity:

$$v_{cr} = \frac{d^2\omega(k_0)}{dk_z^2}(\nabla_z \varphi)_{cr} = \frac{4\pi^2\gamma M}{k_0^2 d}\sqrt{\frac{3(M - \langle M_z \rangle)}{M}}$$

Sun, Nattermann, and Pokrovsky, PRL **116**, 257205 (2016): 420 m/sec

Sonin, PRB **95**, 144432 (2017): 3.6 m/sec

Landau criterion for superfluid transport:

Any perturbation of the current state increases its energy

Sonin, PRB **95**, 144432 (2017)

Perturbation: $m_z = M_z - \langle M_z \rangle, \quad \nabla_z \varphi' = \nabla_z \varphi - K$

Critical phase gradient:

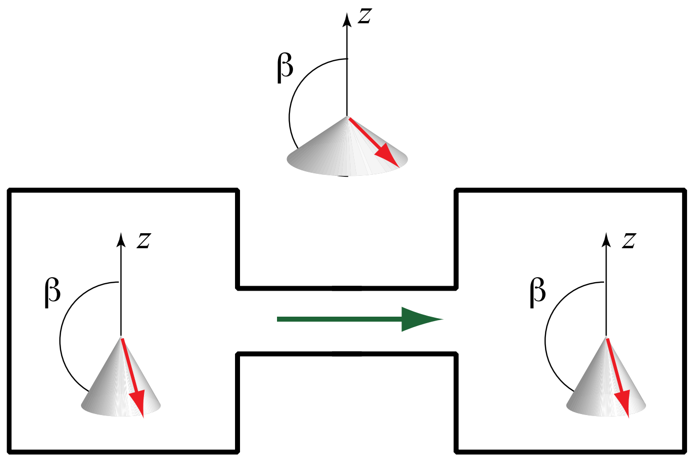
$M - \langle M_z \rangle \propto$ magnon density

$$(\nabla_z \varphi)_{cr} = \sqrt{\frac{3\pi\gamma(M - \langle M_z \rangle)}{\frac{d^2\omega(k_0)}{dk_z^2}}} = \sqrt{\frac{3(M - \langle M_z \rangle)}{M}} \frac{k_0^2 d}{4\pi}$$

Critical magnon group velocity:

$$v_{cr} = \frac{d^2\omega(k_0)}{dk_z^2} (\nabla_z \varphi)_{cr} = \frac{4\pi^2 \gamma M}{k_0^2 d} \sqrt{\frac{3(M - \langle M_z \rangle)}{M}}$$

Superfluid ^{3}He-B



A.S. Borovik-Romanov, Yu.M. Bunkov, V.V. Dmitriev, and Yu.M. Mukharskiy,
JETP Lett. **45**, 124 (1987)

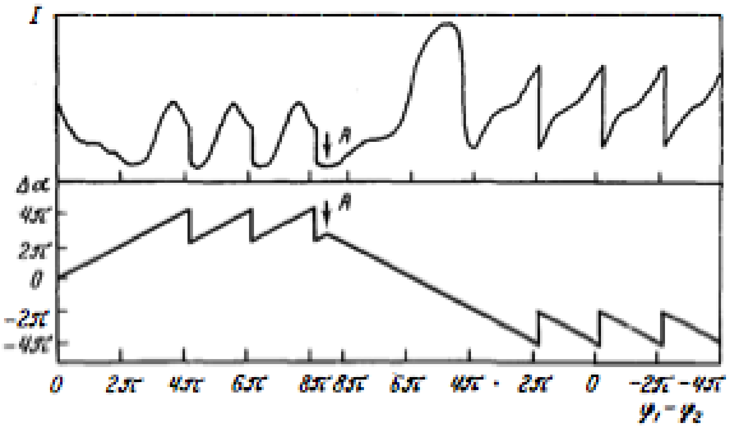


FIG. 2. Signal from the receiving coil and proposed profile of the precession phase difference along the channel. $P = 11$ bar, $\gamma H/2\pi = 460$ kHZ, $T = 0.584\ T_c$, $\omega_H/2\pi = 460.40$ kHz.

Bozhko , Serga, Clausen, Vasyuchka, Heussner, Melkov, Pomyalov, L'vov, and Hillebrands,
Nat. Phys. **12,** 1057 (2016) Supercurrent in a room-temperature Bose-Einstein magnon condensate

the x-axis (see Fig. 2d).

There are two reasons for the x-dependence of the BEC phase φ in our experiment. The first is the already mentioned temperature dependence of ω_c. Within the hot spot of radius R centred at $x = 0$ (that is, for $|x| < R$) the temperature $T(x)$ is higher than the temperature T_0 of the rest of the film (see Fig. 2d). Since in an in-plane magnetized YIG film $d\omega_c(T)/dT < 0$, the BEC frequency in the spot is smaller than outside: $\delta\omega_c(x) = \omega_c(T(x)) - \omega_c(T_0) < 0$. Correspondingly, the phase accumulation $\delta\varphi(x) = \delta\omega_c(x)t$ inside of the spot is smaller than in the surrounding cold film. Therefore, the phase gradient $\partial\delta\varphi(x)/\partial x$ is positive for $x > 0$ and negative for $x < 0$. It means that a thermally induced supercurrent flows out from the spot (mostly in x-direction), as is shown by the red arrows in Fig. 2d:

$$J_T = N_c D_x \frac{\partial(\delta\omega_c t)}{\partial x} \qquad (3)$$

This outflow decreases the magnon BEC density $N_c(x)$ in the spot, $|x| < R$, with respect to that in the cold film, where $N_c(x \gg R) = N_c^0$.

Spatial deviations in the density $N_c(x)$ of the magnon condensate constitute the second reason for the variation of its phase $\partial\varphi/\partial x \neq 0$.

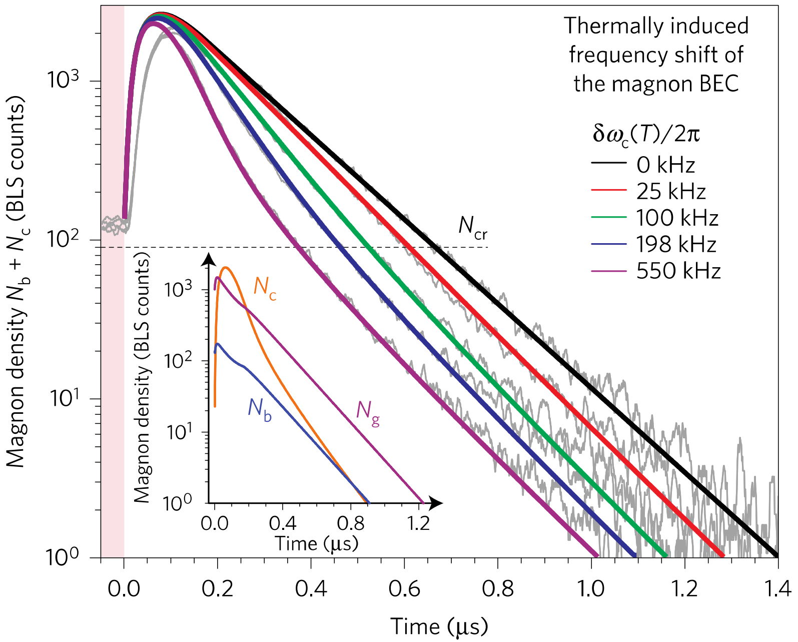
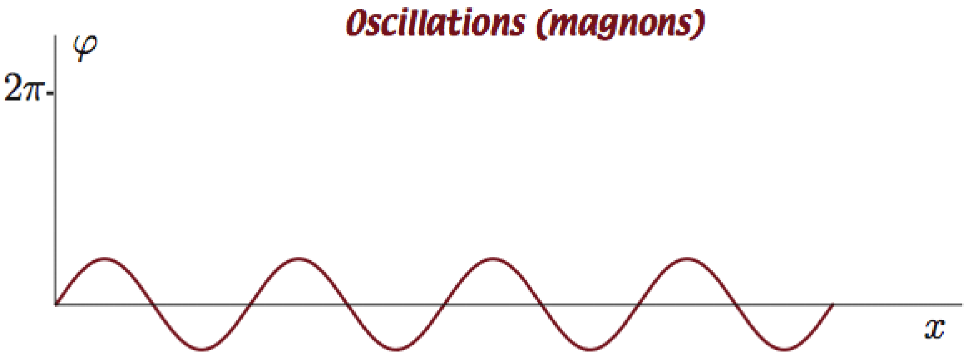
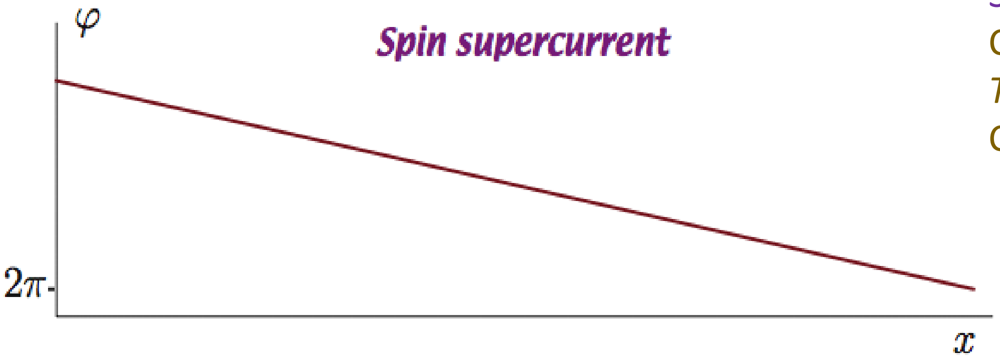
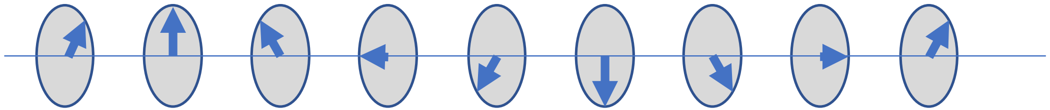


Figure 5 | Theoretically calculated magnon dynamics in a thermal gradient. Theoretical dependencies (coloured lines) of the observable

Phase accumulation $\delta\varphi = \delta\omega_c t < \frac{2\pi}{3}$!







Sonin, JETP (1978), Adv. Phys. 59,181 (2010)
Chen & MacDonald, in: *Universal Themes of Bose-Einstein Condensation* Cambridge University Press, 2017

Ferromagnetic spin-1 BEC

$$\boldsymbol{\psi} = \begin{pmatrix} \psi_x \\ \psi_y \\ \psi_z \end{pmatrix} \qquad \boldsymbol{\psi} = \boldsymbol{m} + i\boldsymbol{n} \qquad \boldsymbol{s} = [\boldsymbol{m} \times \boldsymbol{n}]$$

Triad of 3 orthogonal real unit vectors: $\boldsymbol{m}$, $\boldsymbol{n}$, $\boldsymbol{s}$

Gross-Pitaevskii equations:

$$i\hbar\frac{\partial\boldsymbol{\psi}}{\partial t} = -\frac{\hbar^2\nabla_j^2\boldsymbol{\psi}}{2m} + V|\boldsymbol{\psi}|^2\psi \qquad j_i = -\frac{i\hbar}{2}(\psi_j^*\nabla_i\psi_j - \psi_j\nabla_i\psi_j^*)$$

$$\rho = m\boldsymbol{\psi}^* \cdot \boldsymbol{\psi}$$

$$\boldsymbol{S} = i\hbar[\boldsymbol{\psi} \times \boldsymbol{\psi}^*] \qquad \boldsymbol{s} = \frac{\boldsymbol{S}}{S} \qquad S = \frac{\hbar\rho}{m}$$

Madelung transformation ➡ Hydrodynamics

Hydrodynamical variable: $\rho, \ \boldsymbol{v}_s, \ \boldsymbol{s}$

Euler equation: $$\dot{\boldsymbol{v}}_s + (\boldsymbol{v}_s \cdot \boldsymbol{\nabla})\boldsymbol{v}_s + \boldsymbol{\nabla}\mu_0 + \frac{\hbar^2}{2m^2}\boldsymbol{\nabla} s_i \nabla^2 s_i = 0$$

Continuity equation: $$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0 \qquad \boldsymbol{j} = \rho \boldsymbol{v}_s \qquad \mu_0 = \frac{\hbar^2}{4m^2}\nabla_i \boldsymbol{s} \cdot \nabla_i \boldsymbol{s} + V\rho$$

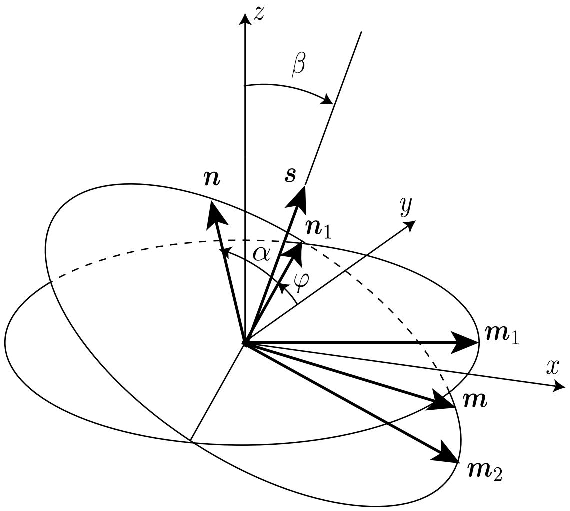
Extended Landau-Lifshitz-Gilbert equation: $$\frac{\partial \boldsymbol{s}}{\partial t} + (\boldsymbol{v}_s \cdot \boldsymbol{\nabla})\boldsymbol{s} - \frac{\hbar}{2m}\left[\boldsymbol{s} \times \nabla^2 \boldsymbol{s}\right] = 0$$

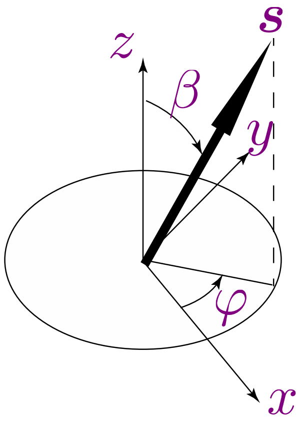
Mermin-Ho theorem: $$\boldsymbol{\nabla} \times \boldsymbol{v}_s = \frac{\hbar}{2m}\epsilon_{ikn} s_i \boldsymbol{\nabla} s_k \times \boldsymbol{\nabla} s_n$$

The spin vector $\boldsymbol{s}$ is an analog of the orbital vector $\boldsymbol{l}$ in ^{3}He-A

Euler angles of the triad: $\alpha, \ \beta, \ \varphi$

$$m_x = \cos\beta\cos\alpha\cos\varphi - \sin\alpha\sin\varphi \qquad n_x = -\cos\beta\sin\alpha\cos\varphi - \cos\alpha\sin\varphi \qquad s_x = \sin\beta\cos\varphi$$
$$m_y = \cos\beta\cos\alpha\sin\varphi + \sin\alpha\cos\varphi \qquad n_y = -\cos\beta\sin\alpha\sin\varphi + \cos\alpha\cos\varphi \qquad s_y = \sin\beta\sin\varphi$$
$$m_z = -\sin\beta\cos\alpha \qquad n_z = \sin\beta\sin\alpha \qquad s_z = \cos\beta$$





Mass current: $\boldsymbol{j} = \rho \boldsymbol{v}_s$

$$\boldsymbol{v}_s = -\frac{\hbar}{m}(\boldsymbol{\nabla}\alpha + \cos\beta\boldsymbol{\nabla}\varphi)$$

Spin current: $\boldsymbol{j}^z = -\frac{\hbar^2\rho}{2m^2}\sin^2\beta\boldsymbol{\nabla}\varphi$

Incompressible superfluids: $c_s \gg c_{sp}$

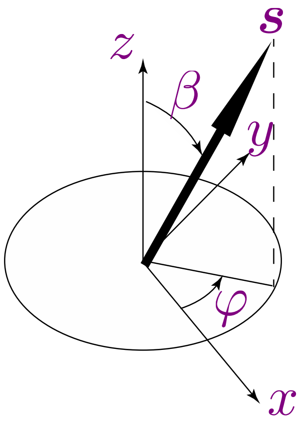
Sonin, arXiv: 1801.01099

$$H = \rho\left\{\frac{v_s^2}{2} + \frac{\hbar^2}{4m^2}\left[\sin^2\beta(\nabla\varphi)^2 + (\nabla\beta)^2\right] + \frac{G(\cos\beta - s_0)^2}{2}\right\}$$

$$H_A = -\gamma H_{ef} S s_z + \frac{\rho G s_z^2}{2}$$

$$s_0 = \frac{\gamma\hbar H_{ef}}{mG}$$

Uniaxial anisotropy



Phase transition at $s_0 = 1$:

Easy-axis anisotropy $s_0 > 1,\ \beta = 0$

Easy-plane anisotropy $s_0 = \cos\beta < 1$

Coexistence of mass and spin superfluidity

Mass superfluidity alone:

Landau critical velocity v_L is equal to the sound wave velocity c_s

Spin superfluidity alone:

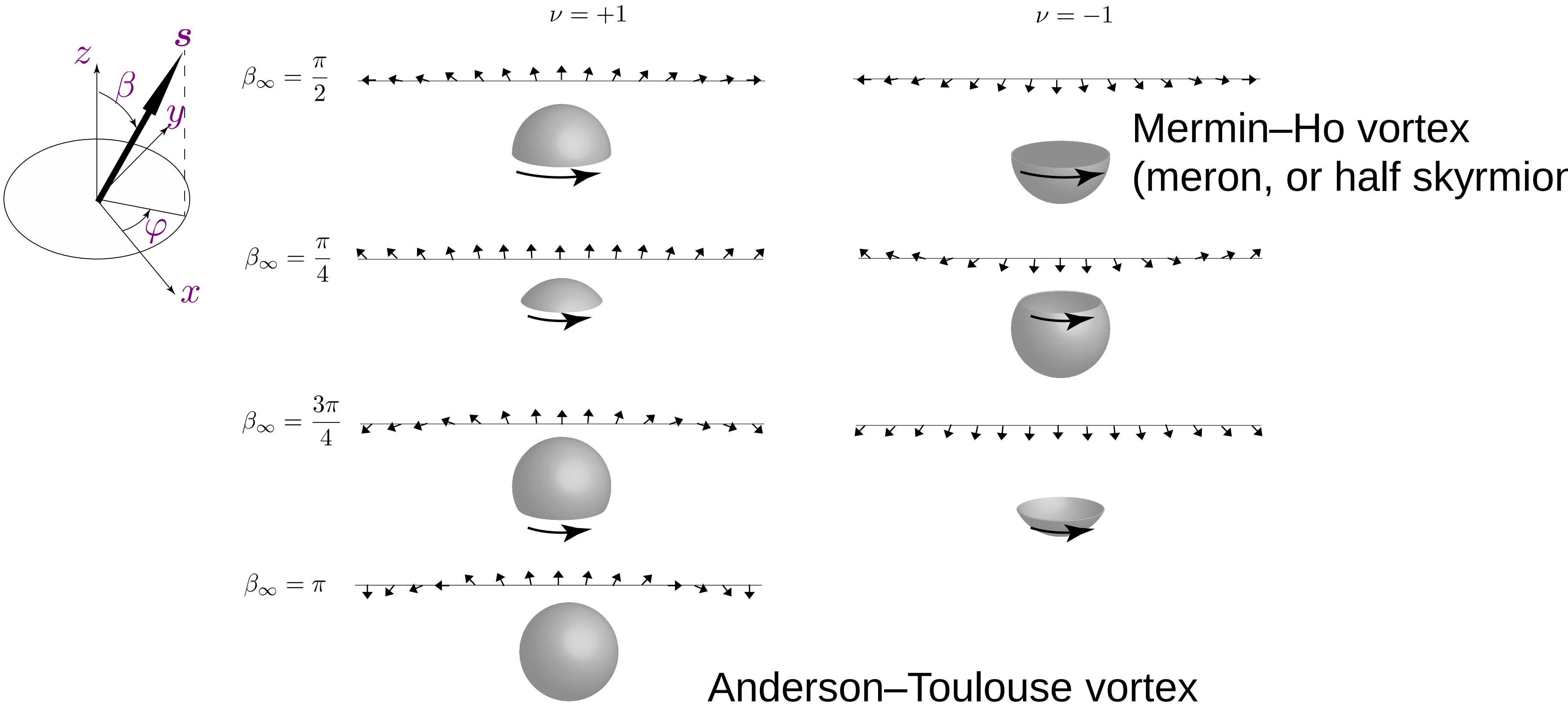
Landau critical velocity v_L is equal to the spin wave velocity c_{sp}

Spin and superfluidity coexist: $v_L = \min(c_s, c_{sp})$

Beattle, Moulder,Fletcher, and Hadzibabic, PRL, **110**, 025301 (2013)

Spin-wave velocity: $c_{sp} = s_\perp \sqrt{\frac{G}{2}}, \quad s_\perp = \sin\beta$ $\qquad \boldsymbol{j}^z = -\frac{\hbar^2 \rho}{2m^2} \sin^2\beta \boldsymbol{\nabla}\varphi$

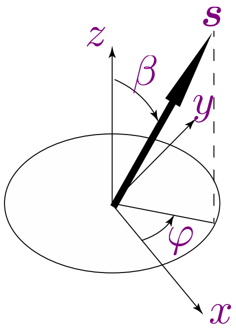
At the phase transition the Landau critical velocity vanishes both for mass and spin superfluidity!

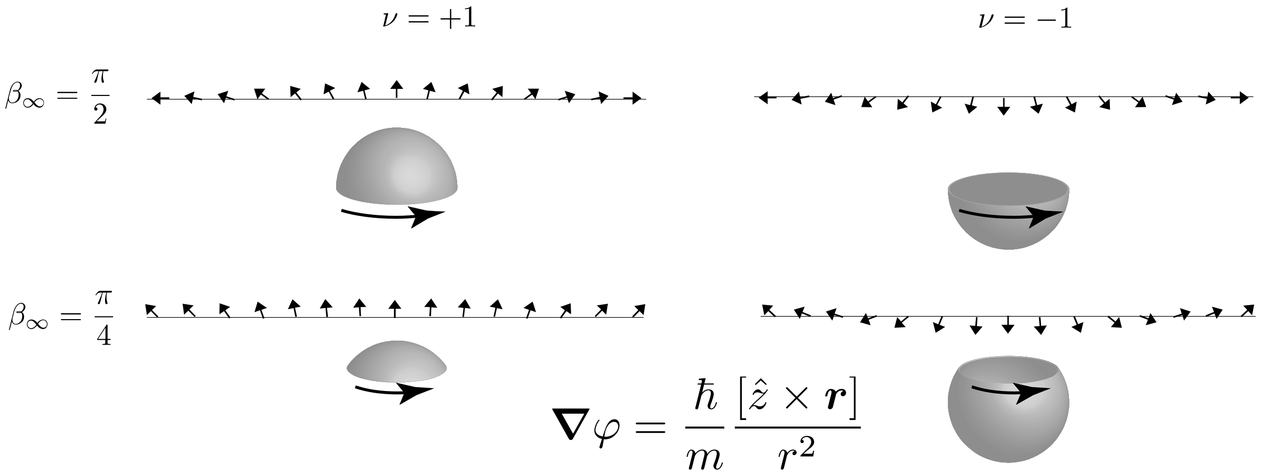


z
$\boldsymbol{s}$
β
y
φ
x
$\nu = +1$
$\nu = -1$
$\beta_\infty = \frac{\pi}{2}$
$\beta_\infty = \frac{\pi}{4}$
$\beta_\infty = \frac{3\pi}{4}$
$\beta_\infty = \pi$
Mermin–Ho vortex
(meron, or half skyrmion
Anderson–Toulouse vortex

Non-singular vortices

Leanhardt, Shin, Kielpinski, Pritchard, and Ketterle, Phys. Rev. Lett. **90**, 140403 (2003)





$$\boldsymbol{\nabla}\varphi = \frac{\hbar}{m}\frac{[\hat{z}\times\boldsymbol{r}]}{r^2}$$

$$\boldsymbol{v}_s = \frac{\hbar(1-\cos\beta_\infty)[\hat{z}\times\boldsymbol{r}]}{mr^2}$$

Circulation: $\dfrac{\hbar(1-\cos\beta)}{m} \rightarrow \dfrac{\hbar\beta_\infty^2}{2m}$

$$\boldsymbol{v}_s = -\frac{\hbar(1+\cos\beta_\infty)[\hat{z}\times\boldsymbol{r}]}{mr^2}$$

Circulation: $-\dfrac{\hbar(1+\cos\beta)}{m}$

Sonin, arXive: 1801.01099:

The upper critical velocity for mass superfluidity is the spin-wave velocity far fro
the phase transition. It does not vanish at the phase transition,
although the Landau critical velocity vanishes.

Instability of non-singular vortices

Shin, Saba, Vengalattore, Pasquini, Sanner, Leanhardt, Prentiss, Pritchard, and Ketterle, PRL. **93**, 160406 (2004)

Anderson–Toulouse vortex is unstable: $$v_s < \frac{\hbar}{m\xi}\left(\frac{\xi_0}{\xi}\right)^{1/3} \ll \frac{\hbar}{m\xi}$$

sample. The theory to be described in this paper shows that the apparent critical field as measured with short pulses may differ considerably from the true critical field that would be observed with sufficiently long pulses.

A theory of the steady-state absorption at low power levels was first given by Kaganov and Tsukernik.[5]

A convenient starting point for a formal theory of the transient growth of spin waves under parallel pumping is the spin-wave formalism developed by Holstein and Primakoff[8] and by Akhiezer.[9] We first express the components of the magnetization vector **M** in terms of creation and annihilation operators at $a^\dagger$ and a. Thus

$$M_x+iM_y=(2g\mu_B M)^{1/2}a^\dagger(1-g\mu_B a^\dagger a/2M)^{1/2}$$

$$M_z=M-g\mu_B a^\dagger a, \qquad (1)$$

where g is the spectroscopic splitting factor, μ_B the

* Supported by the U.S. Air Force under Contract No. AF 19(628)-3861.

† Present address: Department of Electrical Engineering

HERMANN

become

$\kappa_k-\eta_k$ must be sufficiently large and positive for some s

For convenience we express the result (24) in terms

lated to the power absorbed by

$$\bar{P}_{abs}=(8$$

BEC or lasing?

L.V. Keldysh, in: *Bose-Einstein Condensation*, edited by A. Griffin, D.W. Snoke, and S. Stringari, Cambridge University Press, Cambridge, 1995, p. 246:

"The accumulation of a macroscopic number of initially incoherent excitation quanta in a single-magnon mode is lasing"

Rayleigh-Jeans Condensation of Pumped Magnons in Thin-Film Ferromagnets

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We show that the formation of a magnon condensate in thin ferromagnetic films can be explained within the framework of a classical stochastic non-Markovian Landau-Lifshitz-Gilbert equation where the properties of the random magnetic field and the dissipation are determined by the underlying phonon dynamics. We have numerically solved this equation for a tangentially magnetized yttrium-iron garnet film in the presence of a parallel parametric pumping field. We obtain a complete description of all stages of the nonequilibrium time evolution of the magnon gas which is in excellent agreement with experiments. Our calculation proves that the experimentally observed condensation of magnons in yttrium-iron garnet at room temperature is a purely classical phenomenon which should be called Rayleigh-Jeans rather than Bose-Einstein condensation.

Conclusions:

- Spin superfluidity, which was predicted in 70s and observed in the B phase of superfluid ^3He in 80s, still waits its experimental detection in magnetically ordered solids.

- In the ferromagnetic spin-1 BEC of cold atoms mass and spin superfluidity may coexist strongly affecting one another. An unique consequence of their interplay is that the upper critical velocity for mass superfluidity can exceeds the Landau critical velocity.

Thanks