March 26-28 (2018), “Bose condensation and related phenomena”
Bose national center for basic sciences, Kolkata, India

Strong-coupling properties of an ultracold Fermi gas in the BCS-BEC crossover region and application to neutron star EoS

Yoji Ohashi

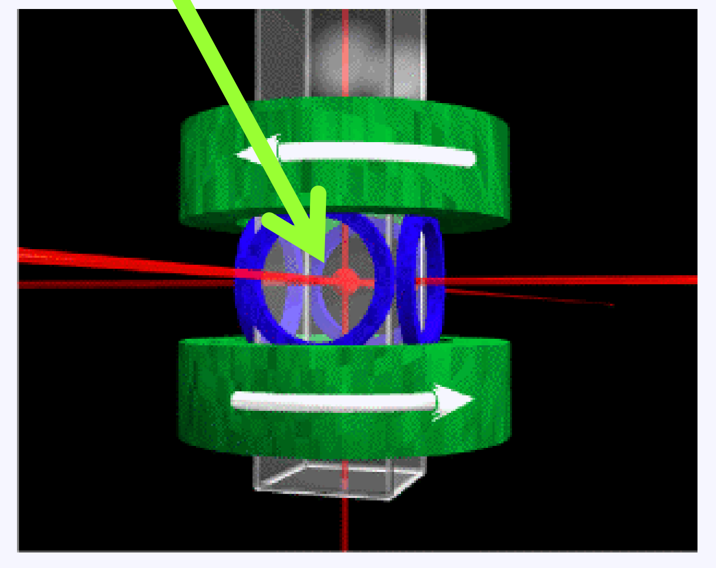
Department of Physics, Keio University, Japan

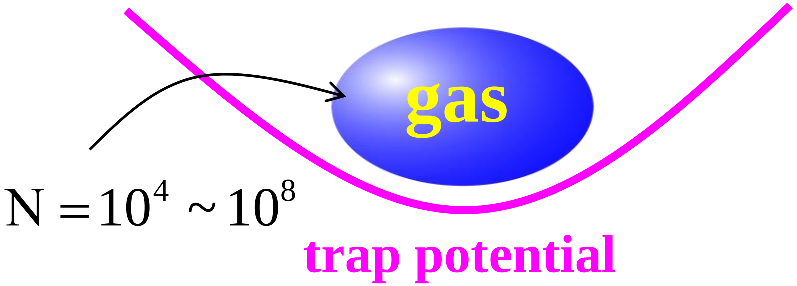
Keio Institute of Pure and Applied Sciences (KiPAS), Japan

- Introduction ▪ ultracold Fermi gas and neutron star
- Strong-coupling theory of an ultracold Fermi gas
    - phase diagram in the BCS-BEC crossover region
    - superfluid properties
- challenge to neutron star EoS
- Summary

Introduction: Ultracold Fermi atomic gas

Fermi atoms (^{6}Li, ^{40}K) are trapped in a magnetic/optical potential, and are cooled down to <O(μK), where various quantum phenomena appear, such as superfluidity.

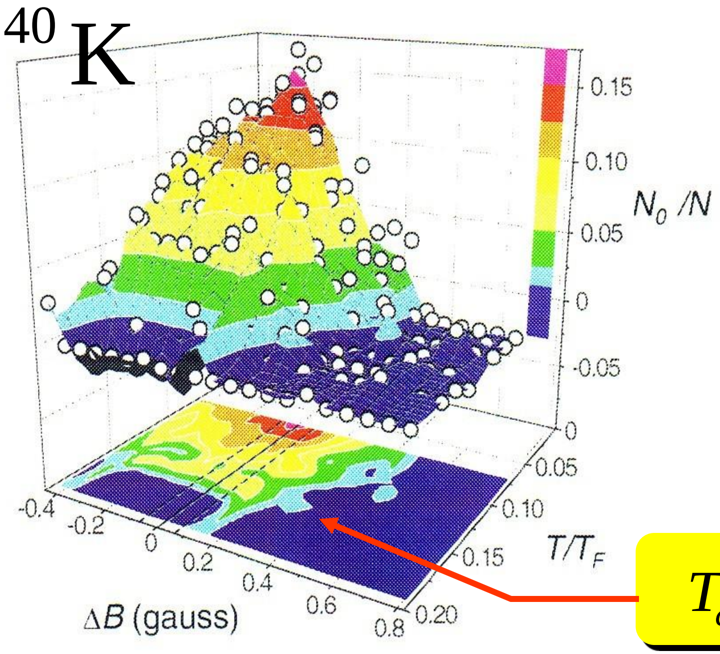




- **Highly clean system**
- **High-tunability of various parameters**
  - interaction strength
  - lattice effects (optical lattice)
  - density
  - temperature
  - statistics (Bose, Fermi)

Quantum simulator for the study of complicated many-body phenomena

Fermion Superfluidity in ^{40}K and ^{6}Li Fermi gases (2004)



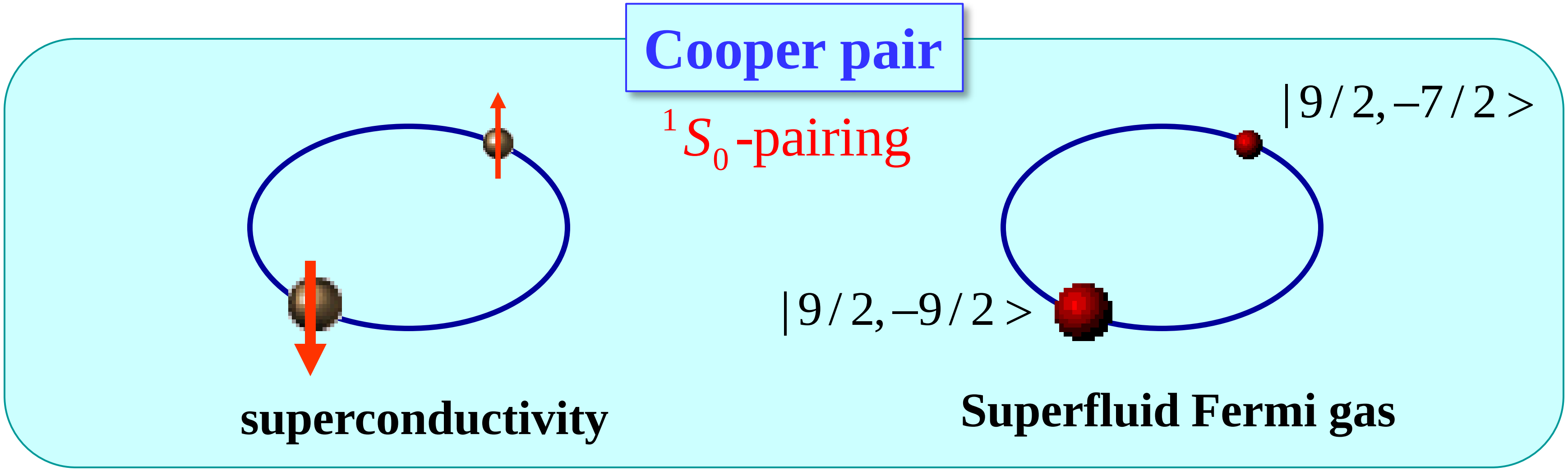
$$|9/2,-7/2> + |9/2,-9/2>$$

$$T_F = 0.35\mu K$$

$$N \sim 10^5$$

C. A. Regal, et al. PRL 92 (2004) 040403.

$$T_c / T_F \sim 0.08 - 0.2 >> 10^{-4} - 10^{-2} (metal)$$



Cooper pair

$^1S_0$-pairing

$|9/2,-7/2>$

$|9/2,-9/2>$

superconductivity

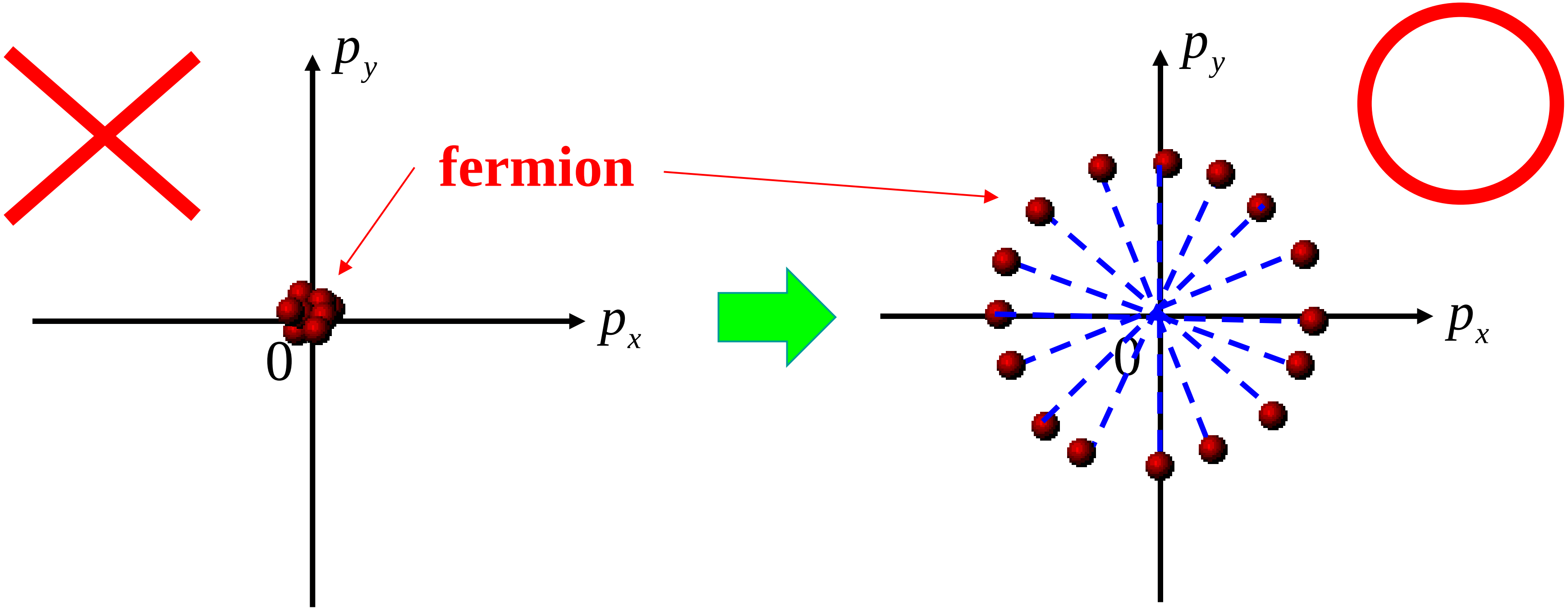
Superfluid Fermi gas

Fermion Superfluidity in ^{40}K and ^{6}Li Fermi gases (2004)

$$\left|\mathrm{BCS}\right\rangle = \prod_{\mathbf{p}}\left[1 + g_{\mathbf{p}} c^{\dagger}_{\mathbf{p}\uparrow} c^{\dagger}_{-\mathbf{p}\downarrow}\right]\left|0\right\rangle = e^{\sum_{\mathbf{p}} g_{\mathbf{p}} c^{\dagger}_{\mathbf{p}\uparrow} c^{\dagger}_{-\mathbf{p}\downarrow}}\left|0\right\rangle$$

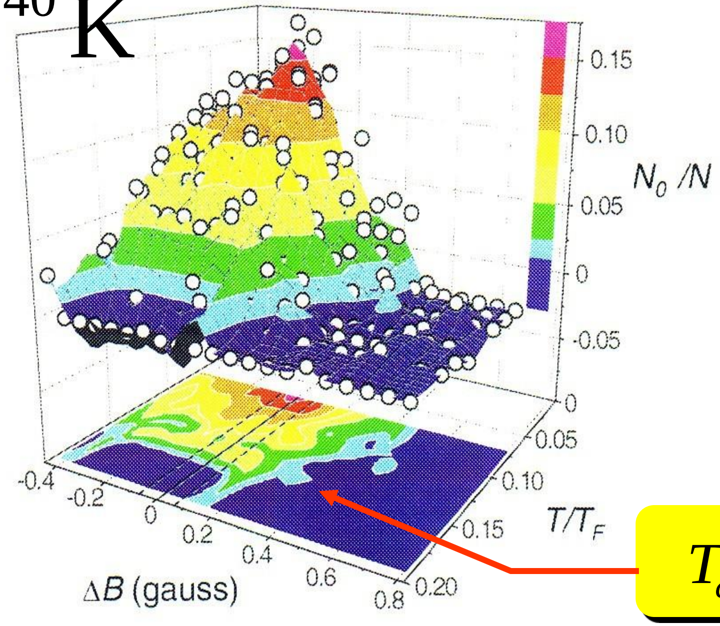
$$\left|\mathrm{BEC}\right\rangle = e^{\sqrt{N_{\mathbf{q}=0}}\, b^{\dagger}_{\mathbf{q}=0}}\left|0\right\rangle$$

BCS = molecular BEC into zero center of mass momentum



Fermion Superfluidity in ^{40}K and ^{6}Li Fermi gases (2004)

^{40}K



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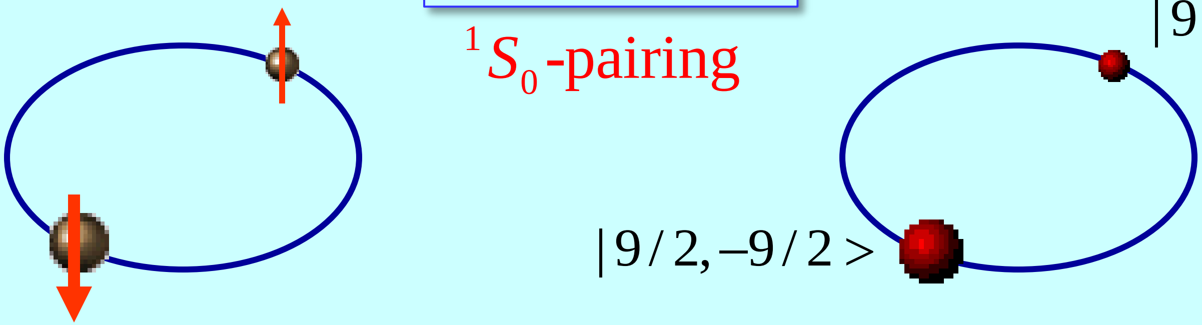

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Superfluid Fermi gas

Fermion Superfluidity in ^{40}K and ^{6}Li Fermi gases (2004)

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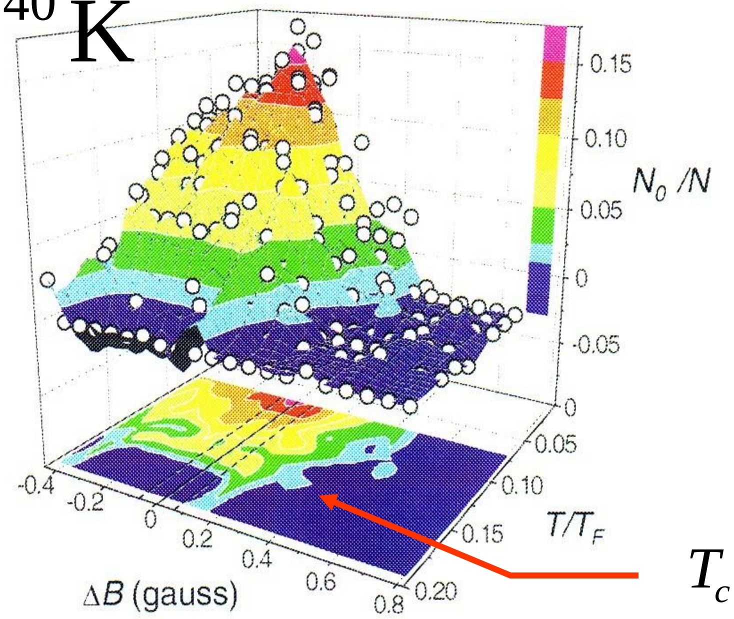

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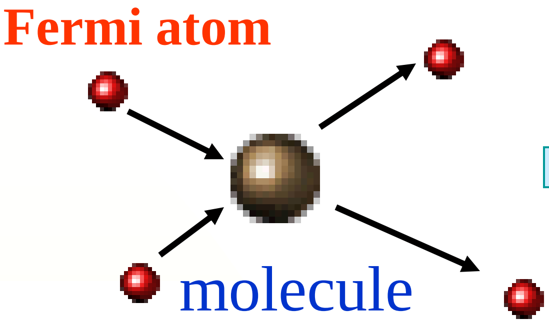
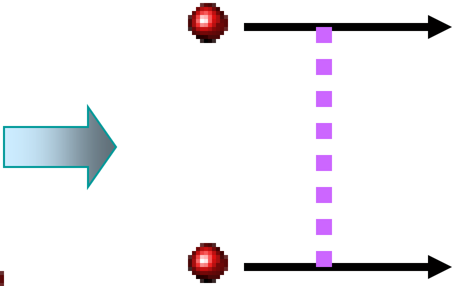
Key words **to understand this atomic Fermi superfluid**

- **Feshbach resonance**
- **BCS-BEC crossover**

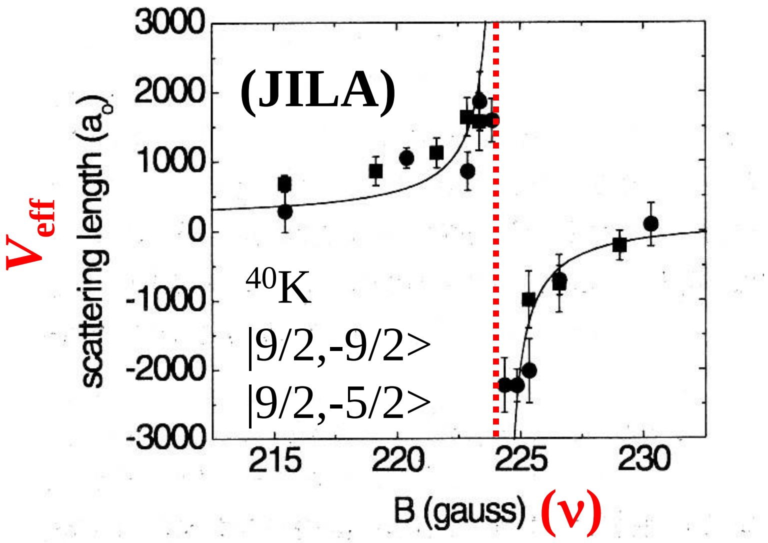
Feshbach resonance



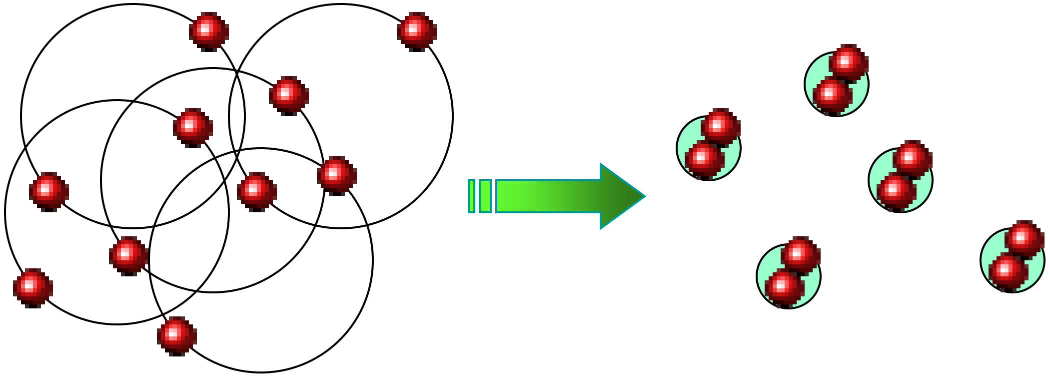


$$V_{eff} = -g^2 \frac{1}{2\nu}$$

tunable by magnetic field



BCS-BEC crossover tuned by a Feshbach resonance

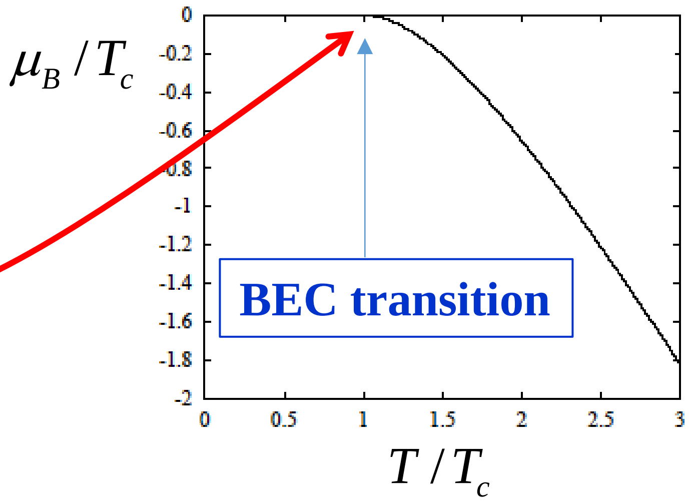


Essence of BCS-BEC crossover

Bose-Einstein condensation (BEC) of an ideal Bose gas

$$N = \sum_{\mathbf{q}} \frac{1}{e^{\beta(\varepsilon_{\mathbf{q}} - \mu_B)} - 1}$$

$$T_{BEC} = \frac{2\pi}{(\zeta(3/2))^{2/3}} \frac{n^{2/3}}{m}$$



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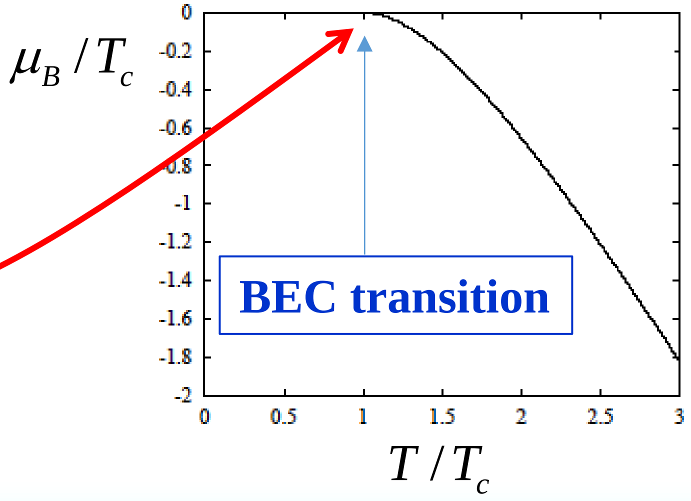
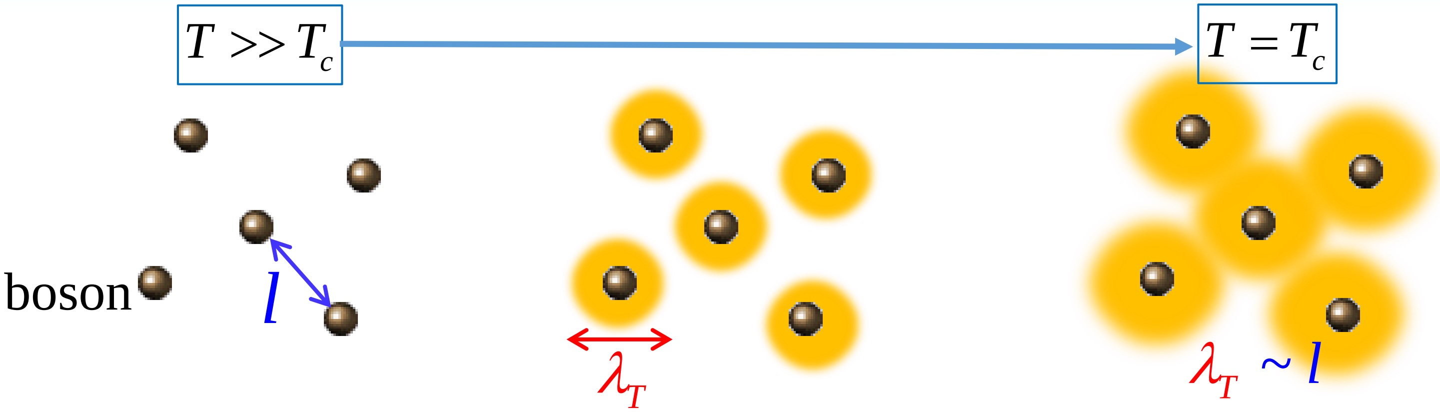
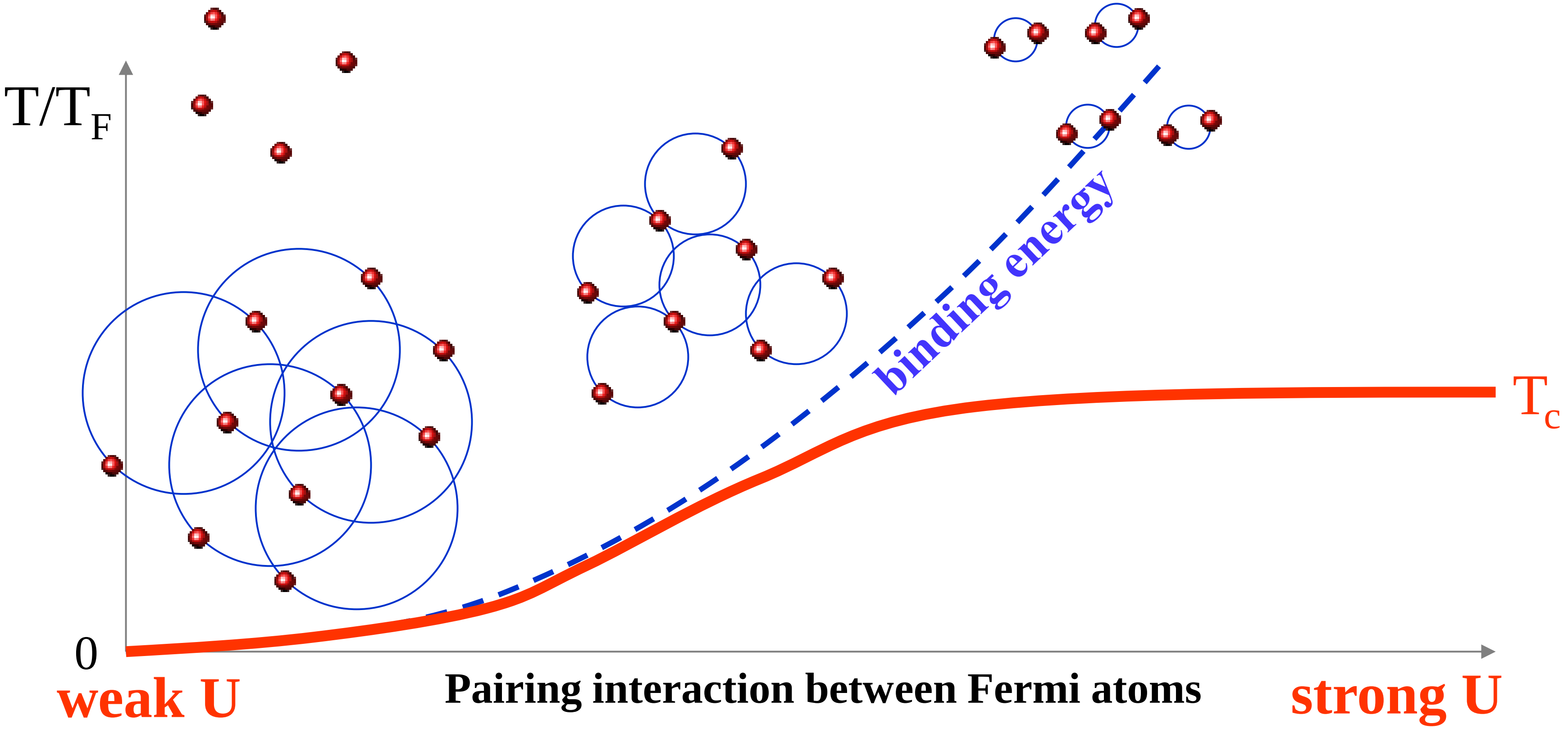


Thermal de Broglie length: $\lambda_T = \dfrac{h}{\sqrt{2\pi m T}}$ ← **quantum (statistical) size of a particle**



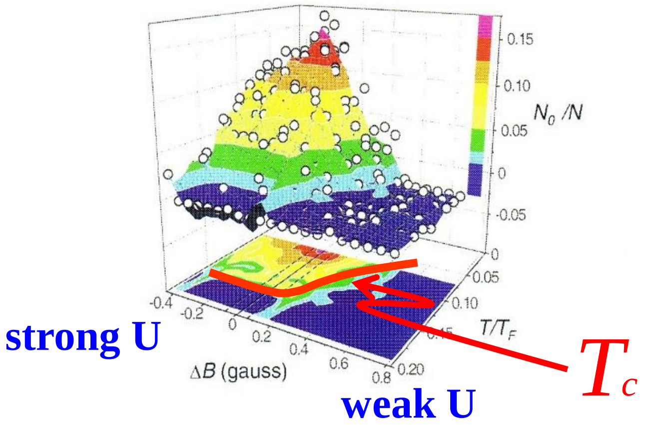
BEC occurs, when the quantum size of a particle reaches the inter-particle distance.

Essence of BCS-BEC crossover

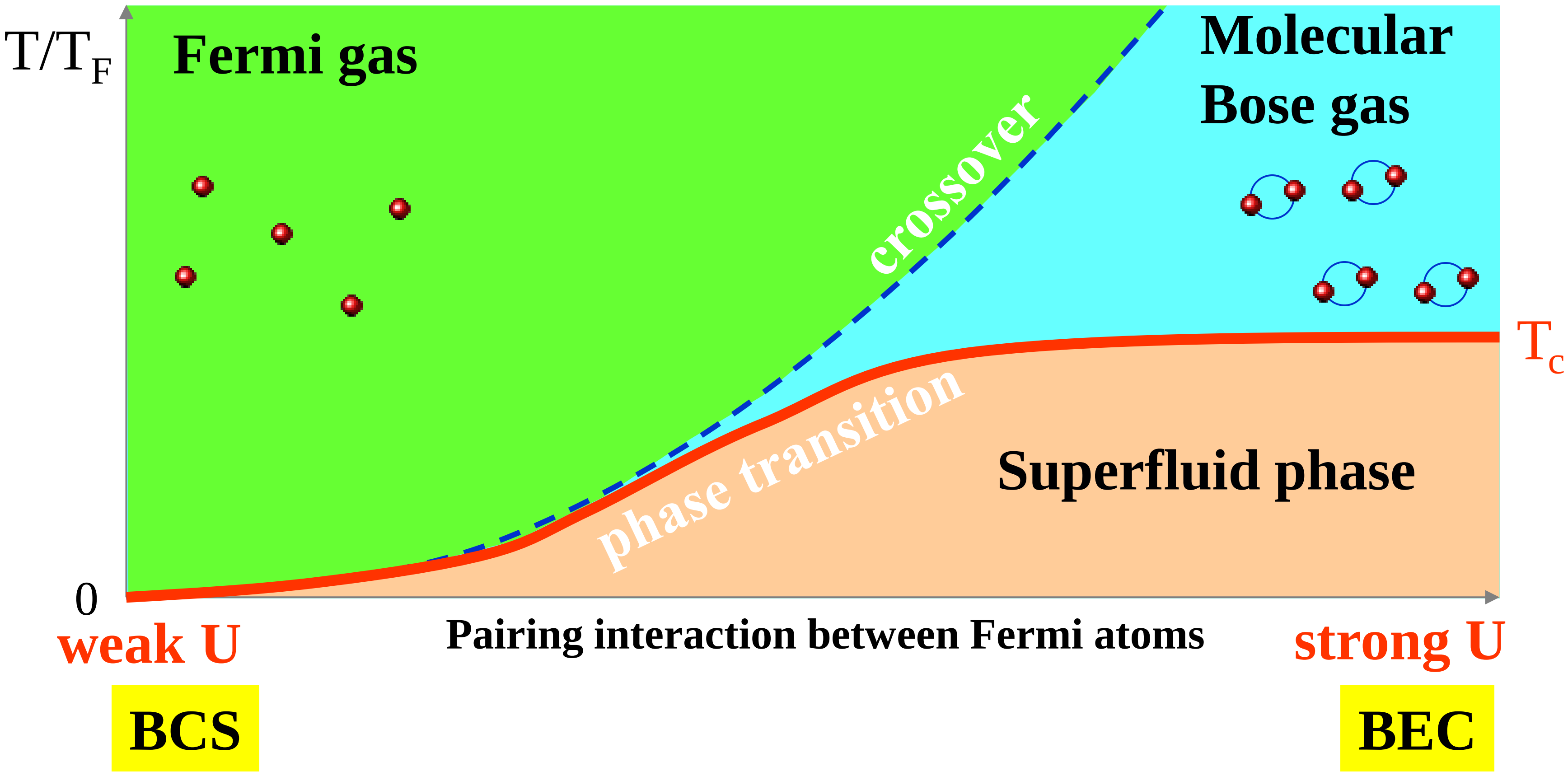


BCS

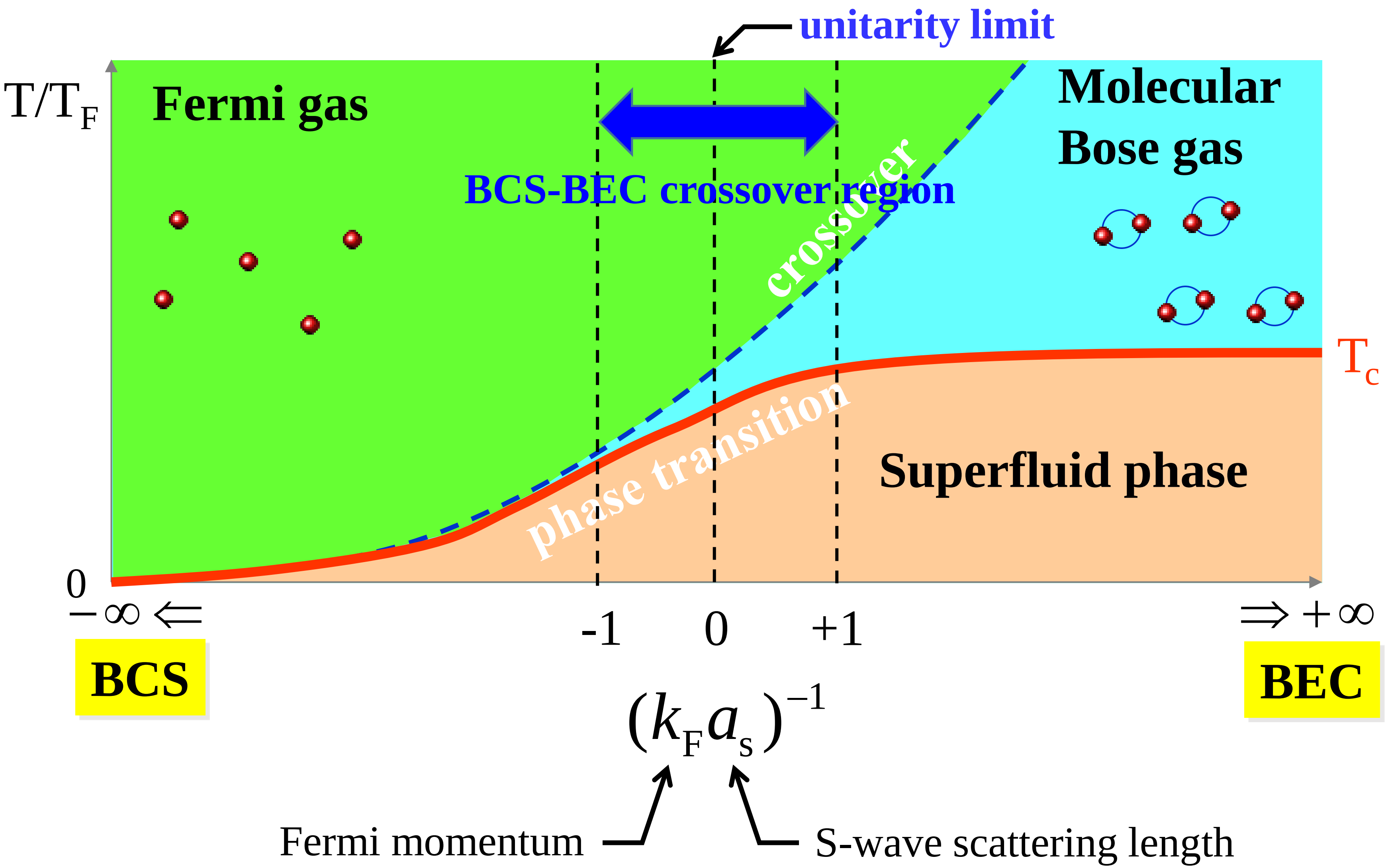
BEC



Phase diagram of ultracold Fermi gas



Phase diagram of ultracold Fermi gas



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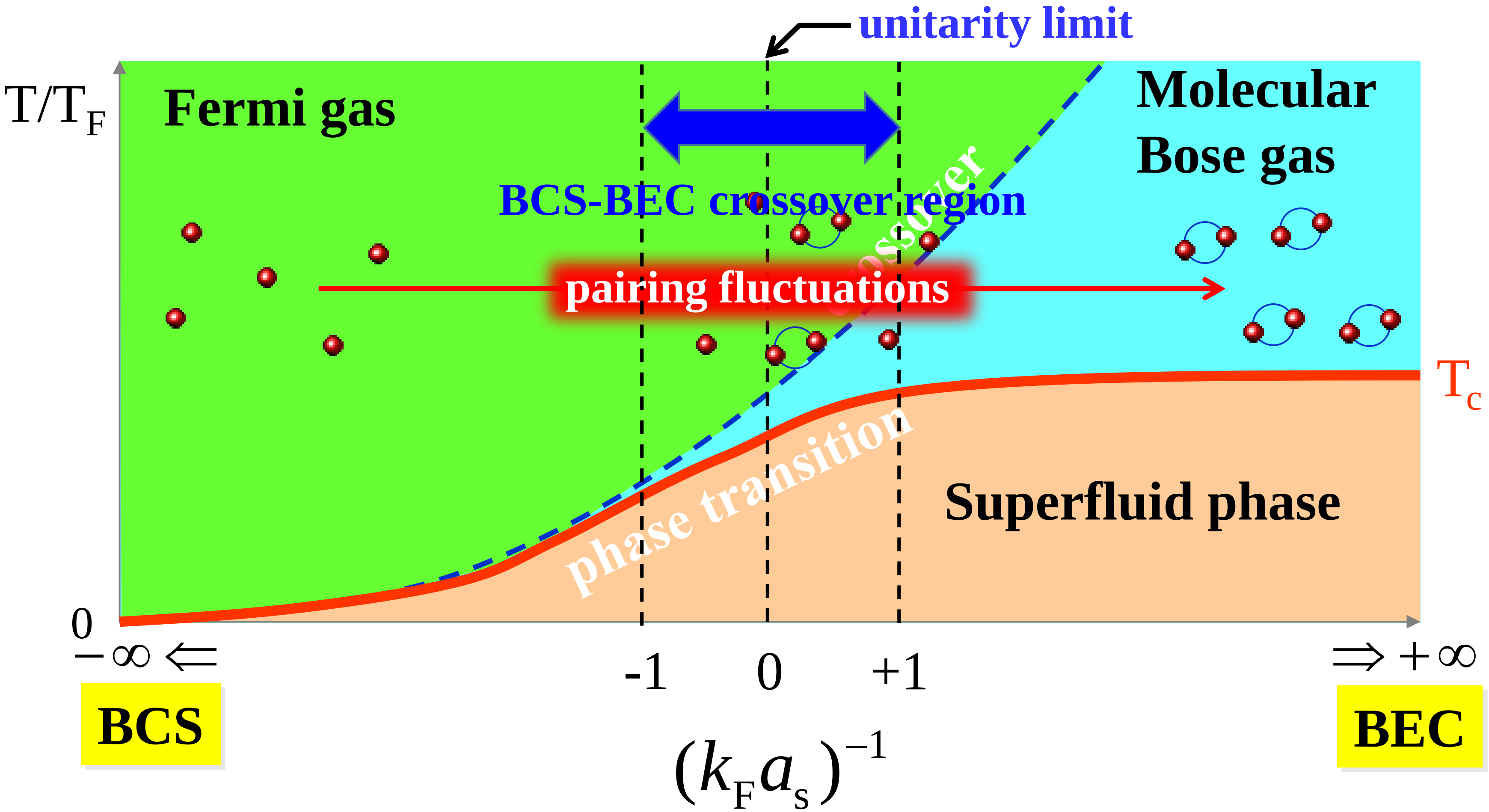
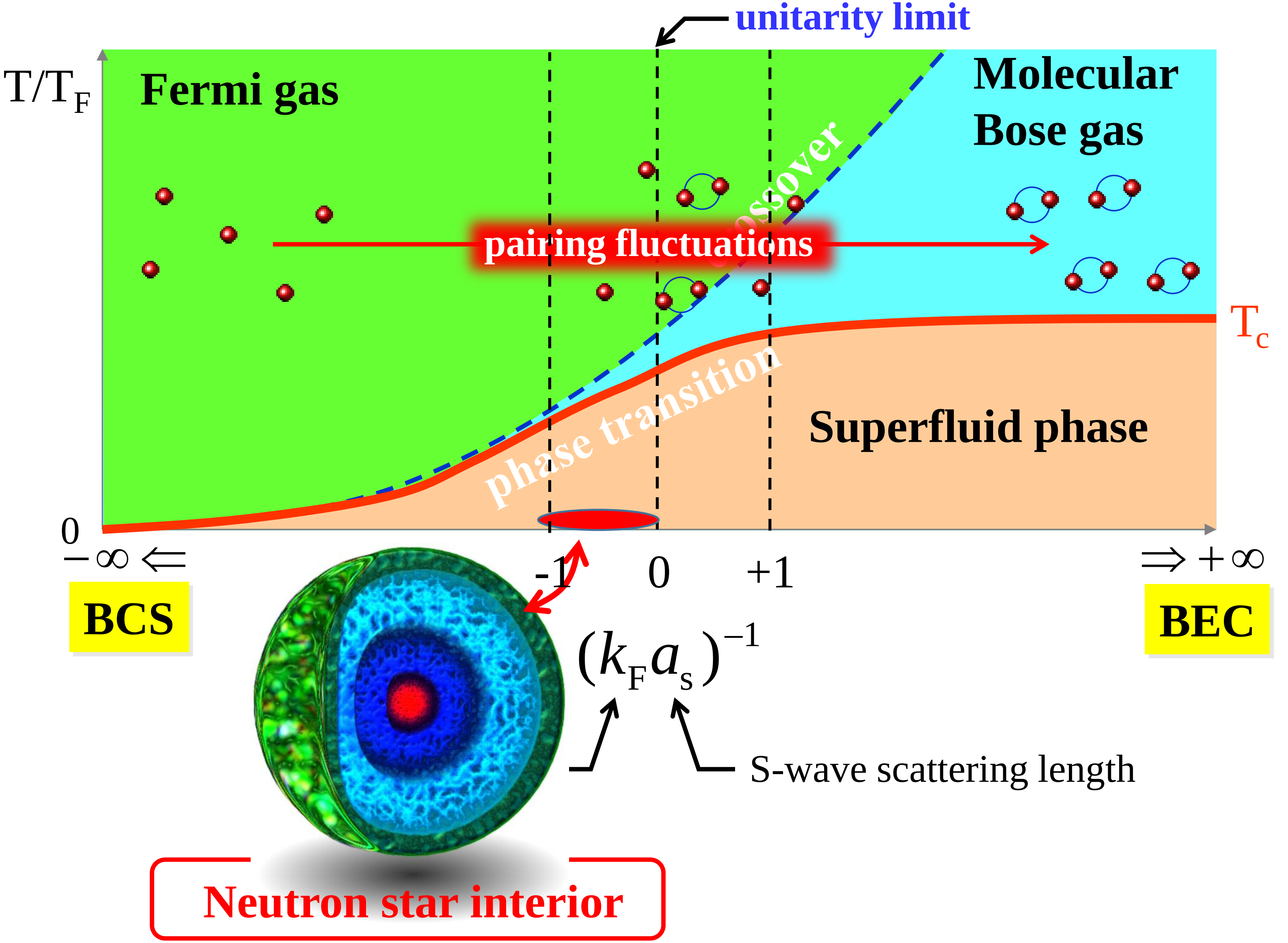


$$(k_F a_s)^{-1}$$

Fermi momentum

S-wave scattering length

Phase diagram of ultracold Fermi gas

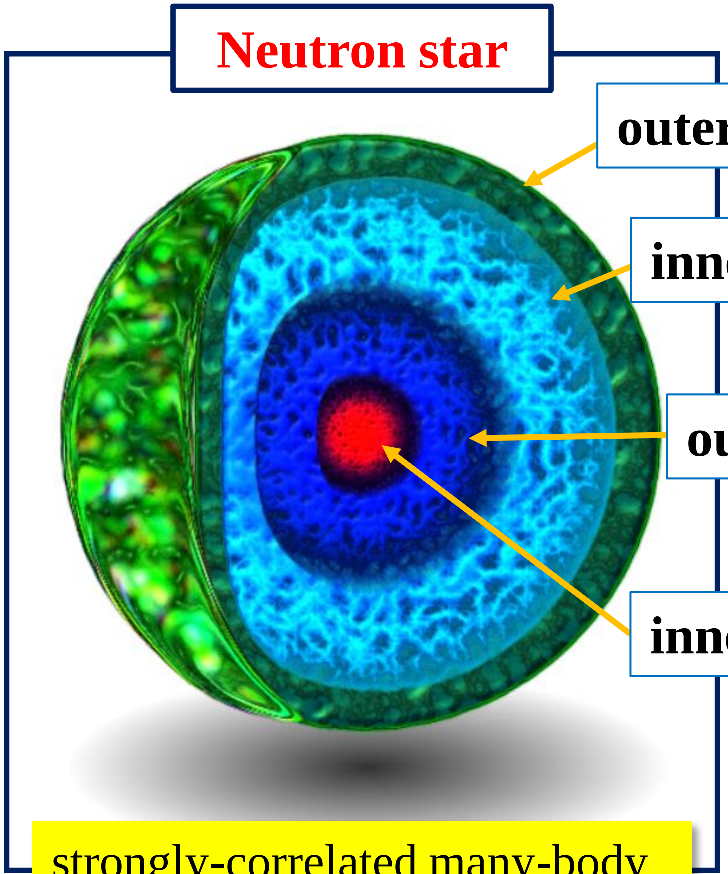


Neutron Star = “extreme state of matter in the universe”

Radius～10 Km

Mass～Solar mass

Current understanding (?) of neutron star interior



	ρ / ρ_0	
outer crust	$\lesssim 0.001$	Fe ions with drip electrons
inner crust	$\lesssim 0.5$	Nuclei with drip neutrons
outer core	$\lesssim 2$	Neutron liquid (> 95%)
inner core	$\gtrsim 2$	Hyperons (Λ,Ξ, Σ ?)) π-condensate, quark matter?

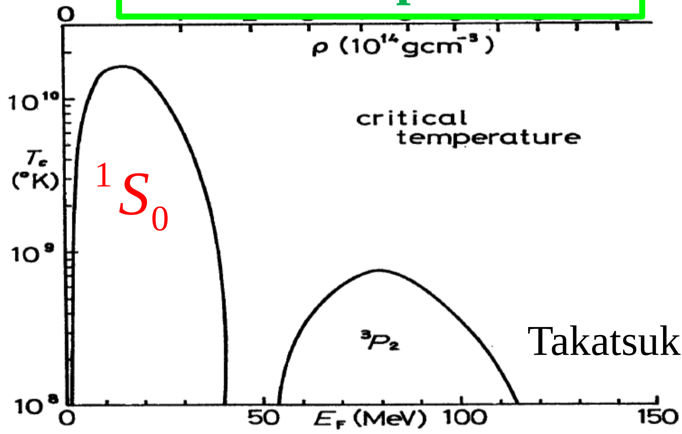
Neutron-superfluid ?

strongly-correlated many-body system with very high density

$$\rho \leq 5 \sim 10 \rho_0$$

↑

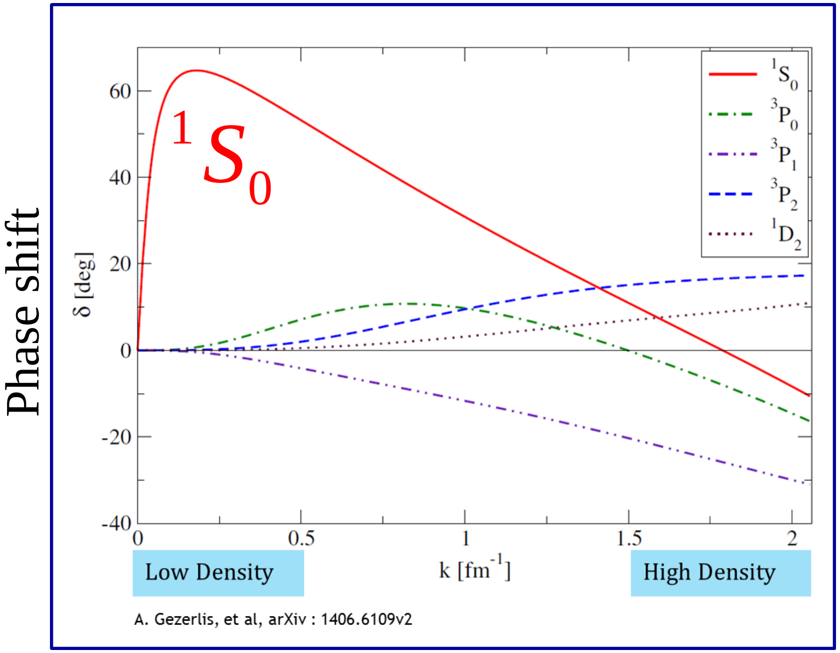
nuclear saturation density =0.16fm^{-3}



Takatsuka (1972)

Neutron star as a strongly correlated Fermi system

N-N interaction channels



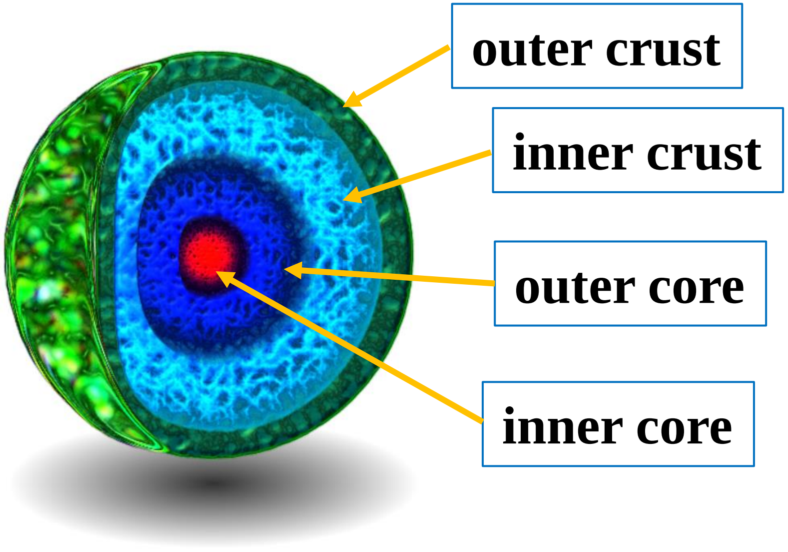
A. Gezerlis, et al, arXiv : 1406.6109v2

Neutron-neutron s-wave scattering length

$$a_s^{NN} = -18.5\text{fm}$$

$$T / T_F << 1$$

Neutron superfluid



outer crust

inner crust

outer core

inner core

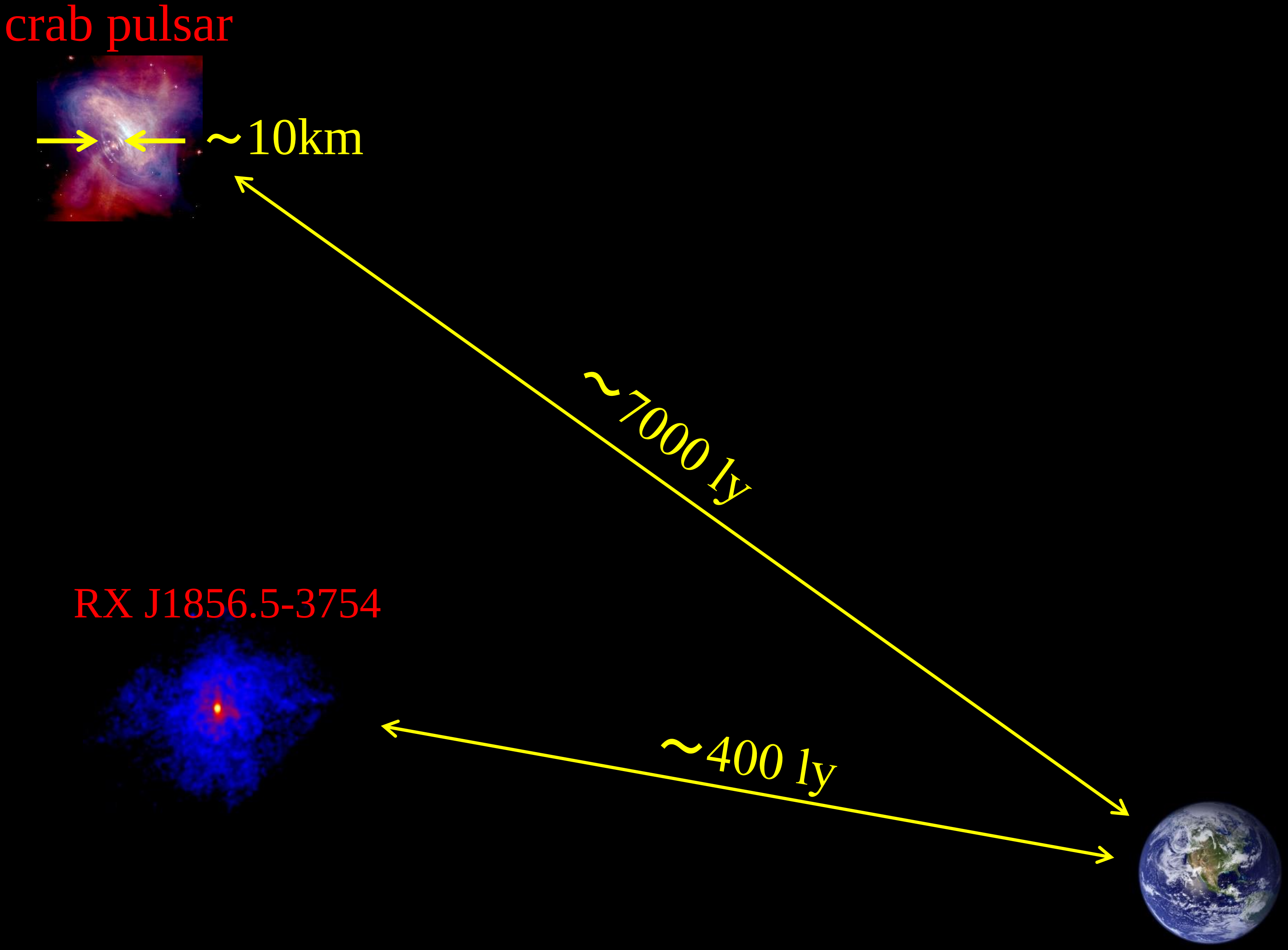
$$(k_F a_s^{NN})^{-1} = 0.05 << 1!$$

(typical value of k_F~1fm^{-1})

Fermi gas near the unitary limit

Many-body effects are important!

Where are neutron stars?



crab pulsar

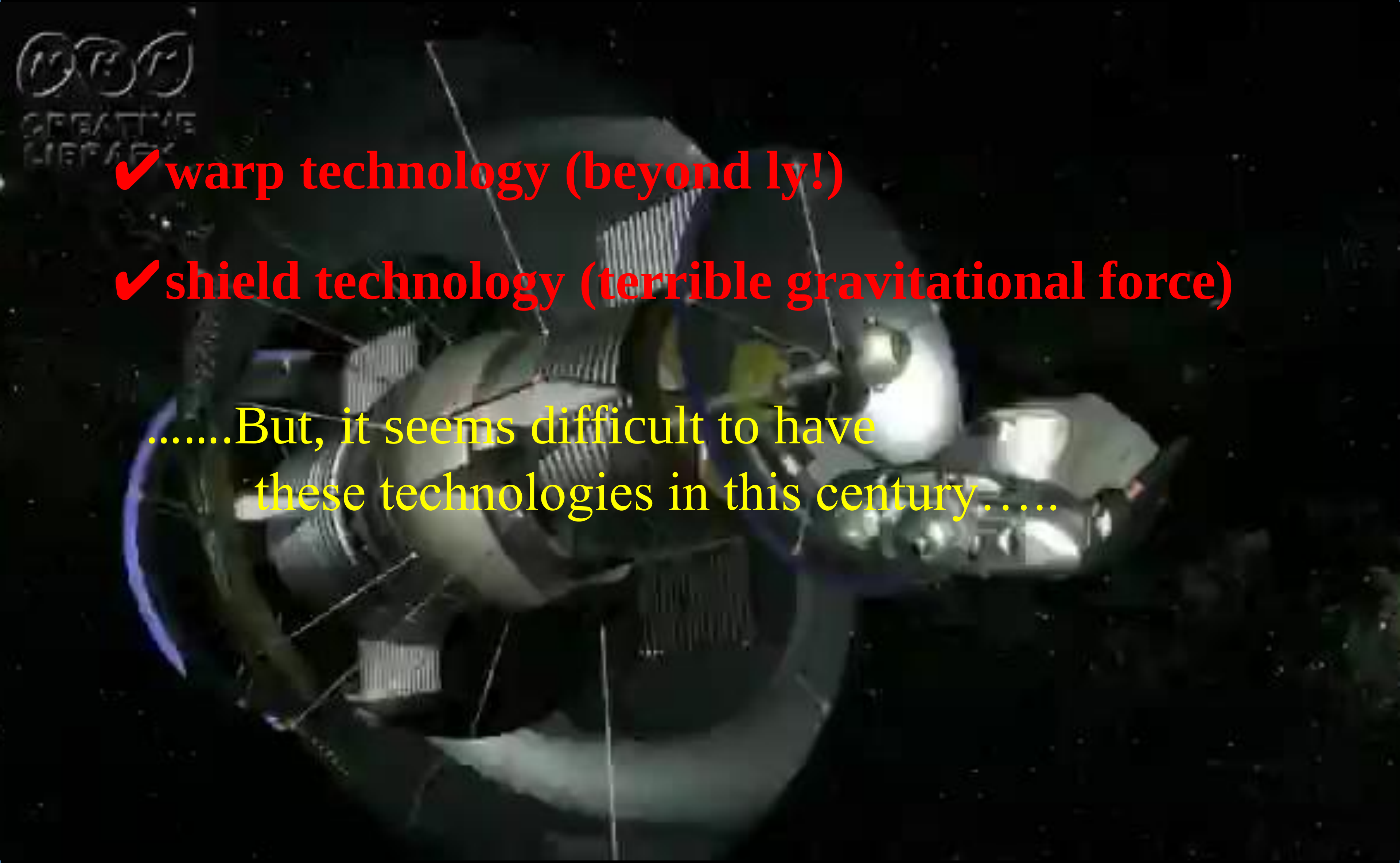
~10km

~7000 ly

RX J1856.5-3754

~400 ly

The best way: Boldly go where no one has gone before!

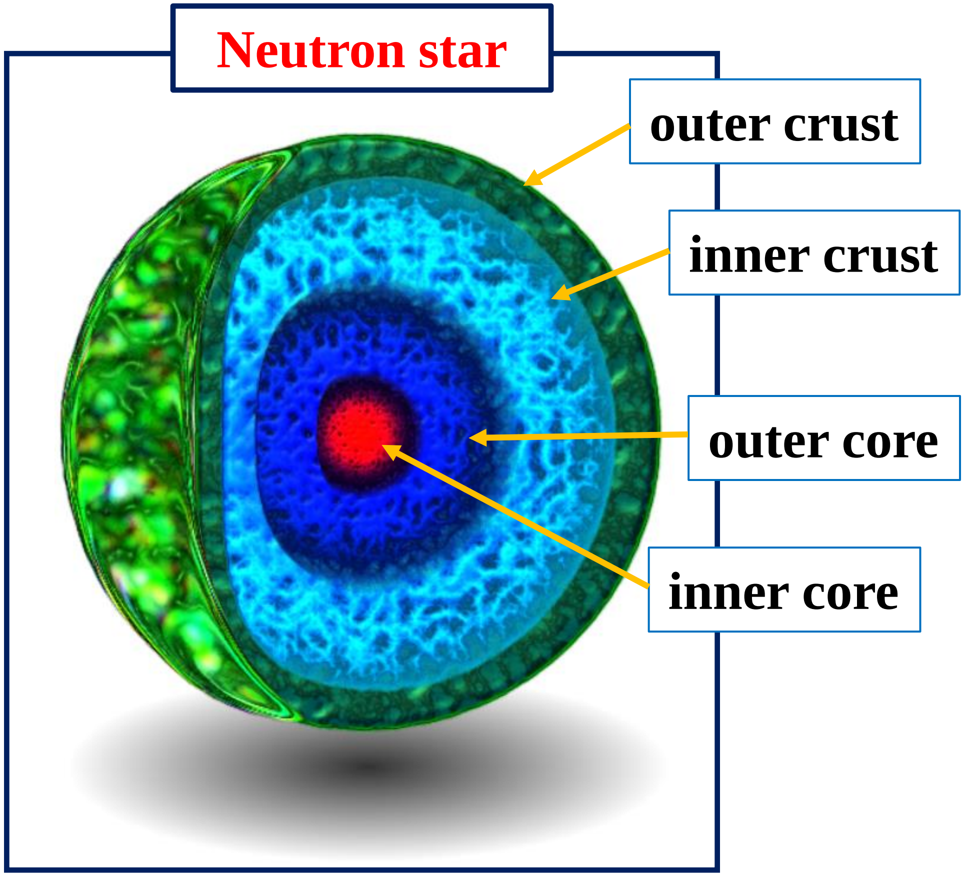


✔warp technology (beyond ly!)

✔shield technology (terrible gravitational force)

…….But, it seems difficult to have
these technologies in this century…..

Current practical approach by human beings



theorists on the earth

↓

Equation of state (EoS)

⇕

internal structure

\+

TOV eq.

(Tolman-Oppenheimer-Volkoff)

↓

"Mass-radius (MR)" relation

Obserbavle! *Obserbavle!*

experimentalists on the earth

Standard approach to “Neutron star EoS”

Phase shift data of NN interaction

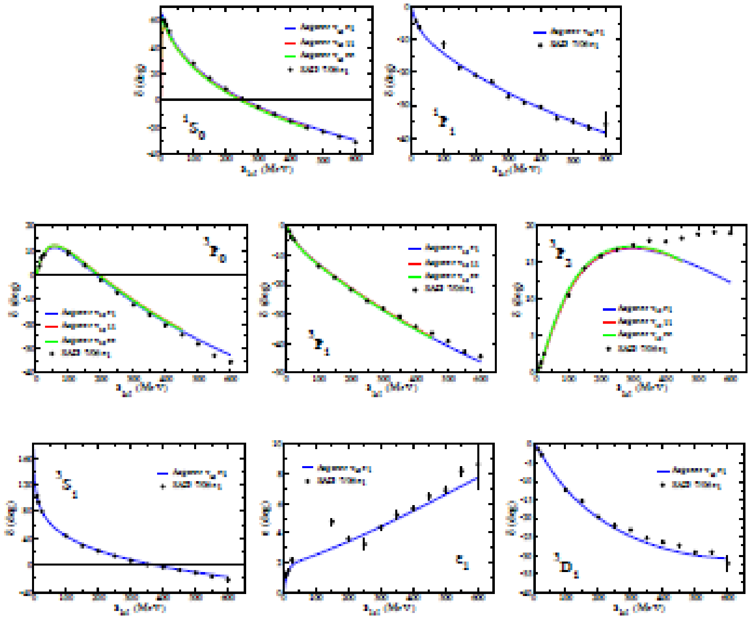


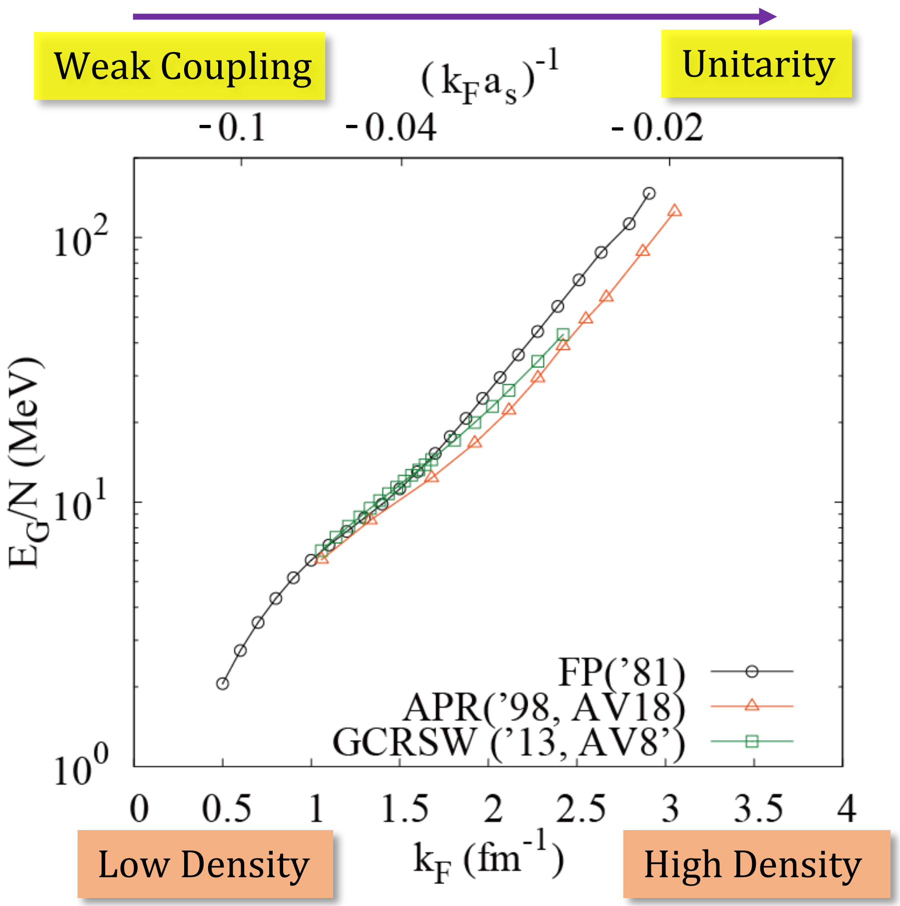
Fig. 1. Phase shifts of AV18 nucleon-nucleon potential. Experimental phase shifts are from the SAID Partial-Wave Analysis Facility (gwdac.phys.gwu.edu).

S. Gandolfi et al, Eur. Phys. J. A 50 (2014)

effective interaction potential with “32” fitting parameters (AV18)

QMC, variational calculation, ….

EoS in the intermediate density region



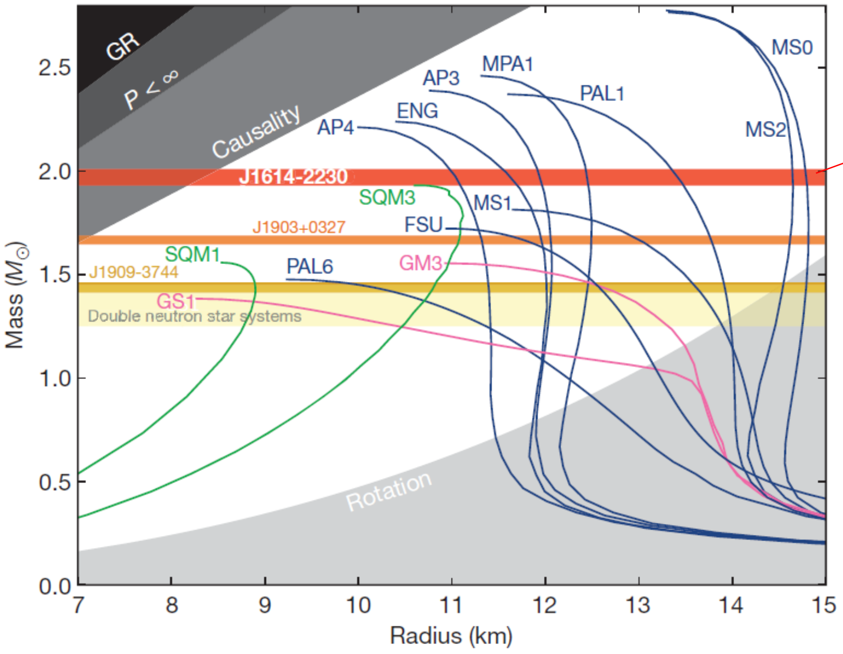
FP(`81) : B.Friedman et al, Nucl. Phys, A 361 (1981) 502

APR(`98, AV18) : A. Akmal et al, Phys. Rev. C 58 (1998)

GCRSW(`13, AV8) : S. Gandolfi et al, Eur. Phys. J. A 50 (2014)

Importance of MR-relation: “two-solar mass” problem

EoS+TOV→***MR* relation**

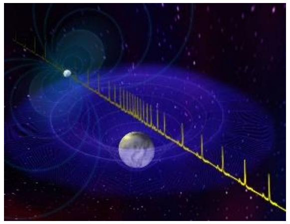


M

R

Demorest, Nature 467 (2010) 1081

precise determination of neutron star mass by using the Shapiro-delay effect



$M = 1.97 M_\odot$

Importance of MR-relation: “two-solar mass” problem

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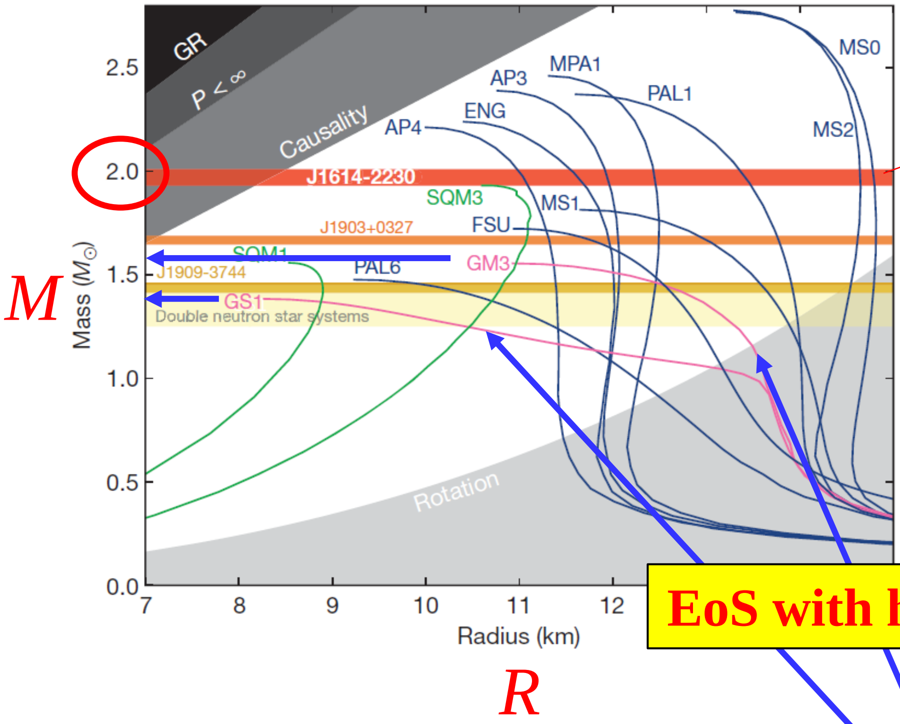
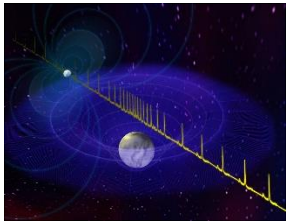


EoS with hyperons

Demorest, Nature 467 (2010) 1081

precise determination of neutron star mass by using the Shapiro-delay effect



$M = 1.97 M_\odot$

Too heavy!!

“two-solar-mass problem”

Softening of neutron star by Λ-hyperons expected in the inner core ($\rho > 2\rho_0$) suppresses the upper limit of neutron star mass down to below 2M$_\odot$.

This problem requires us to re-consider the neutron star EoS….

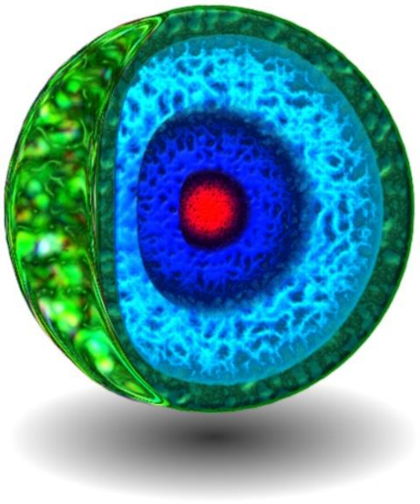
Approach to neutron star interior from the earth

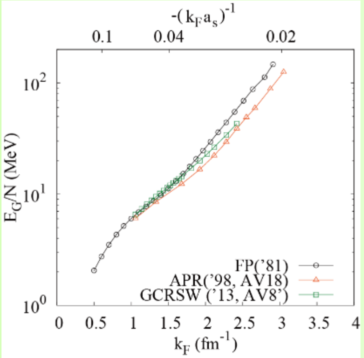
many-body physics

few-body physics

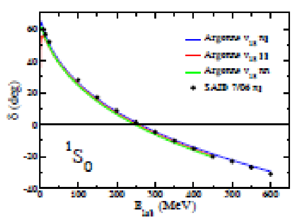
Theoretical challenge

Experimental support (phase shift data→AV18)





No experimental support!



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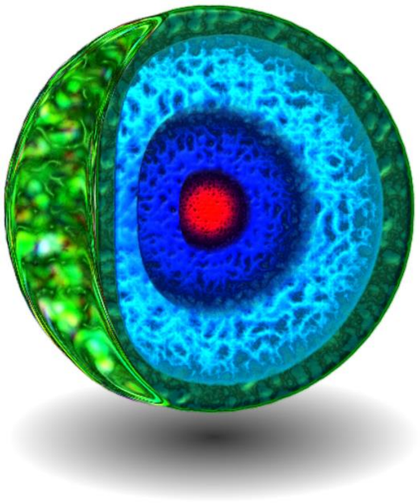

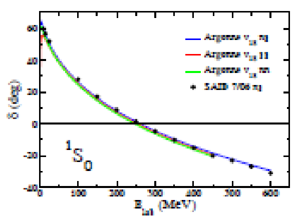

reduction of
"theoretical ambiguity"

Theoretical challenge

Experimental support

Approach to neutron star interior from the earth

many-body physics

few-body physics

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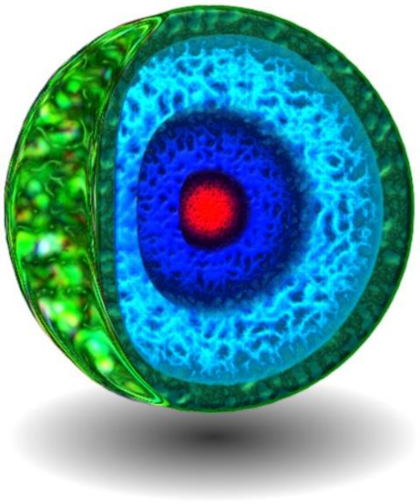
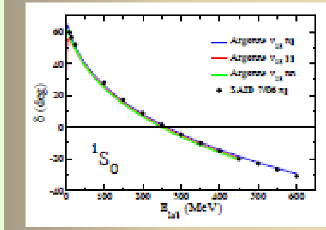
Theoretical challenge





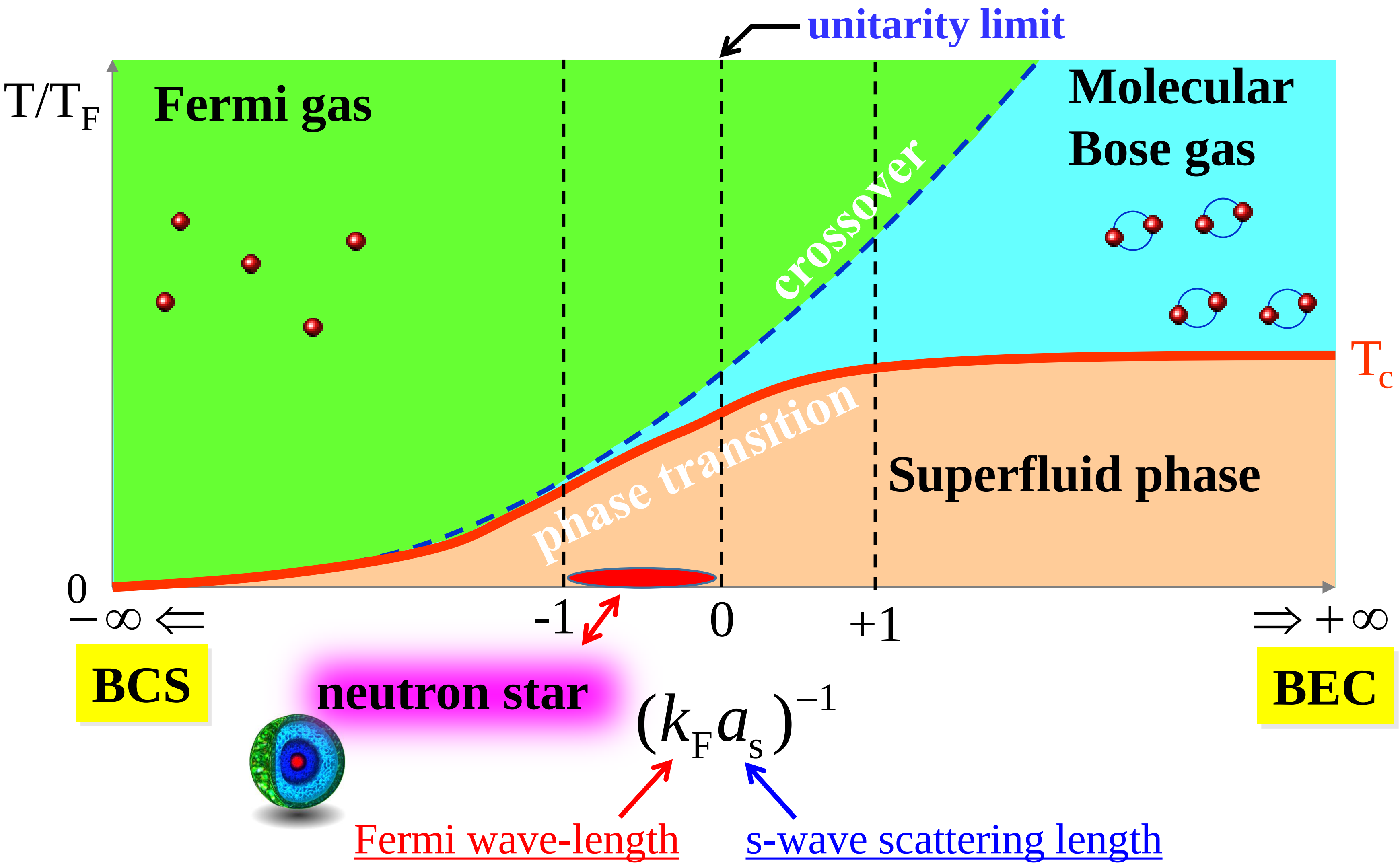
reduction of
“theoretical ambiguity”

Theoretical challenge

Experimental support

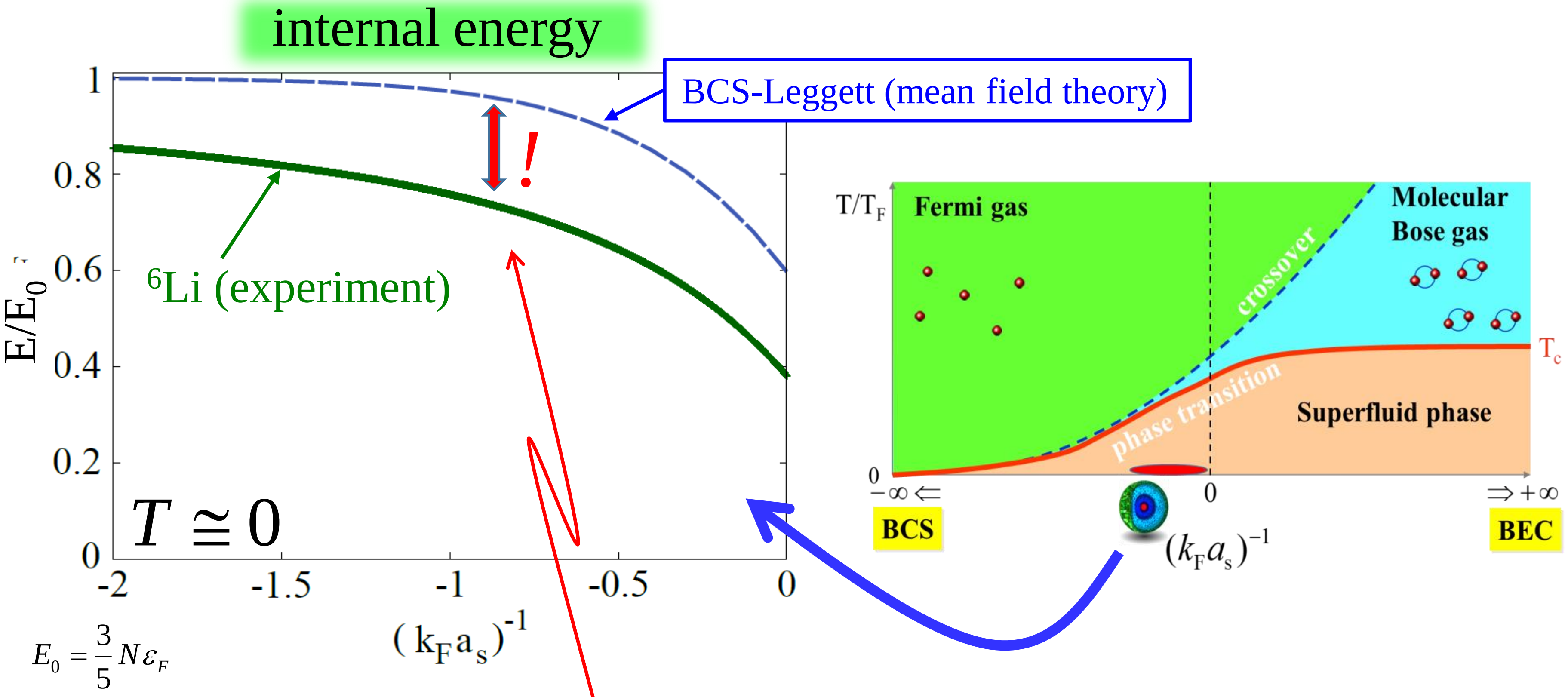
Ultracold Fermi gas

Phase diagram of ultracold Fermi gas



Fermi gas		**tunable**
neutron star	**tunable**	$a_s^{NN} = -18.5\text{fm} < 0$

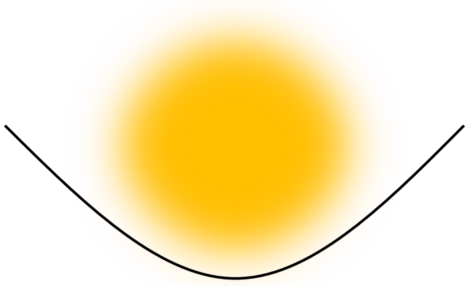
EoS observed in the "neutron-star regime" of a superfluid ^{6}Li Fermi gas



Usually, it is believed that the BCS-Leggett theory (consisting of the mean-field BCS gap equation and the BCS number equation) can describe the BCS-BEC crossover at T=0. However, this result clear shows that, *even at T=0* (where thermal fluctuations are absent), strong-coupling corrections beyond the mean-field level are crucial.

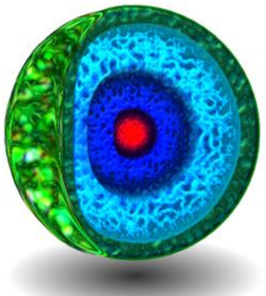
Crucial difference between cold Fermi gas and neutron star

The magnitude of <u>effective range</u> r_e is VERY different between the two systems.



ultracold Fermi gas

$$p_F r_e \sim 0$$



neutron star

$$p_F r_e \lesssim 3$$

$$(r_e = 2.7\text{fm})$$

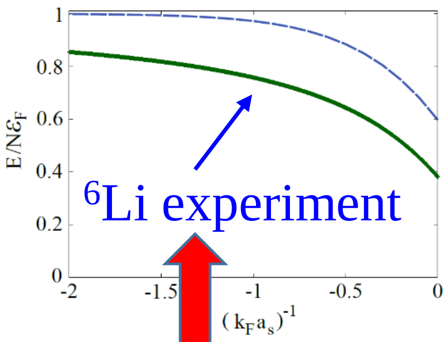
We need to *theoretically* include effects of <u>a finite effective range</u> r_e, to approach neutron star EoS starting from cold Fermi gas physics.

Our strategy to reach neutron star

We first construct a reliable strong-coupling theory which can *quantitatively* explain the observed EoS in a superfluid ^{6}Li Fermi gas. We then extend this theory to include a finite effective range (r_e=2.7fm), to examine to what extent we can discuss the neutron star EoS in the low density (crust) region by this novel approach.

Ultracold Fermi gas

$r_e = 0$



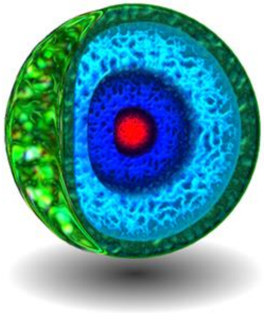
^{6}Li experiment

① **Strong-coupling theory with r_e=0**

\+ ② $r_e = 2.7\text{fm}$

theory

③

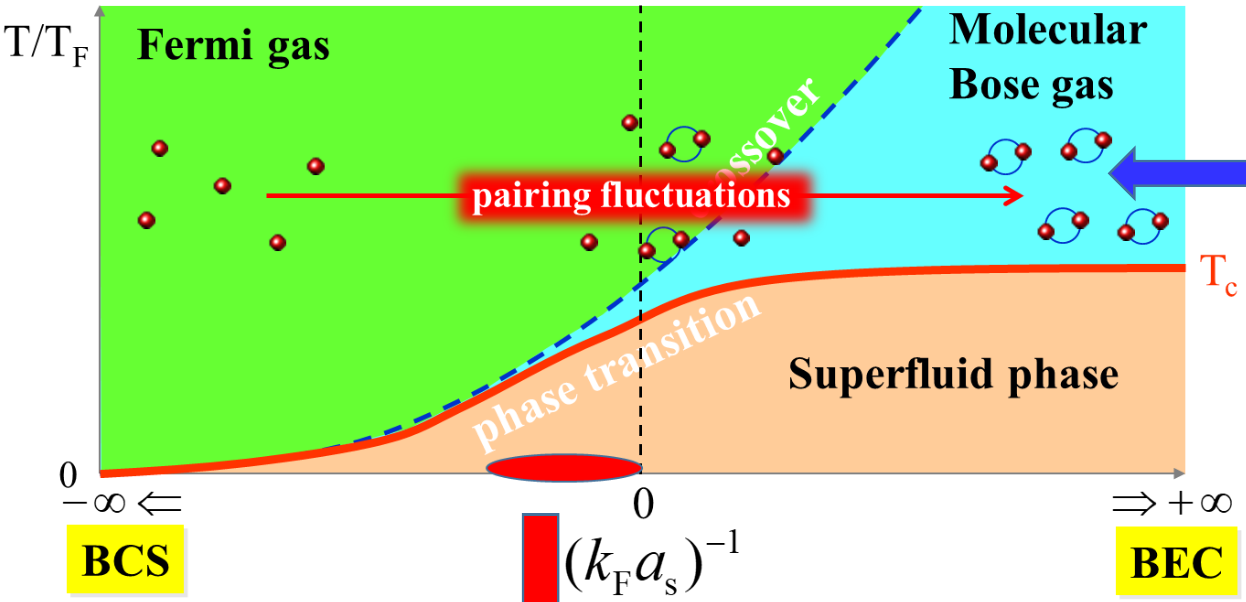


Neutron star EoS

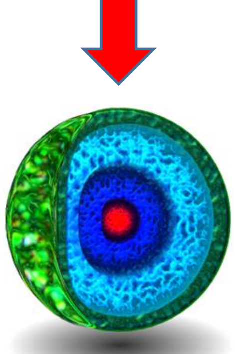
Today’s talk

Strong-coupling properties of an ultracold Fermi gas in the BCS-BEC crossover region and application to neutron-star EoS”

Fermi atomic gas (^{6}Li, ^{40}K)



(1) BCS-BEC crossover



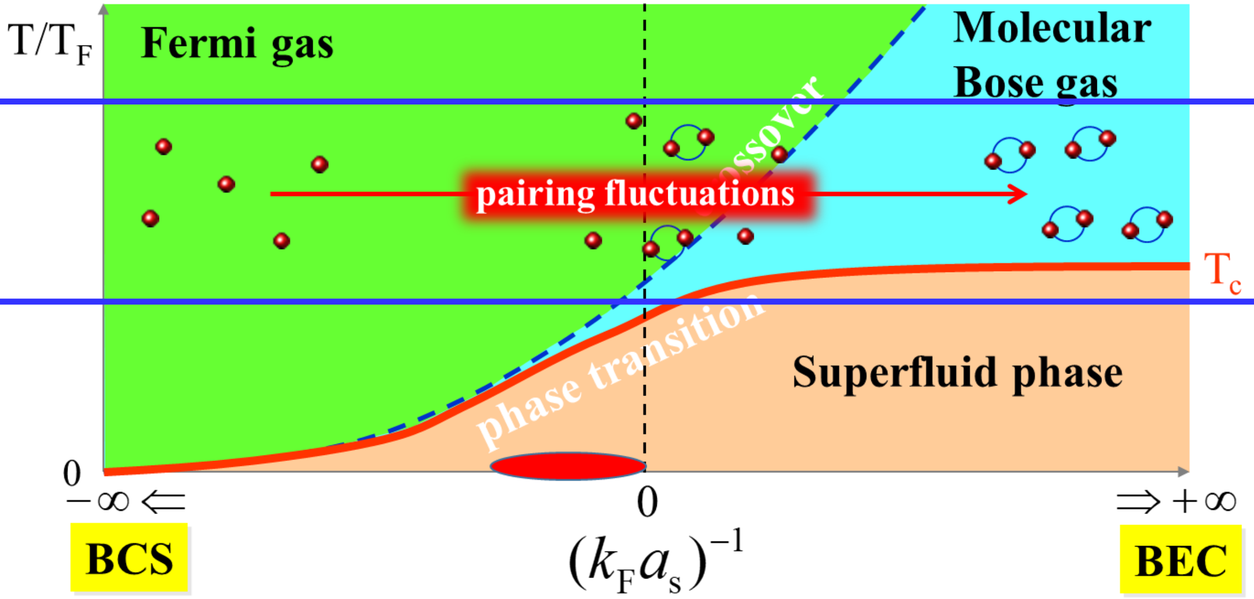
(2) Application to NS-EoS

Neutron star (low-density region)

BCS-BEC crossover in a gas of Fermi atoms with a Feshbach resonance

~~Effective range~~

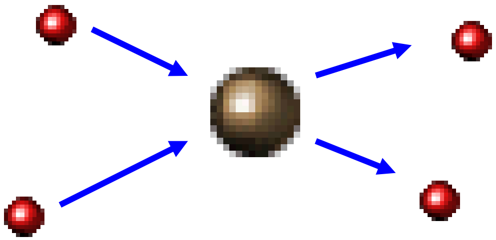
$$r_e = 0$$



Formulation: model ultracold Fermi gas ($T > T_c$)

$$H = \sum_{\mathbf{p},\sigma} (\varepsilon_{\mathbf{p}} - \mu) c^{\dagger}_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - U \sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} c^{\dagger}_{\mathbf{p}+\mathbf{q}/2\uparrow} c^{\dagger}_{-\mathbf{p}+\mathbf{q}/2\downarrow} c_{-\mathbf{p}'+\mathbf{q}/2\downarrow} c_{\mathbf{p}'+\mathbf{q}/2\uparrow}$$

- uniform gas is assumed.
- σ : two atomic hyperfine states = pseudospin ↑,↓
- U : effective pairing interaction associated with the F.R.



$\Rightarrow \quad U$

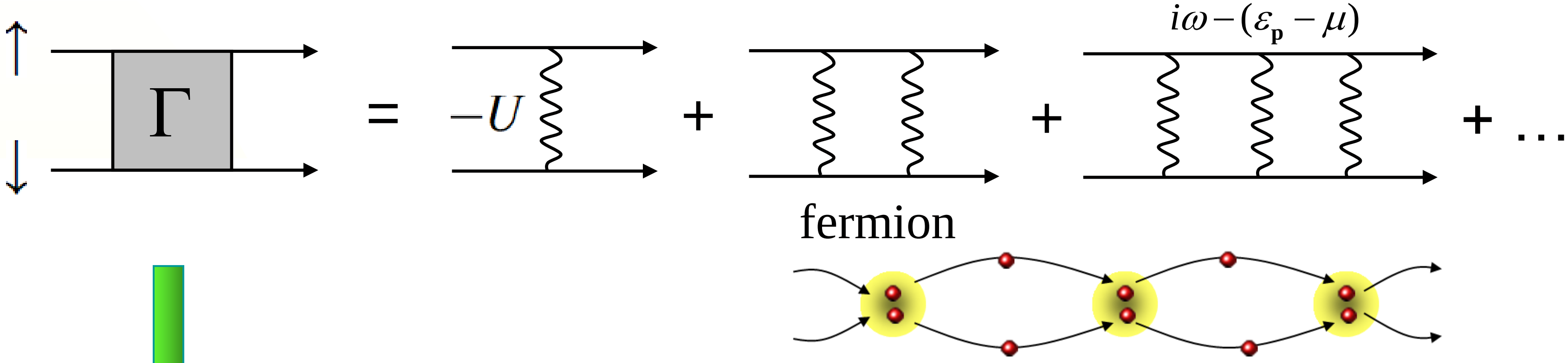
Feshbach resonance

We treat *U* as a tunable parameter.

Strong-coupling Formalism: Nozirères-Schmitt-Rink (NSR)

T_c: Thouless criterion

$$G = \frac{1}{i\omega - (\varepsilon_{\mathbf{p}} - \mu)}$$



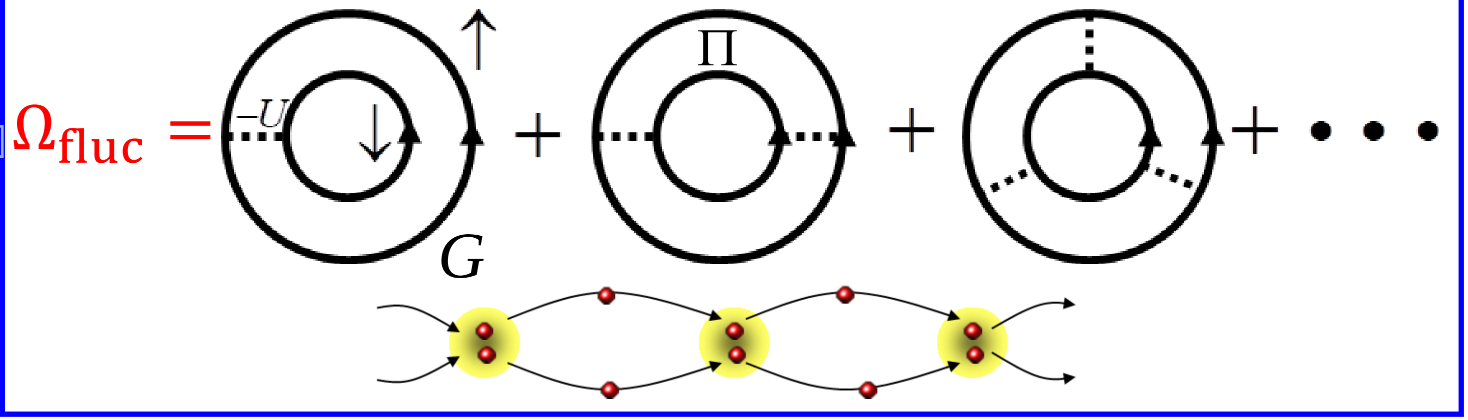
pole of Γ at $q=\omega=0$ → T_c -equation

$$1 = U \sum_{\mathbf{p}} \frac{\tanh \frac{\beta}{2}(\varepsilon_p - \mu)}{2(\varepsilon_p - \mu)}$$

μ remarkably deviates from the Fermi energy in the BCS-BEC crossover.

Strong-coupling Formalism: Nozirères-Schmitt-Rink (NSR)

Thermodynamic potential: $\Omega = \Omega_{MF} + \Omega_{fluc}$



$$N = -\frac{\partial \Omega}{\partial \mu}$$

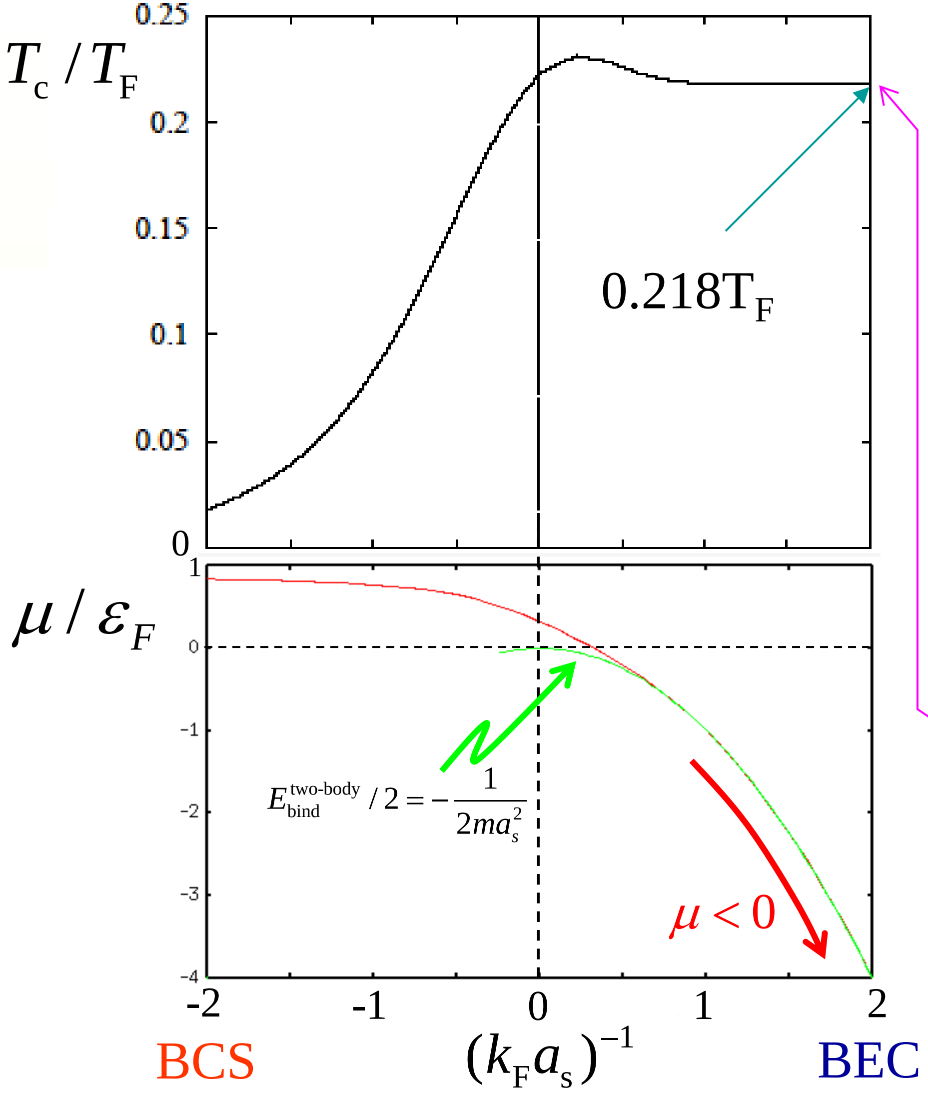
We solve the number equation (*N*), together with the *T*$_c$-equation, to determine *T*$_c$ and μ self-consistently.

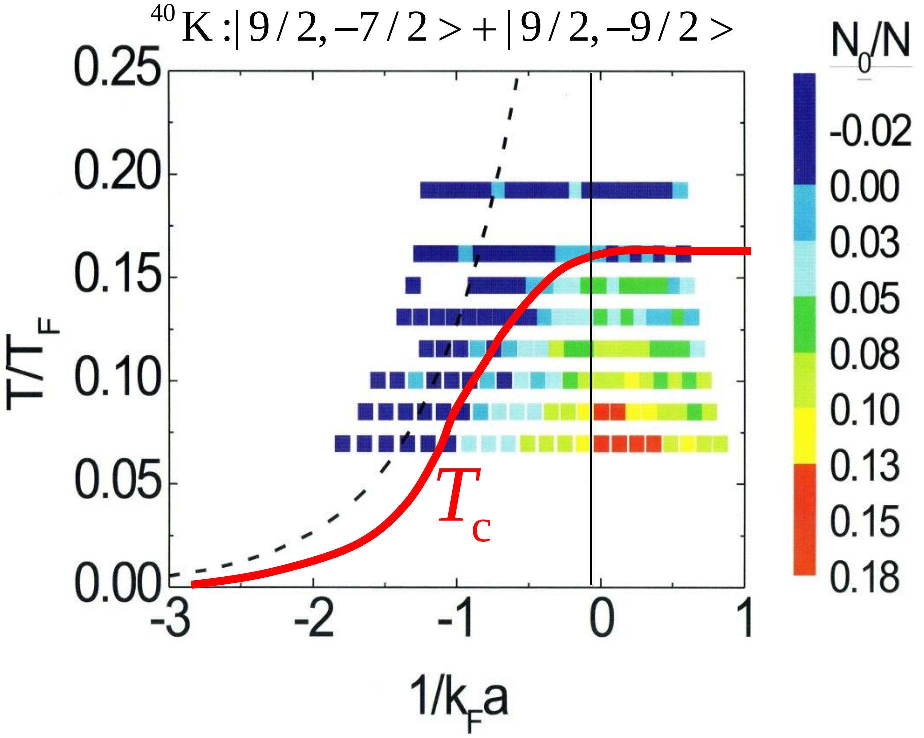
$$E = \Omega - T\left(\frac{\partial \Omega}{\partial T}\right)_{V,\mu} - \mu\left(\frac{\partial \Omega}{\partial \mu}\right)_{V,T}$$

Thermodynamic quantities

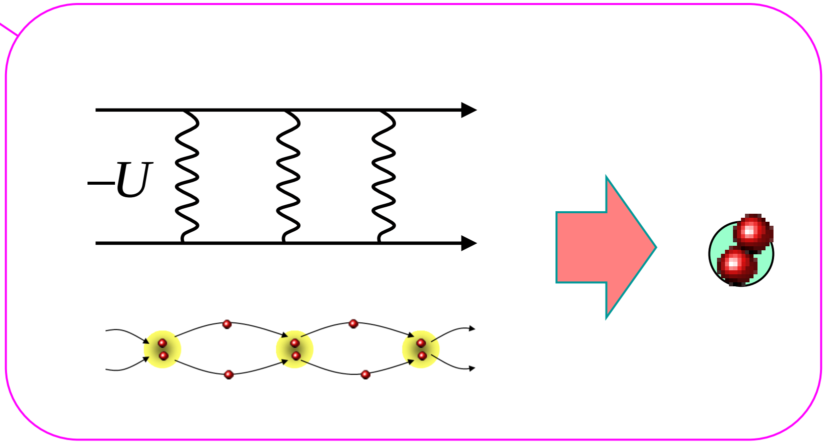
$$C_V = \left(\frac{\partial E}{\partial T}\right)_V$$

Self-consistent solutions for T_c and $\mu(T_c)$

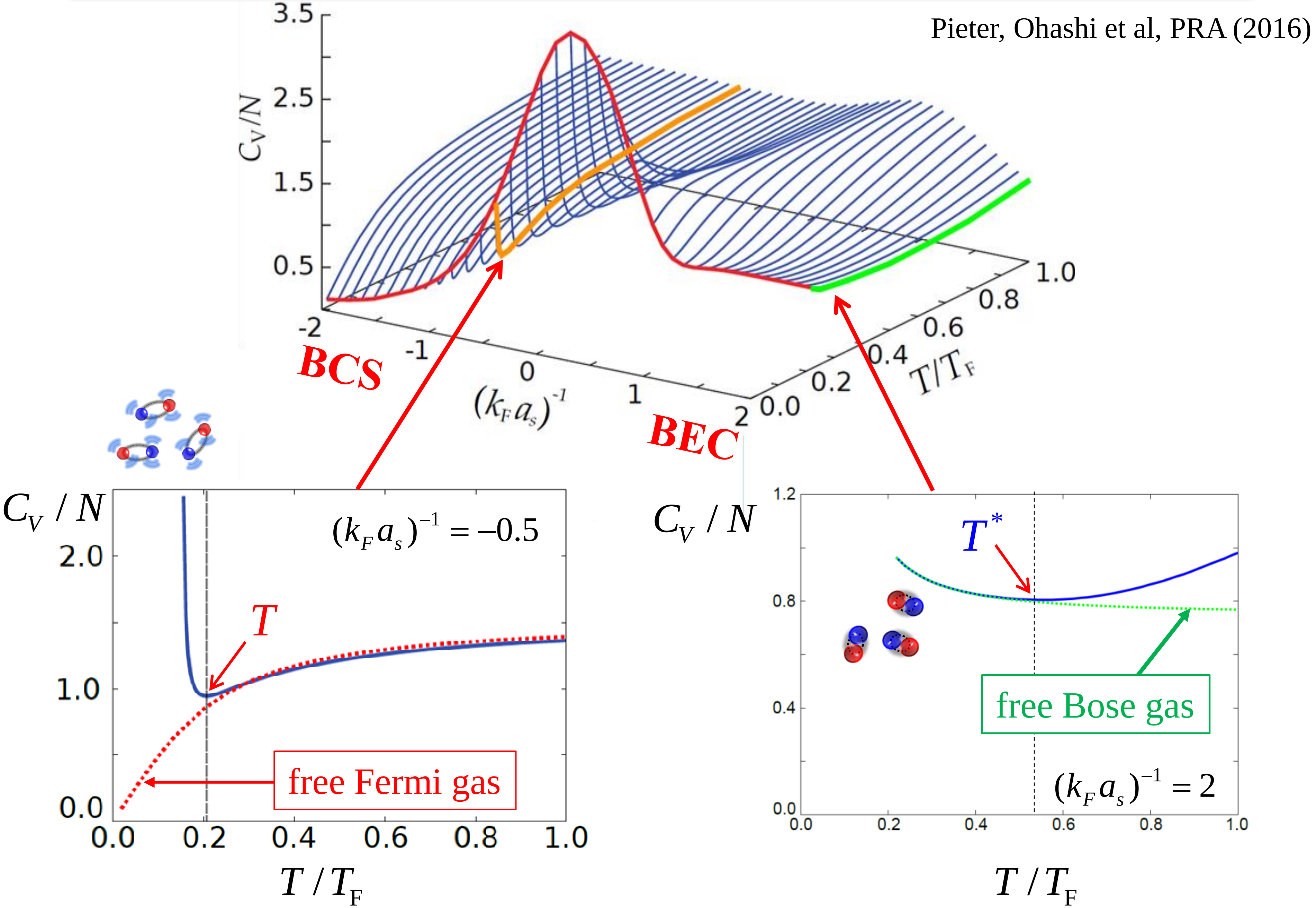




C. A. Regal, et al. PRL 92 (2004) 040403.

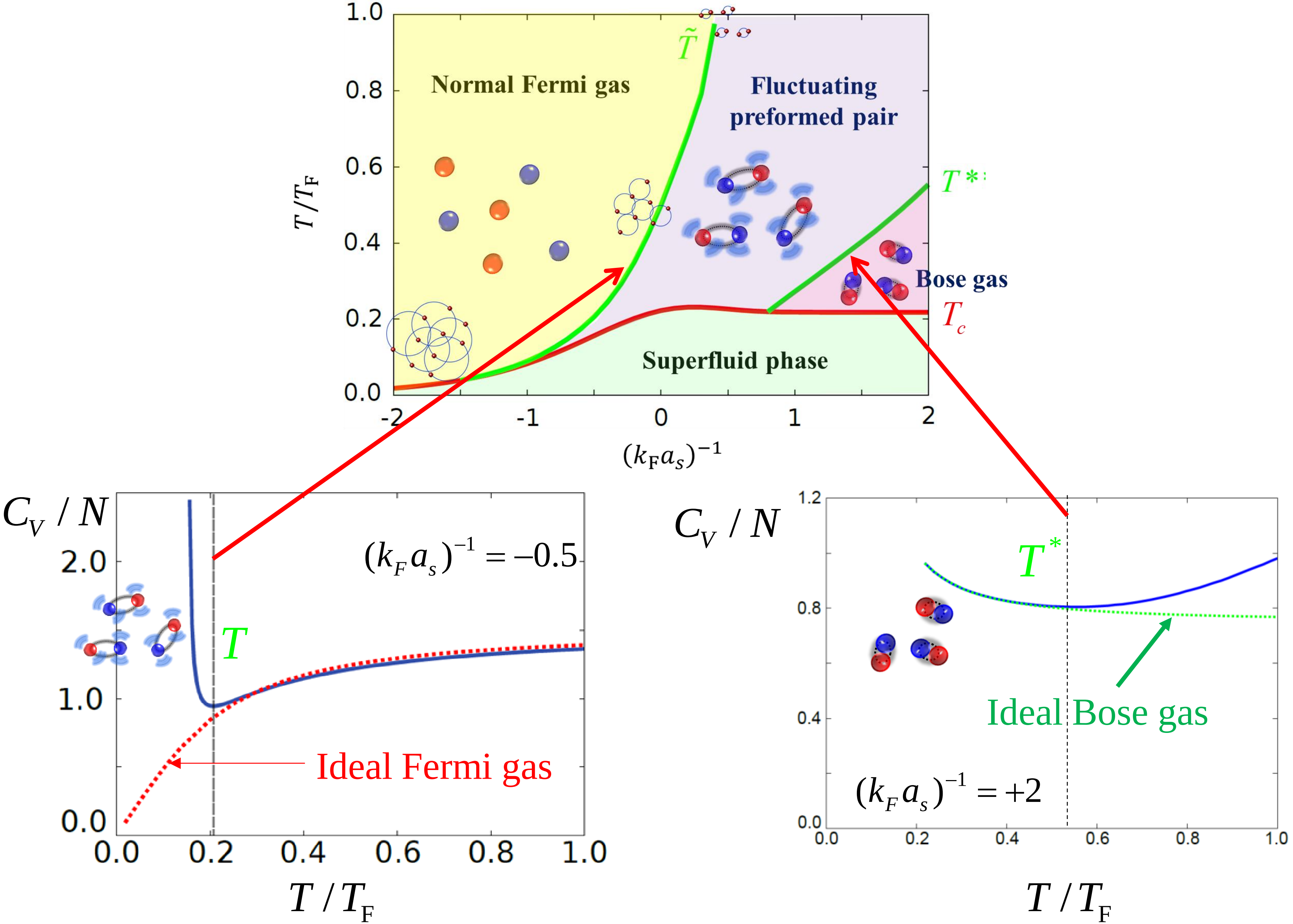


Specific heat C_v in the BCS-BEC crossover above T_c

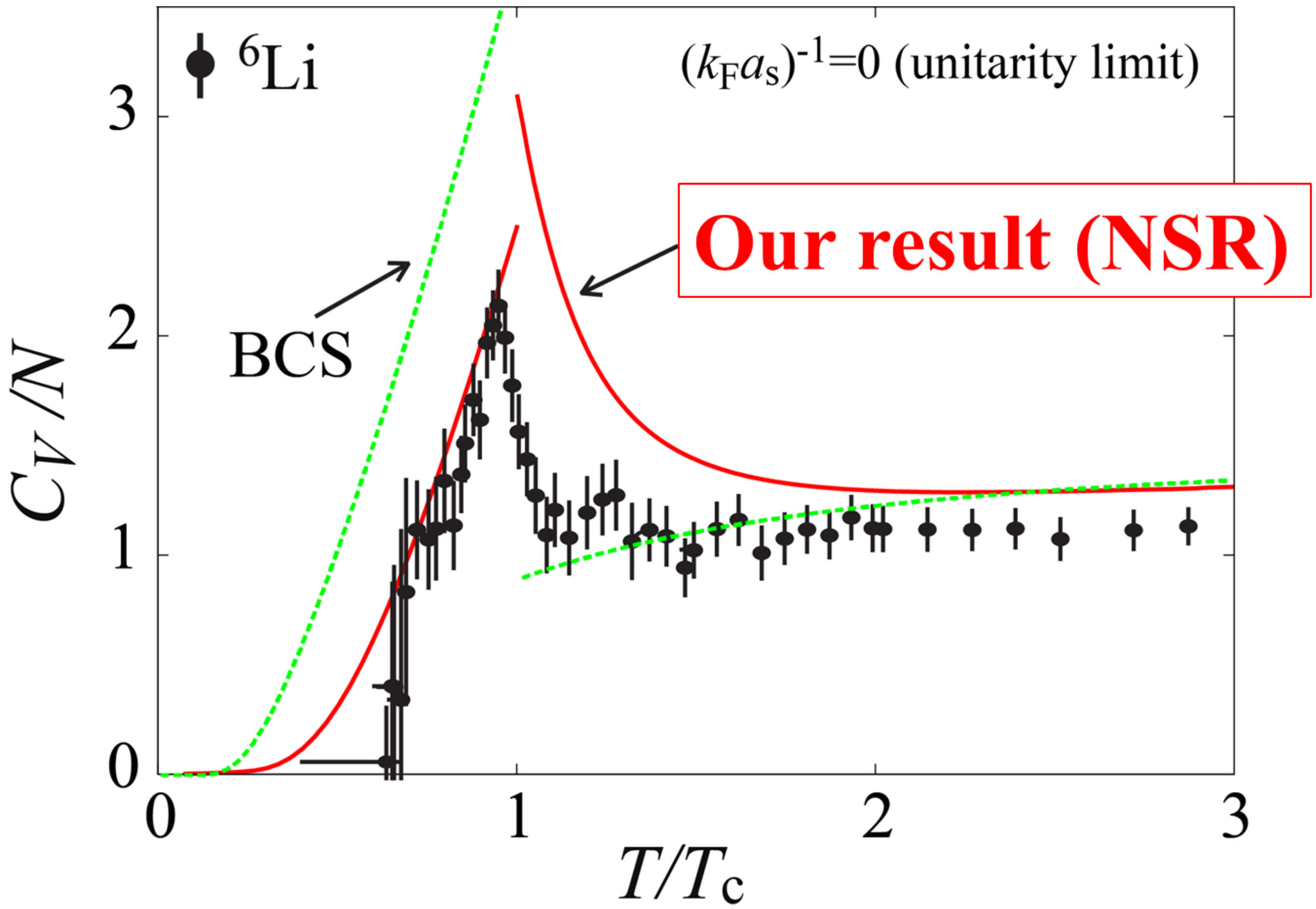


Pieter, Ohashi et al, PRA (2016)

Phase diagram of an ultracold Fermi gas in the crossover region



Comparison with recent experiment on a ^{6}Li Fermi gas

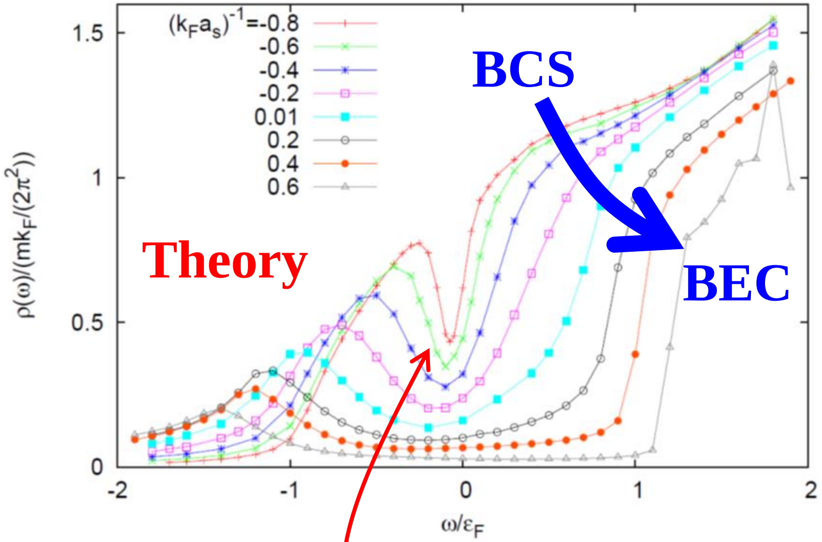


^{6}Li data: M. Ku, et al., Science 335, 563 (2012).

Pseudogap phenomenon in the BCS-BEC crossover region

Normal-state density of states

$T = T_c : \Delta = 0!$



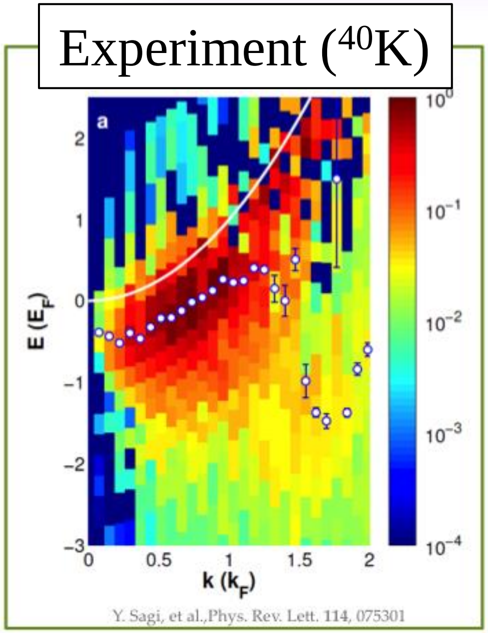
Tsuchiya, Ohashi, PRA

peudogap

Dissociation energy of preformed Cooper pair above Tc.

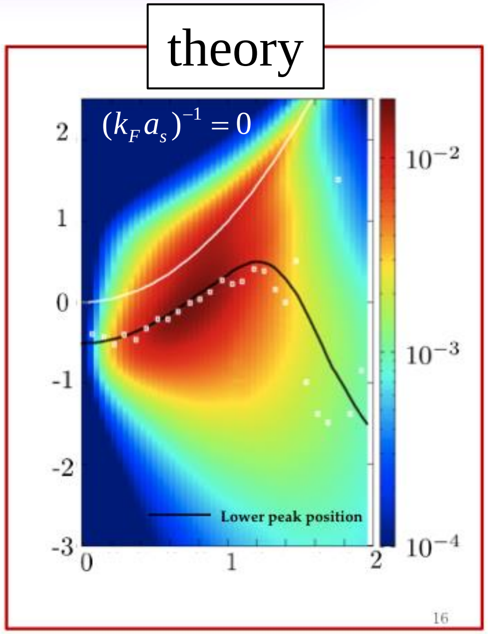
Photoemission spectrum

Experiment (^{40}K)



Y. Sagi, et al, PRL 114, 075301

theory

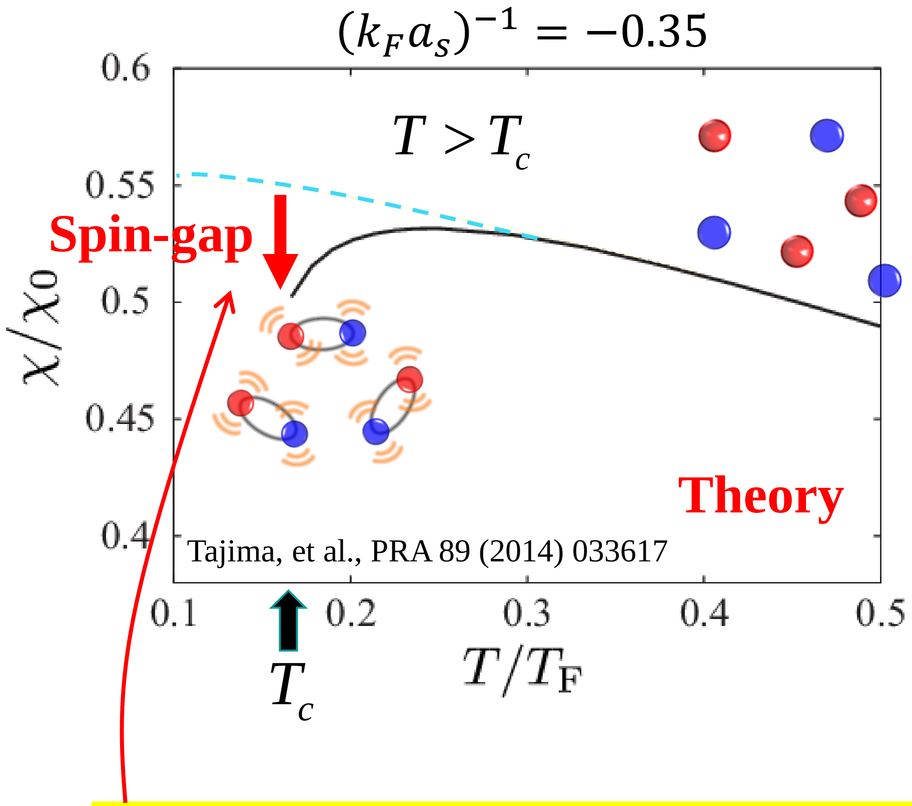


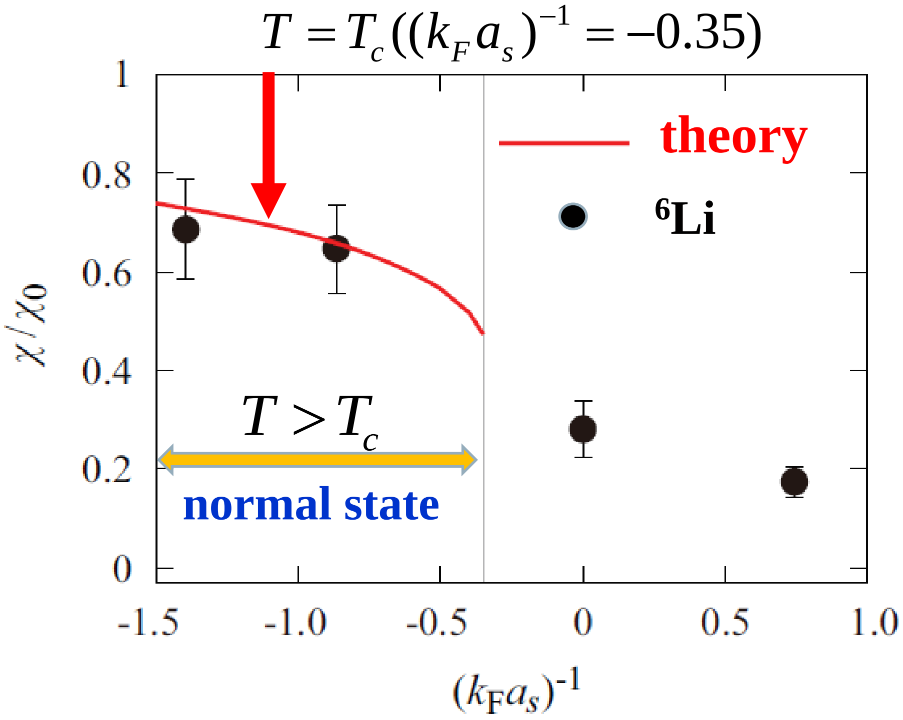
Ota, Ohashi et al, PRA (2016)

Spin-gap phenomenon in the BCS-BEC crossover region

calculated spin-susceptibility

Observed susceptibility (^{6}Li)



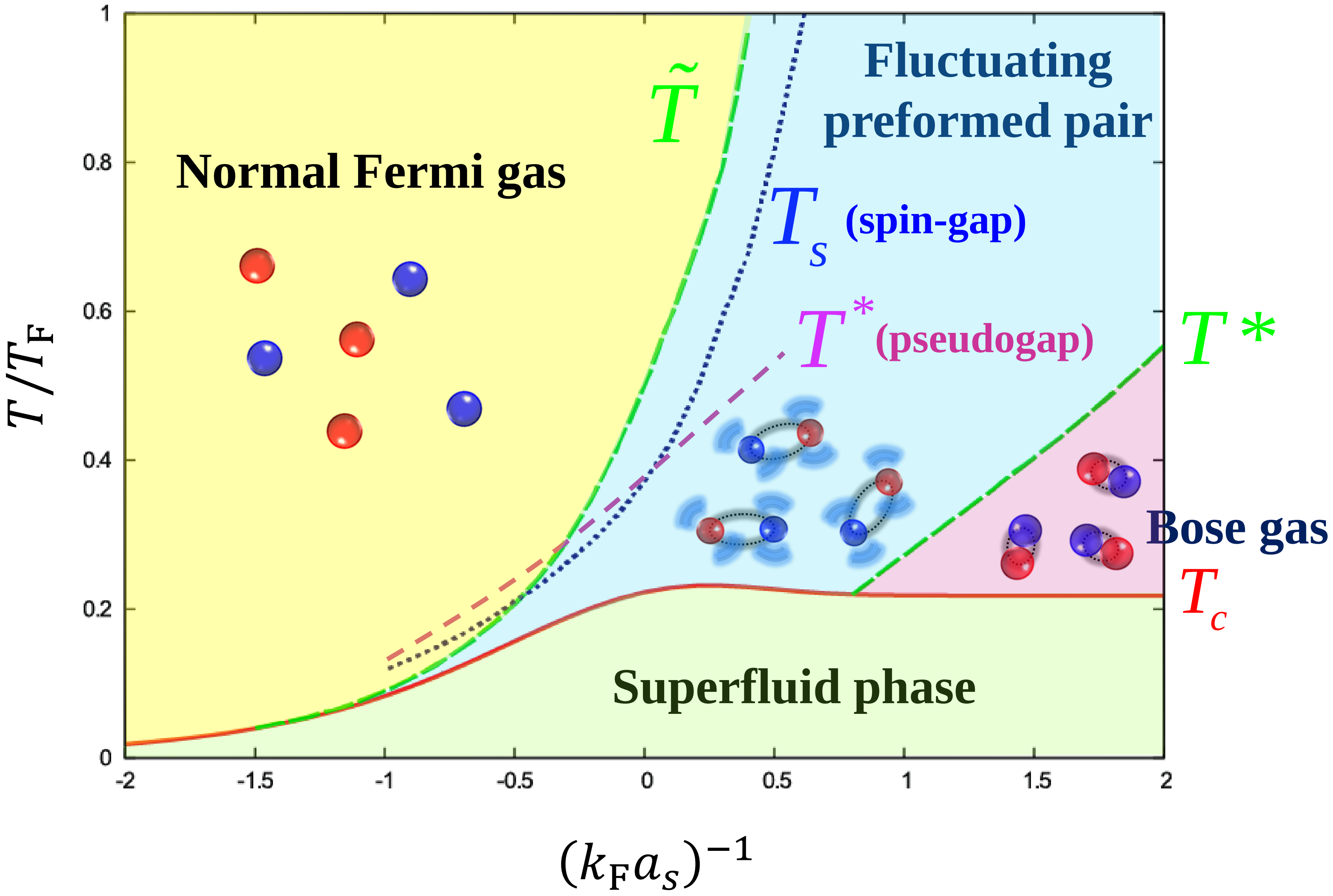


^{6}Li data: Sanner et.al, PRL 106 (2011) 010402

Theory: Kashimura et al., PRA 86 (2012) 043622

Formation of <u>spin-singlet</u> preformed pairs suppresses spin susceptibility.

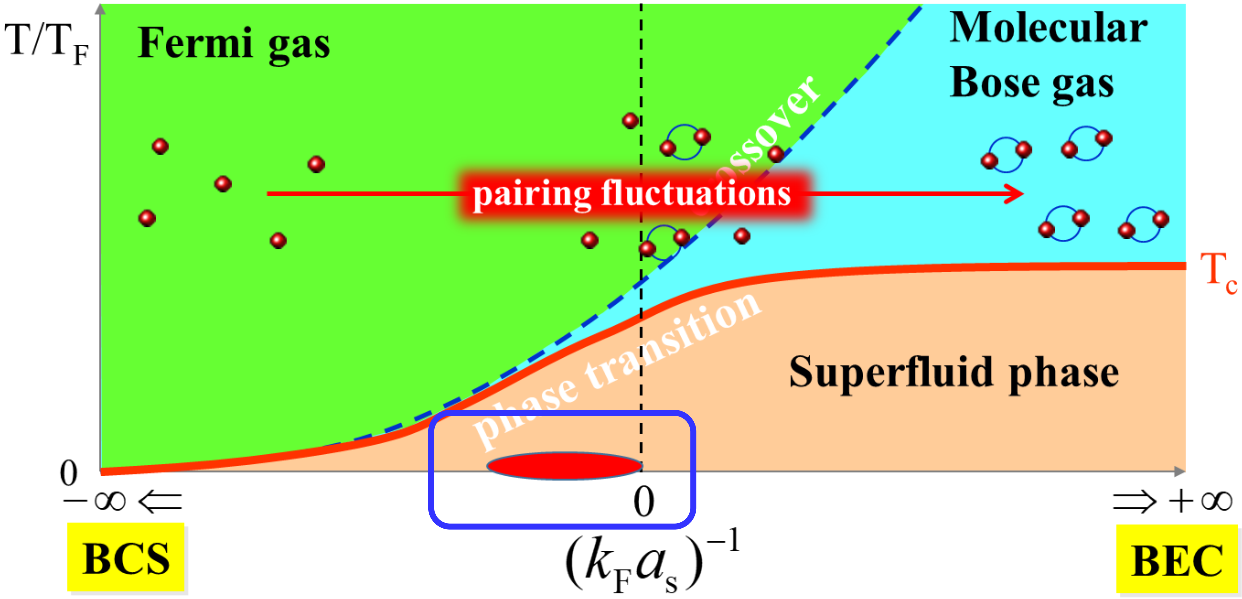
Comparison with spin-gap and pseudogap



Superfluid properties of an ultracold Fermi gas in the BCS-BEC crossover region

~~Effective range~~

$$r_e = 0$$



Model ultracold Fermi gas (normal state)

BCS Hamiltonian

$$H = \sum_{\mathbf{p},\sigma} (\varepsilon_{\mathbf{p}} - \mu) c^{\dagger}_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - U \sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} c^{\dagger}_{\mathbf{p}+\mathbf{q}/2\uparrow} c^{\dagger}_{-\mathbf{p}+\mathbf{q}/2\downarrow} c_{-\mathbf{p}'+\mathbf{q}/2\downarrow} c_{\mathbf{p}'+\mathbf{q}/2\uparrow}$$

- $c_{\mathbf{p}\sigma}$: Fermi atom ($\uparrow, \downarrow$: pseudospins describing atomic hyperfine states)

- U : tunable s-wave pairing interaction → $\frac{4\pi a_s}{m} = -\frac{U}{1 - U\sum_{\mathbf{p}} \gamma_{\mathbf{p}}^2 / (2\varepsilon_{\mathbf{p}})}$

Model ultracold Fermi gas (superfluid state)

BCS Hamiltonian in the Nambu representation

$$H = \sum_{\mathbf{p}} \Psi^{\dagger}_{\mathbf{p}} \left[(\varepsilon_{\mathbf{p}} - \mu)\tau_3 - \Delta \tau_1 \right] \Psi_{\mathbf{p}} - \frac{U}{2} \sum_{\mathbf{q}} \left[\rho_1(\mathbf{q})\rho_1(-\mathbf{q}) + \rho_2(\mathbf{q})\rho_2(-\mathbf{q}) \right]$$

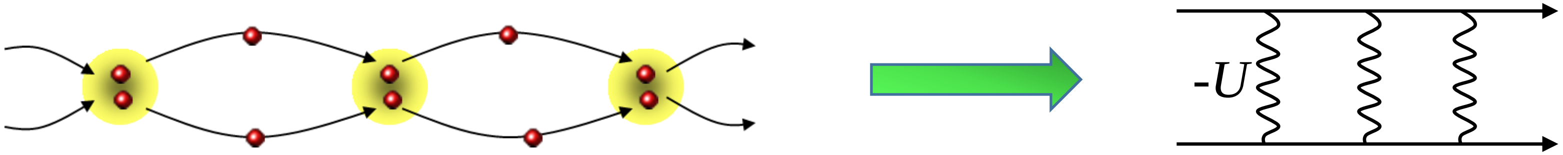
- $\Psi_{\mathbf{p}} = \begin{pmatrix} c_{\mathbf{p}\uparrow} \\ c^{\dagger}_{-\mathbf{p}\downarrow} \end{pmatrix}$: Nambu field ($\uparrow, \downarrow$: pseudospins describing atomic hyperfine states)
- Δ : superfluid order parameter
- U : tunable s-wave pairing interaction ➡ $\dfrac{4\pi a_s}{m} = -\dfrac{U}{1 - U \sum_{\mathbf{p}} \gamma_{\mathbf{p}}^2 / (2\varepsilon_{\mathbf{p}})}$
- $\rho_j(\mathbf{q}) = \sum_{\mathbf{p}} \gamma_{\mathbf{p}} \Psi^{\dagger}_{\mathbf{p}+\mathbf{q}/2} \tau_j \Psi_{\mathbf{p}-\mathbf{q}/2}$: generalized density operator

$$\gamma_{\mathbf{p}} = \frac{1}{\sqrt{1 + (p / p_c)^2}}$$

➡ $r_e \cong \dfrac{2}{p_c} \to 0 \Leftrightarrow p_c = \infty$

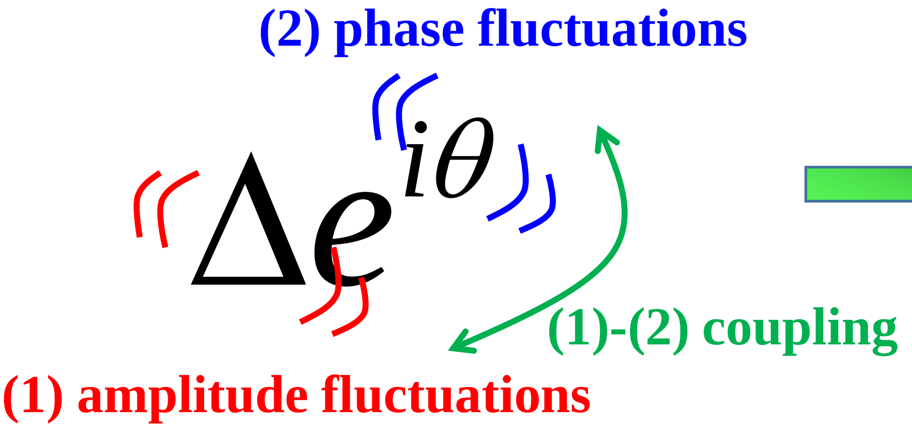
Inclusion of strong coupling corrections beyond mean-field BCS theory

- normal phase ($T > T_c$): "pairing" fluctuations



- superfluid phase ($T < T_c$): fluctuations of "Δ"

$$H = \sum_{\mathbf{p}} \Psi_{\mathbf{p}}^{\dagger}\left[(\varepsilon_{\mathbf{p}} - \mu)\tau_3 - \Delta\tau_1\right]\Psi_{\mathbf{p}} - \frac{U}{2}\sum_{\mathbf{q}}\left[\underbrace{\rho_1(\mathbf{q})\rho_1(-\mathbf{q})}_{\text{amplitude fluc.}} + \underbrace{\rho_2(\mathbf{q})\rho_2(-\mathbf{q})}_{\text{phase fluc.}}\right]$$



(2) phase fluctuations

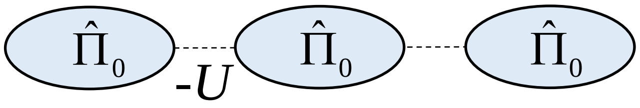
$\Delta e^{i\theta}$

(1)-(2) coupling

(1) amplitude fluctuations

$$\hat{\Pi} = \begin{pmatrix} \langle \rho_1\rho_1 \rangle & \langle \rho_1\rho_2 \rangle \\ \langle \rho_2\rho_1 \rangle & \langle \rho_2\rho_2 \rangle \end{pmatrix}$$

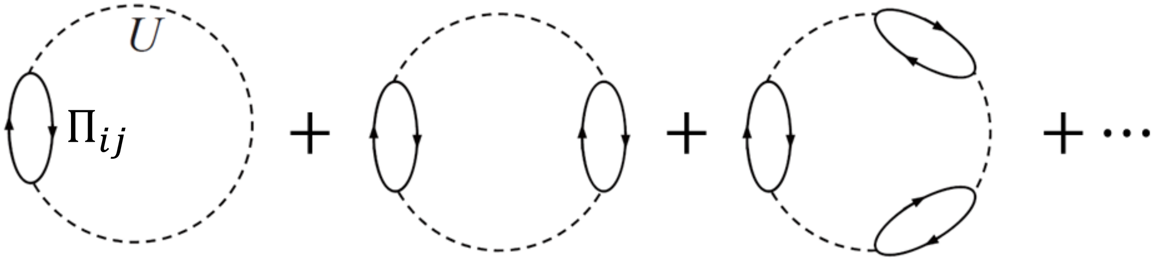
=



Construction of Gaussian fluctuation (NSR) theory below Tc

Thermodynamic potential: $\Omega = \Omega_{\text{MF}} + \Omega_{\text{fluc}}$

$\Omega_{\text{fluc}} =$



$$\Pi_{ij} = \frac{1}{\beta}\sum_{p} \mathrm{Tr}\left[\gamma_{\mathbf{p}}\tau_i \hat{G}(p+\frac{q}{2})\gamma_{\mathbf{p}}\tau_j \hat{G}(p-\frac{q}{2})\right]$$

$$\hat{G}(p) = \frac{1}{i\omega_n - (\varepsilon_{\mathbf{p}} - \mu)\tau_3 + \Delta\tau_1}$$

Number Equation

$$N = N_{\text{MF}} - \left(\frac{\partial \Omega_{\text{fluc}}}{\partial \mu}\right)_{V,T}$$

Solve Δ and μ

Gap Equation

$$1 = -\frac{4\pi a_s}{m}\sum_{\mathbf{p}} \gamma_{\mathbf{p}}^2 \left[\frac{\tanh(E_{\mathbf{p}}/2T)}{2E_{\mathbf{p}}} - \frac{1}{2\varepsilon_{\mathbf{p}}}\right]$$

$$E_{\mathbf{p}} = \sqrt{\xi^2 + \Delta^2}$$

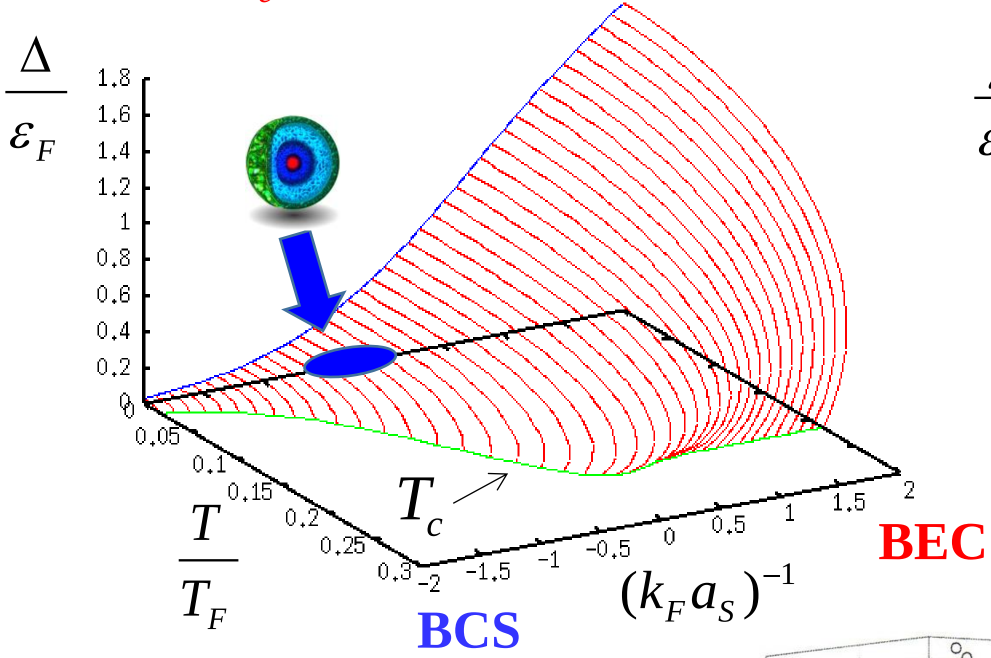
Internal Energy

$$E = E_{MF} + \Omega_{\text{fluc}} - T\left(\frac{\partial \Omega_{\text{fluc}}}{\partial T}\right)_{V,\mu} - \mu\left(\frac{\partial \Omega_{\text{fluc}}}{\partial \mu}\right)_{V,T}$$

Self-consistent solutions for Δ and μ in the crossover region

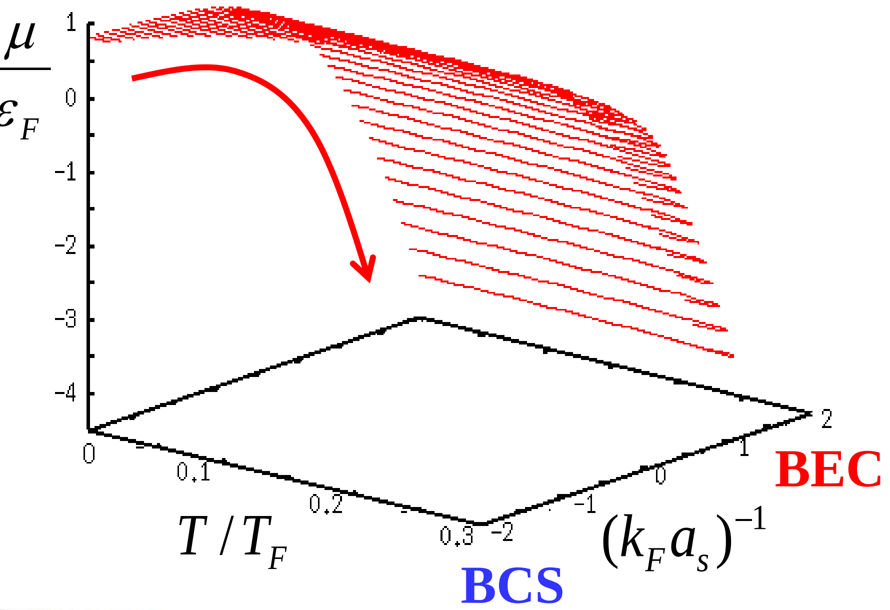
superfluid order parameter

$r_e = 0\,(\text{cold atom})$

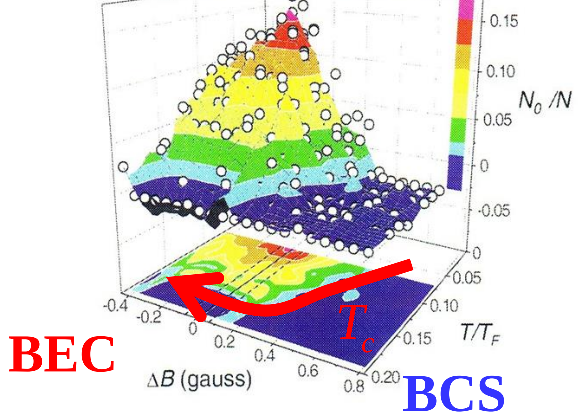


Fermi chemical potential

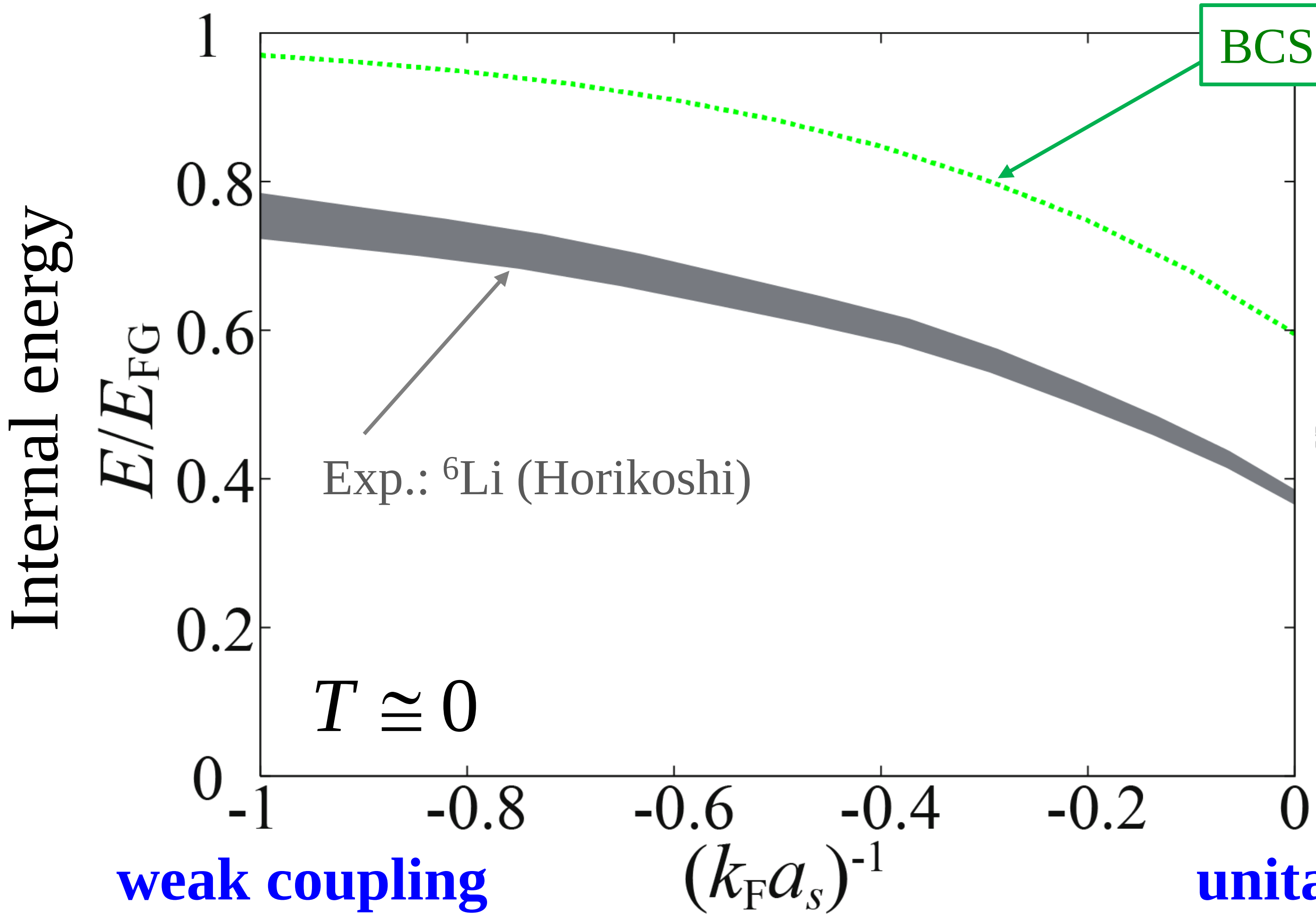
$r_e = 0\,(\text{cold atom})$



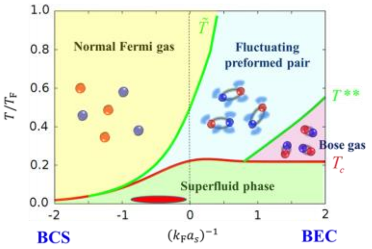
Fukushima, Ohashi, PRA (2007)



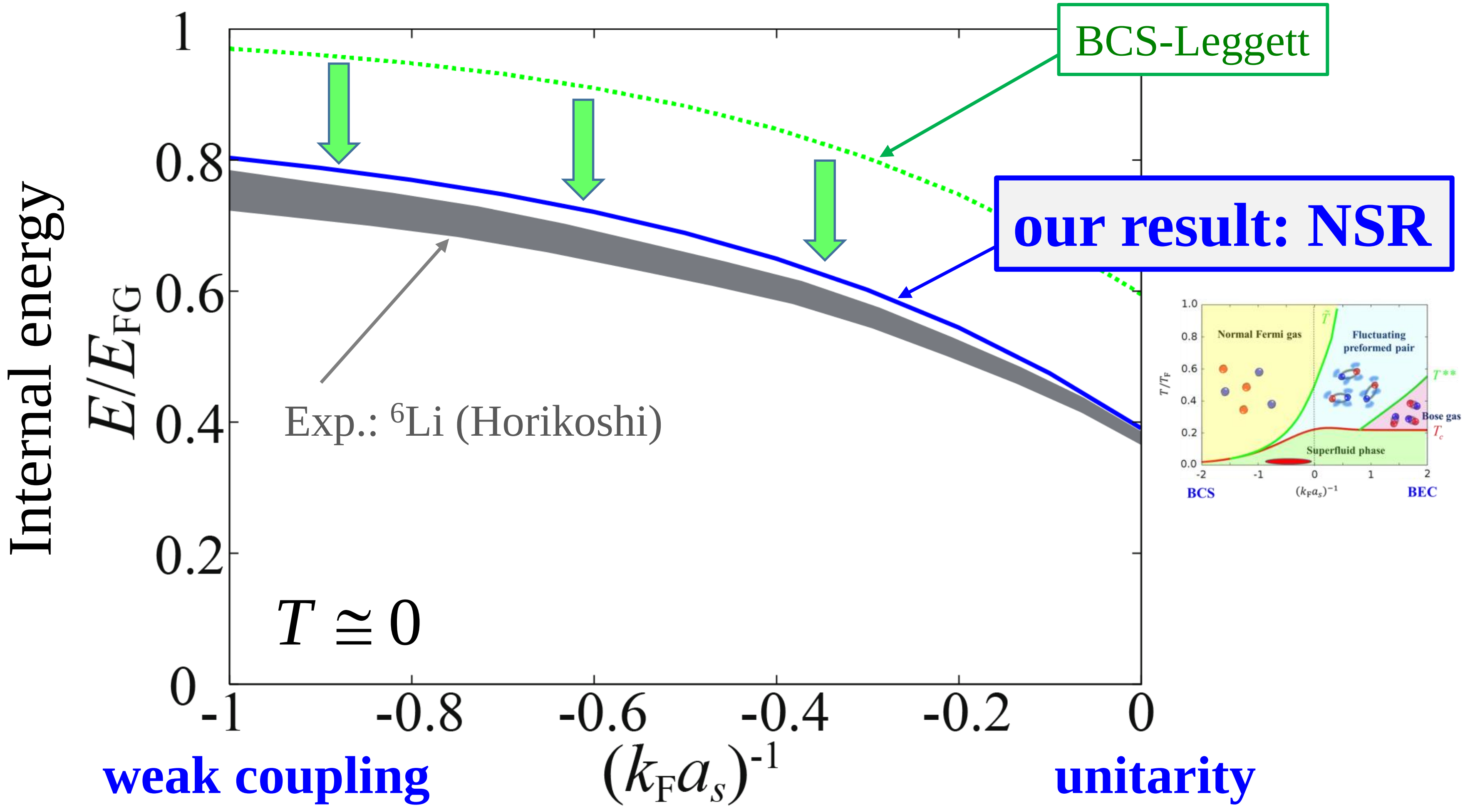
EoS: Superfluid Fermi gas



weak coupling **unitarity**



EoS: Superfluid Fermi gas



Inclusion of superfluid fluctuations is crucial for the quantitative evaluation of the internal energy in the unitary regime ($(k_F a_s)^{-1} << 0$), even at $T=0$.

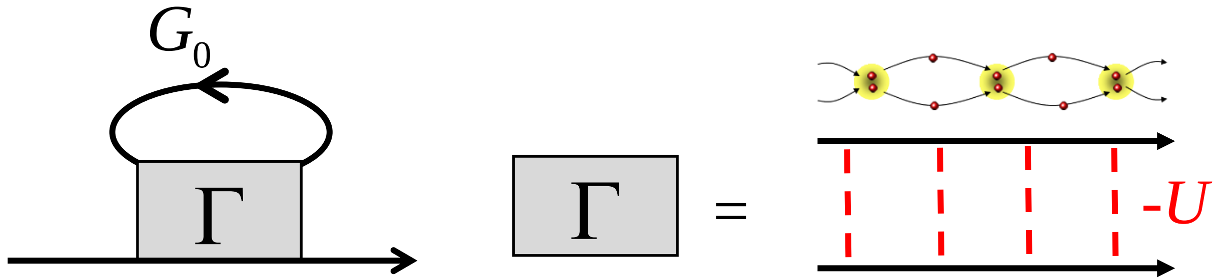
Diagrammatic representation of NSR theory and its extension

- Green's function to reproduce the NSR results (T >T$_c$)

$$G_{\rm NSR} = G_0 + G_0 \Sigma G_0 \qquad G_0 = \frac{1}{i\omega_n - \xi_{\mathbf{p}}}$$

self-energy describing pairing fluctuations

$$\Sigma =$$

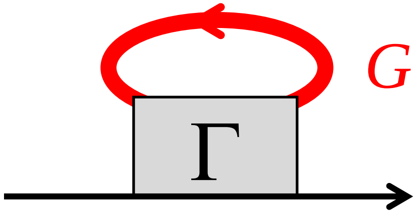


- T-matrix approximation (TMA)

$$G_{\rm TMA} = G_0 + G_0 \Sigma G_0 + G_0 \Sigma G_0 \Sigma G_0 + \ldots = \frac{1}{G_0^{-1} - \Sigma}$$

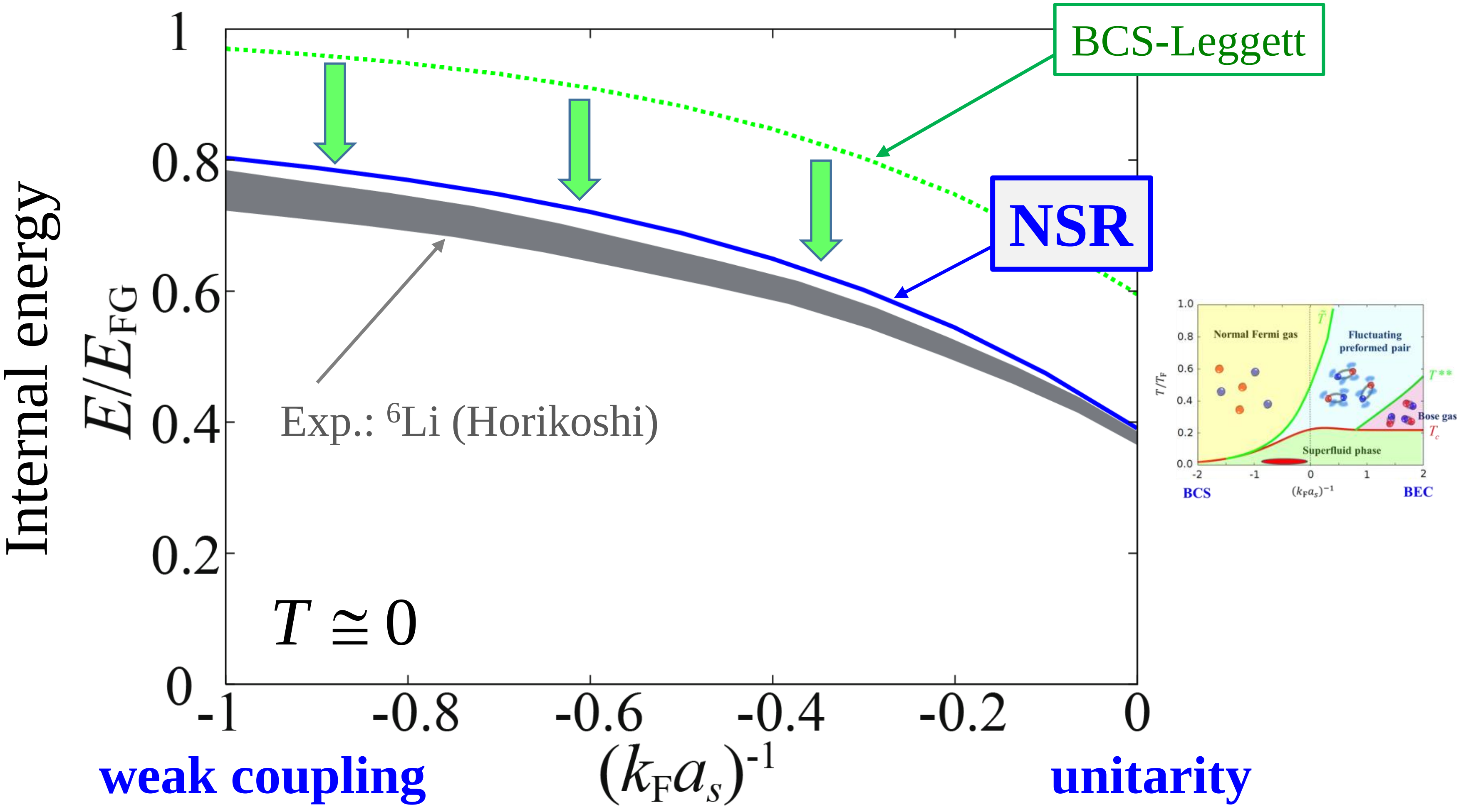
- Extended T-matrix approximation (ETMA)

$$\Sigma_{\rm ETMA} =$$



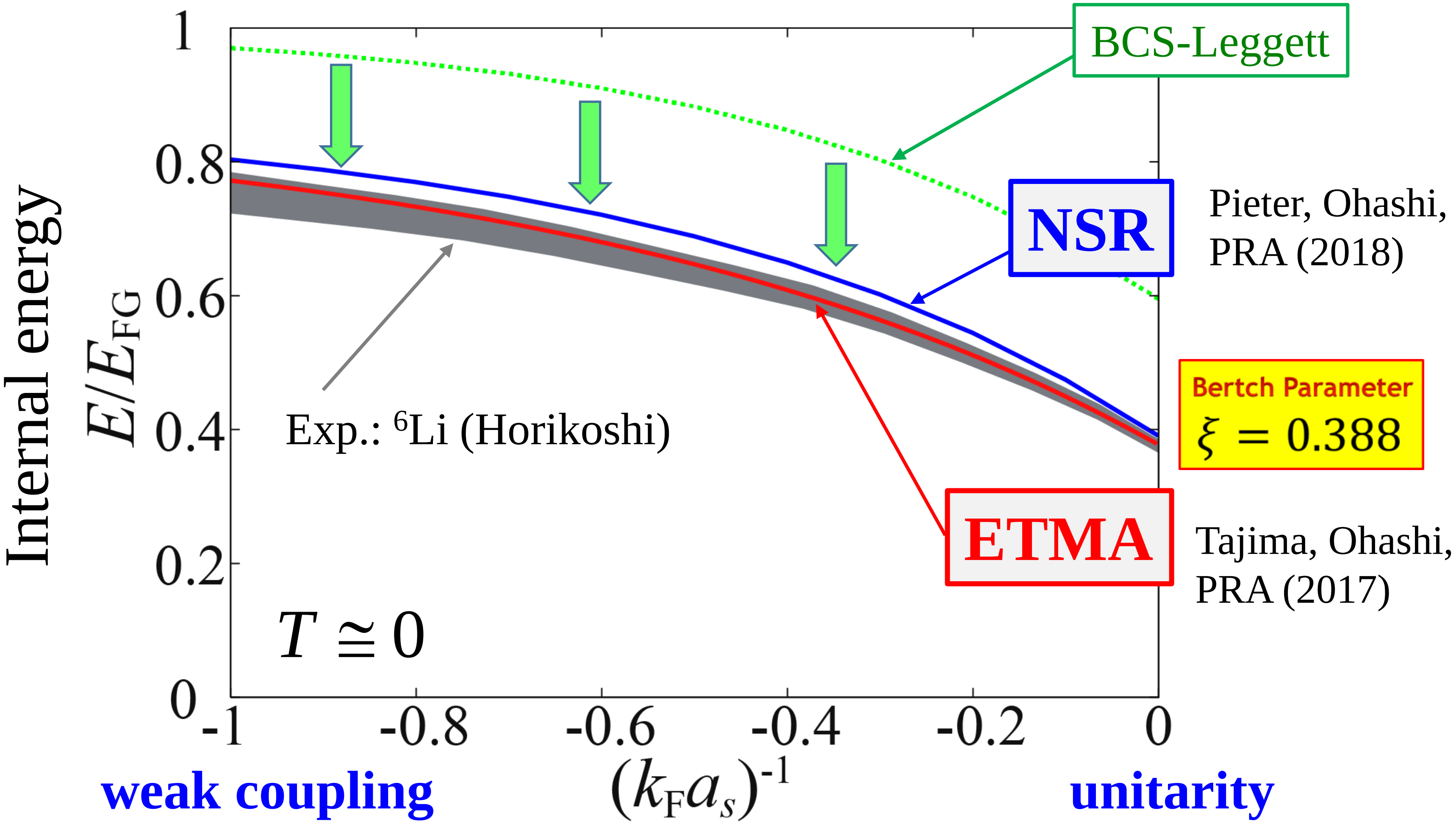
$$G = \frac{1}{i\omega_n - \xi_{\mathbf{p}} - \Sigma}$$

EoS: Superfluid Fermi gas



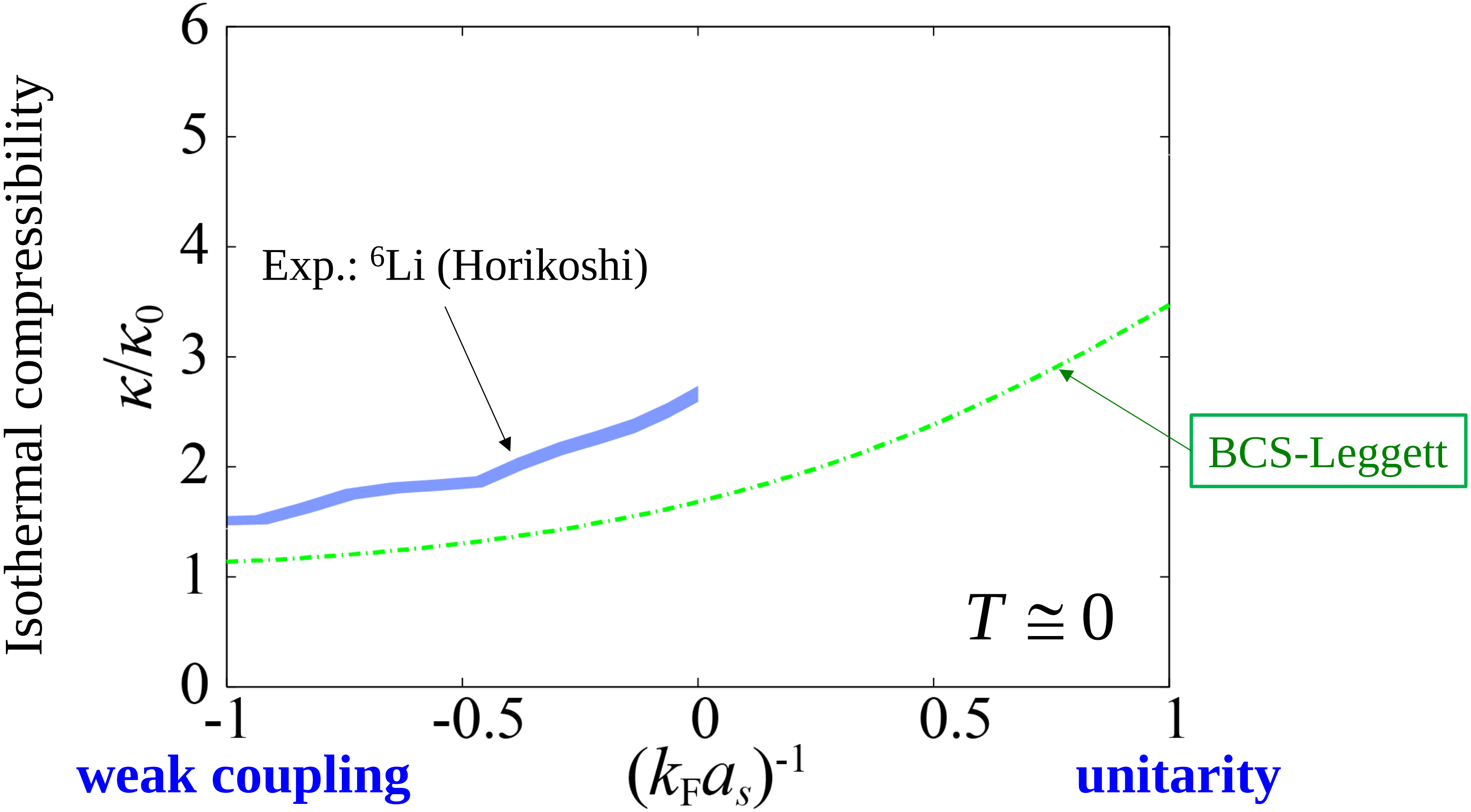
Inclusion of superfluid fluctuations is crucial for the quantitative evaluation of the internal energy in the unirary regime ($(k_F a_s)^{-1} << 0$), even at $T=0$.

EoS: Superfluid Fermi gas



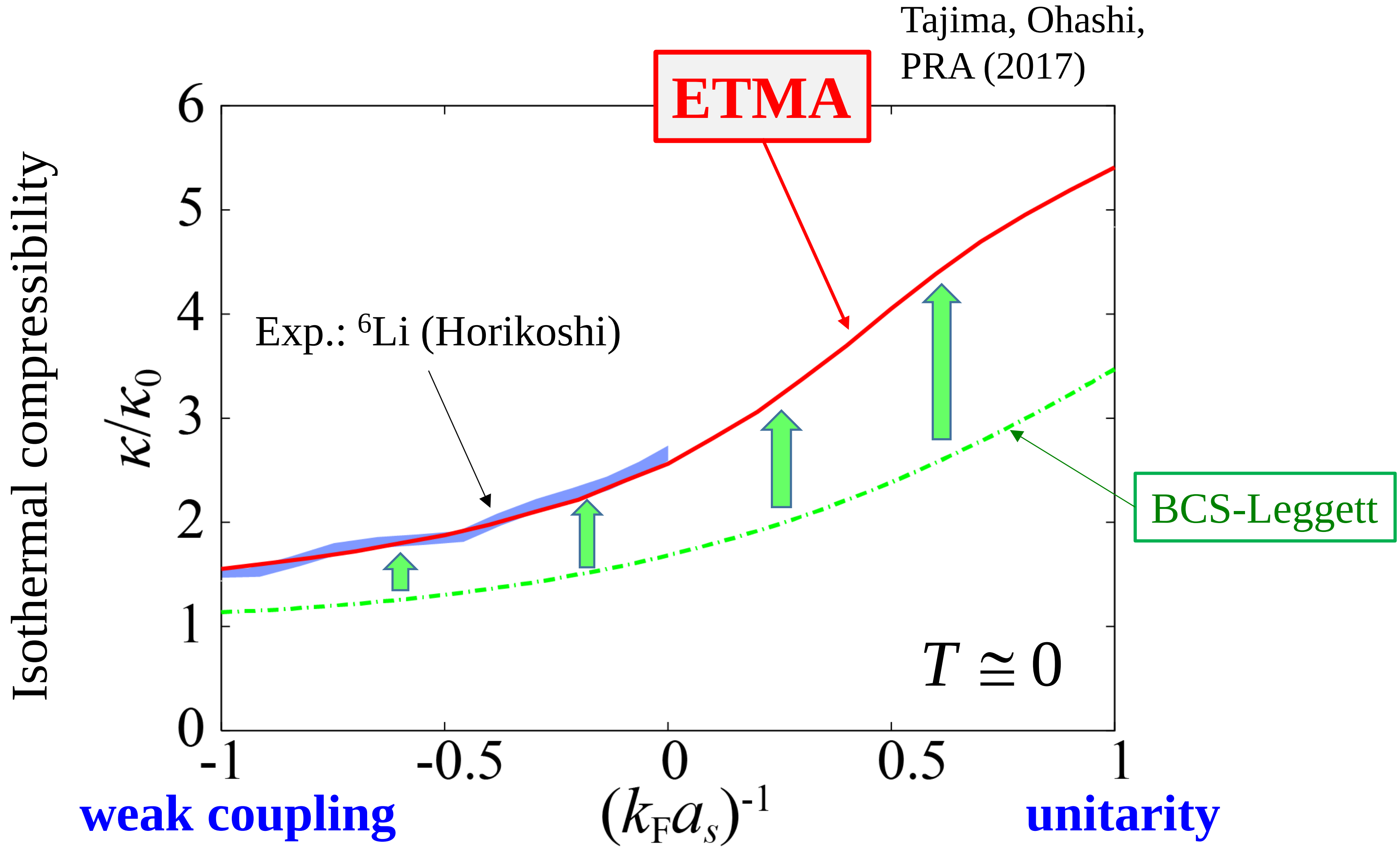
Our strong-coupling calculation agrees well with the observed EoS in a ^{6}Li superfluid Fermi gas, indicating the importance of superfluid fluctuations in the unitary regime ($(k_F a_s)^{-1} \sim 0$), even at $T=0$.

Compressibility κ in a superfluid Fermi gas at T=0



weak coupling **unitarity**

Compressibility κ in a superfluid Fermi gas at T=0

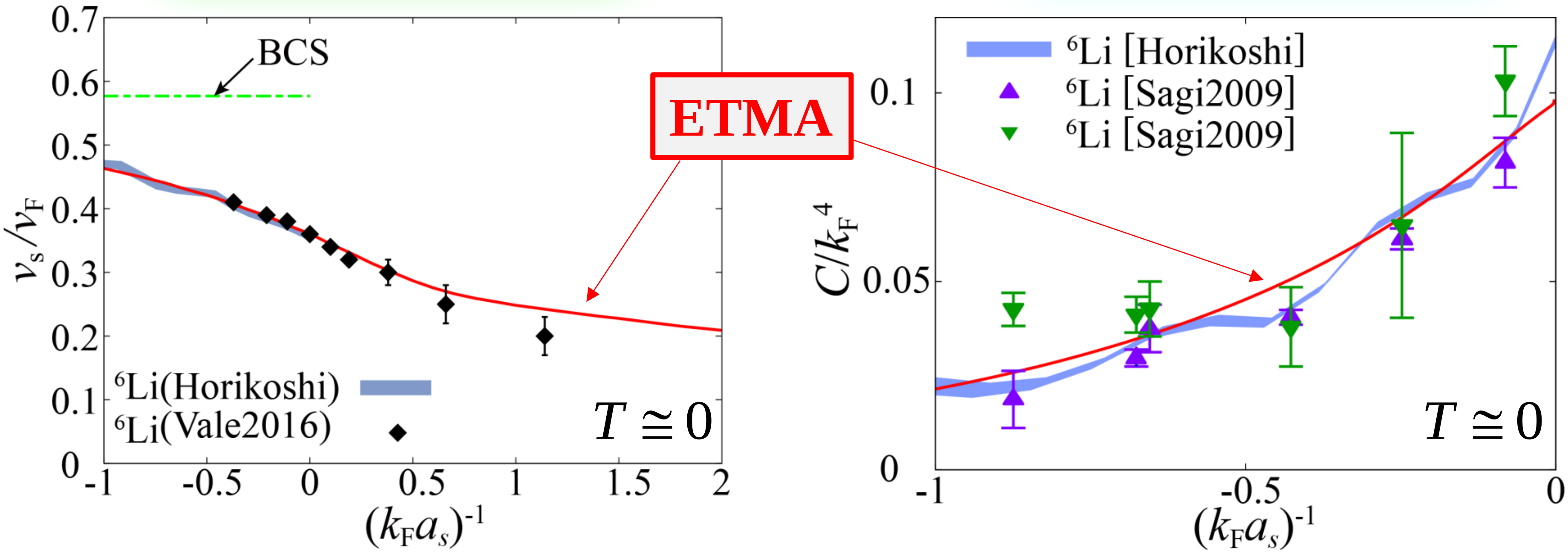


Superfluid fluctuations enhances the compressibility at *T*=0.

Other ground state quantities in a superfluid Fermi gas

Tajima, Ohashi, PRA (2017)

Sound velocity **Tan's contact**

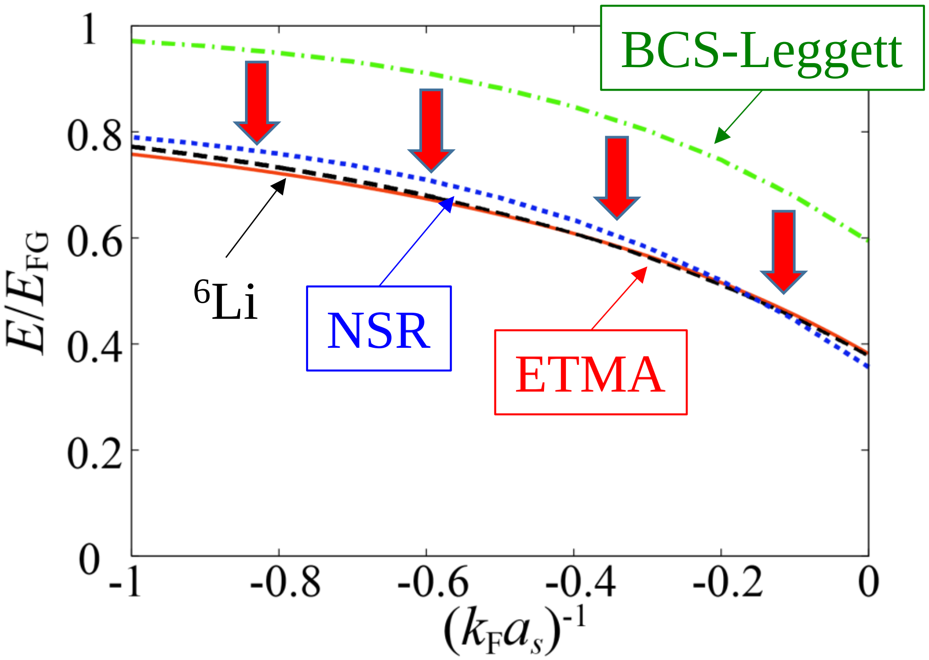


$$E = \sum_{\mathbf{p},\sigma} \varepsilon_{\mathbf{p}} \left[n_{\mathbf{p}} - \frac{C}{p^4} \right] + \frac{C}{4\pi m a_s}$$

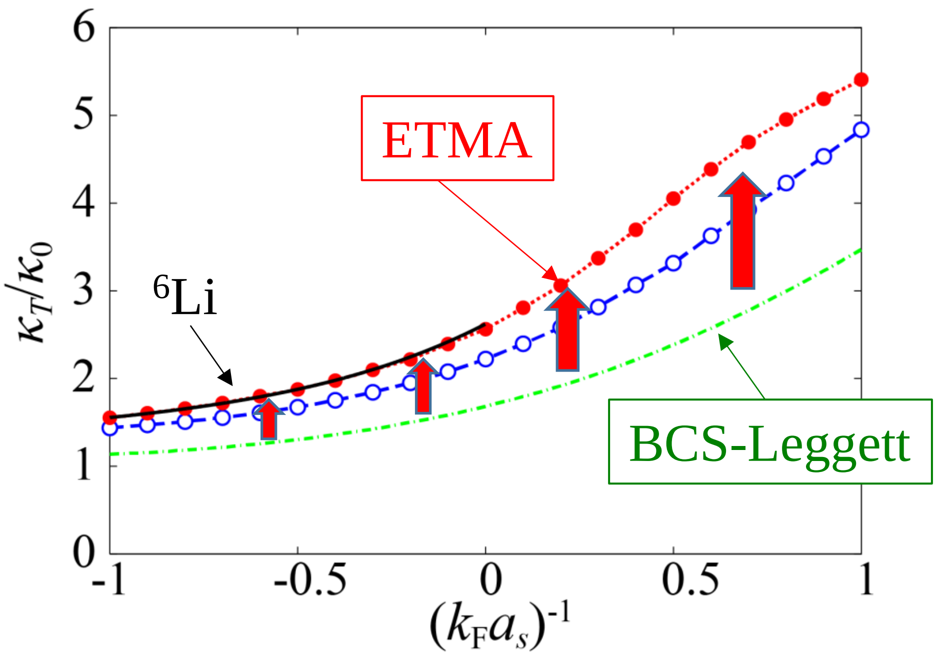
Inclusion of superfluid fluctuations enables us to quantitatively discuss superfluid properties in this regime.

Origin of many-body corrections at T=0

internal energy (EoS)

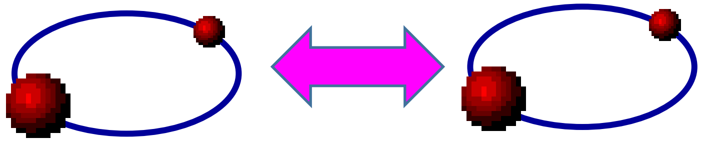


compressibility



Tajima, Ohashi, PRA (2017)

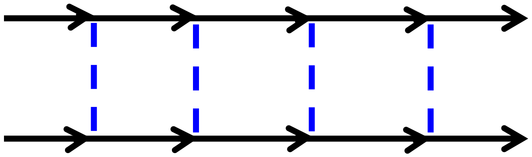
Effective repulsive interaction between Cooper pairs

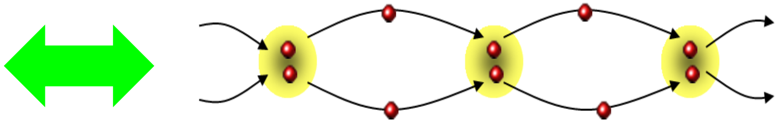


Origin of many-body corrections at T=0

pairing fluctuations

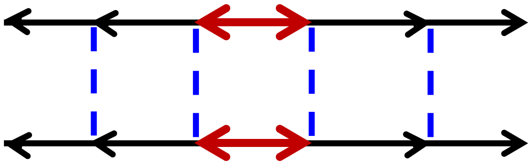
$T > T_c$





$+$

$T < T_c$

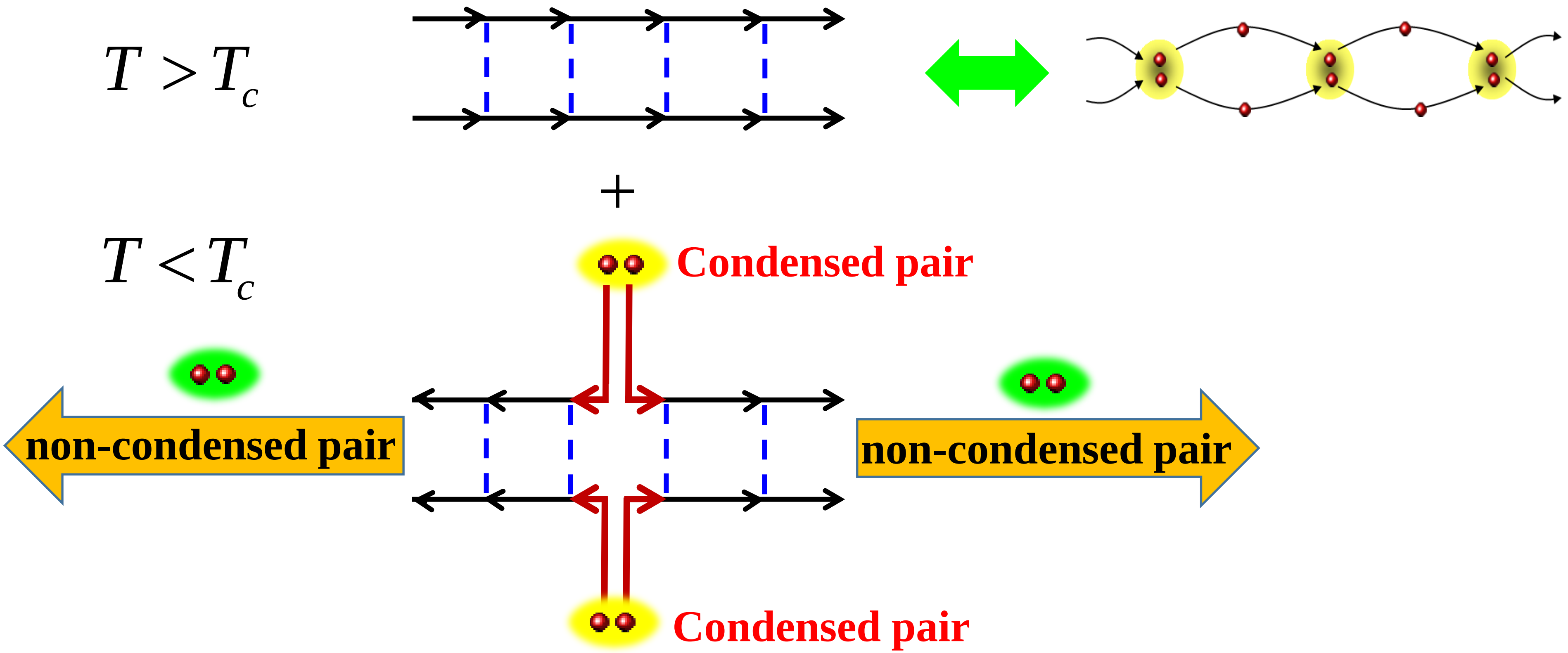


$\leftrightarrow \sim \left\langle c^{\dagger}_{\mathbf{p}\uparrow} c^{\dagger}_{-\mathbf{p}\downarrow} \right\rangle$

Anomalous Green’s function

Origin of many-body corrections at T=0

pairing fluctuations



$T > T_c$

$+$

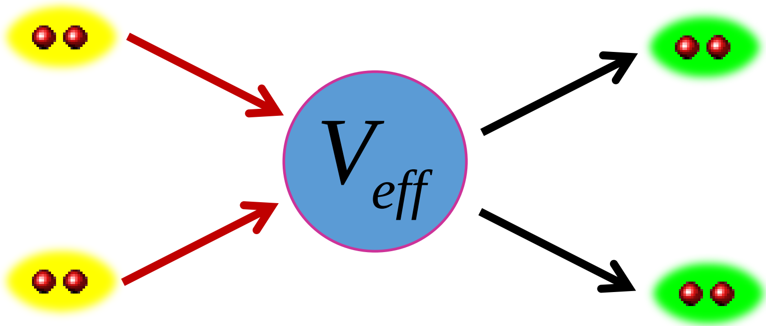
$T < T_c$

Condensed pair

non-condensed pair

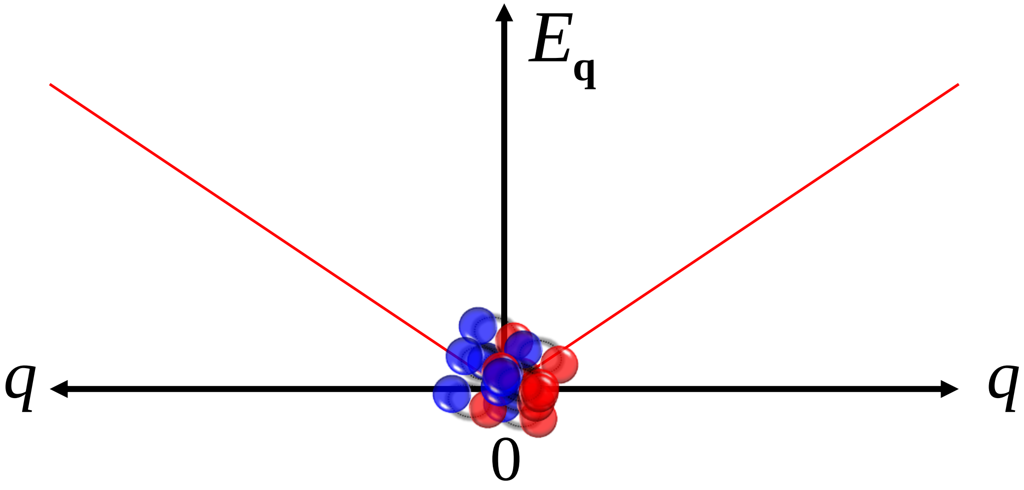
non-condensed pair

Condensed pair



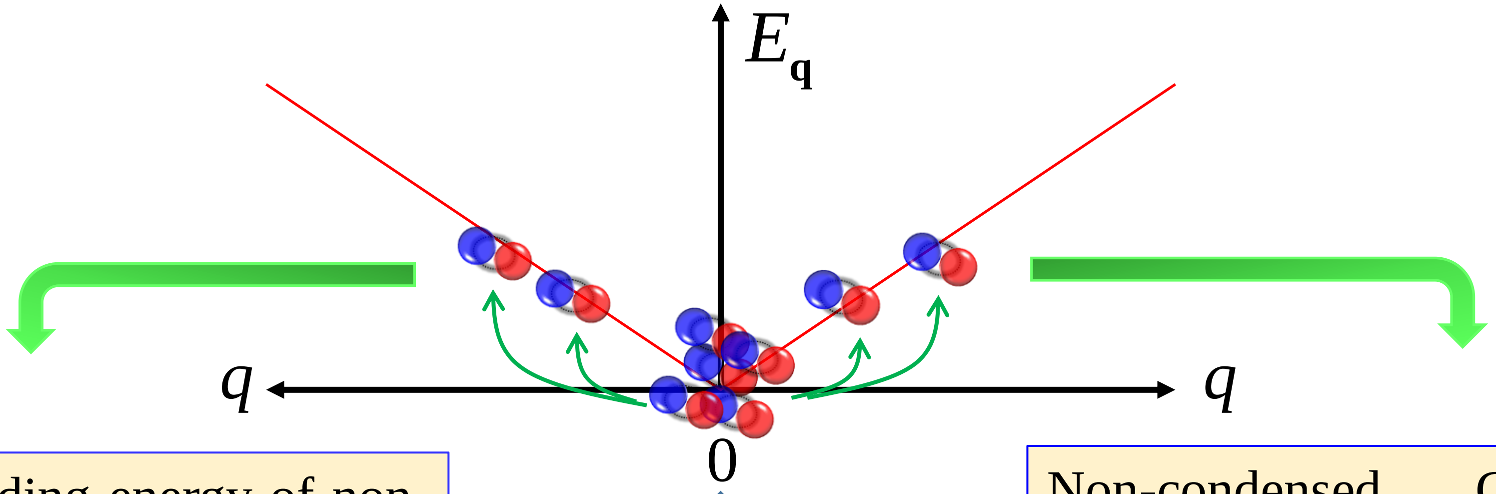
: repulsive interaction

Strong-coupling effects: quantum depletion



Strong-coupling effects: quantum depletion

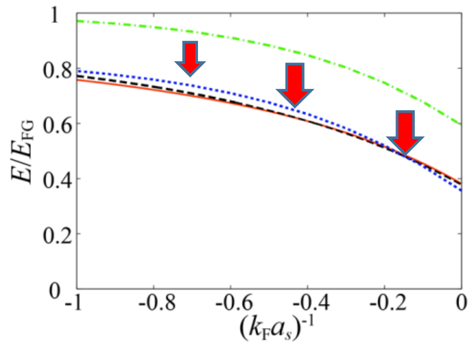
Some Cooper pairs are kicked out from the condensate, because of this *repulsive interaction* between them.

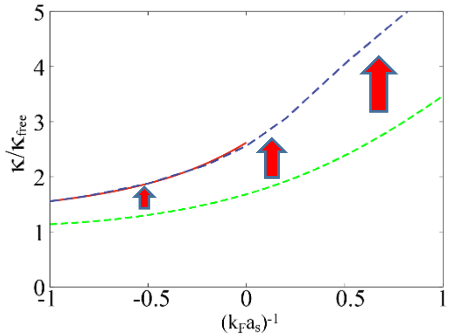


The binding energy of non-condensed pairs lowers the internal energy.

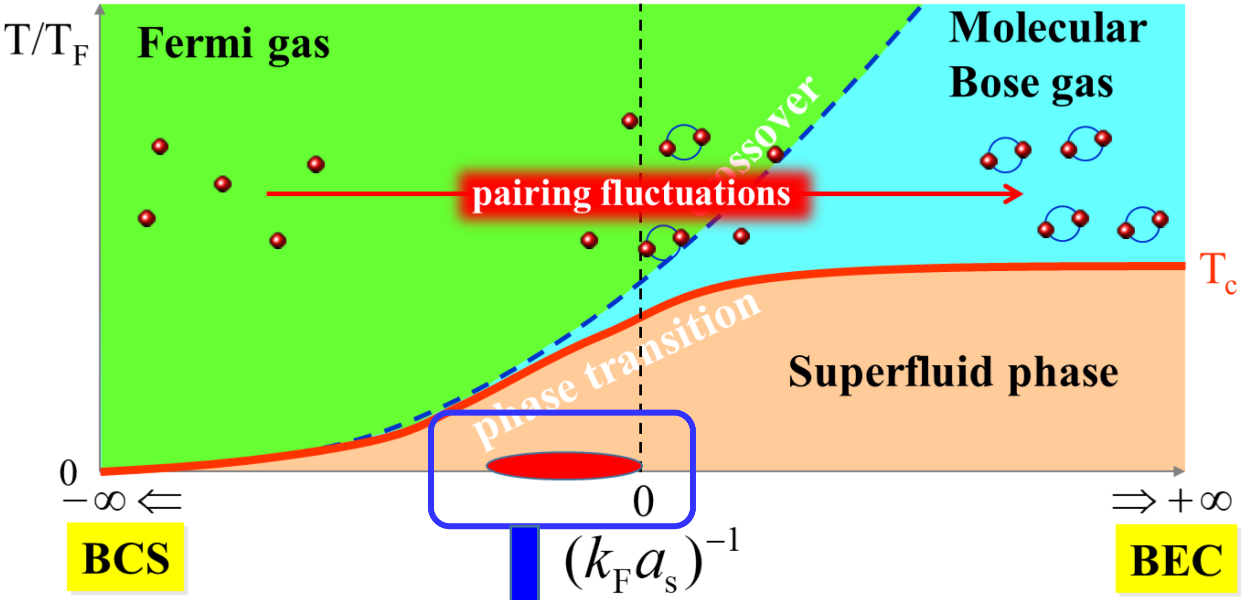
BCS theory

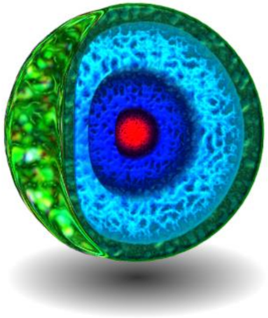
Non-condensed Cooper-pair molecules enhance the bosonic character of the system, leading to κ ↑.





Application to neutron-star EoS in the low-density region





Effective range

$$r_e = 2.7\text{fm}$$

Superfluid Fermi gas ($r_{\rm eff}$=0) → crust regime of neutron star ($r_{\rm eff}$=2.7 fm)

BCS Hamiltonian in the Nambu representation

$$\boxed{H=\sum_{\mathbf{p}}\Psi^{\dagger}_{\mathbf{p}}\left[(\varepsilon_{\mathbf{p}}-\mu)\tau_3-\Delta\tau_1\right]\Psi_{\mathbf{p}}-\frac{U}{2}\sum_{\mathbf{q}}\left[\rho_1(\mathbf{q})\rho_1(-\mathbf{q})+\rho_2(\mathbf{q})\rho_2(-\mathbf{q})\right]}$$

- $\Psi_{\mathbf{p}}=\begin{pmatrix} c_{\mathbf{p}\uparrow} \\ c^{\dagger}_{-\mathbf{p}\downarrow} \end{pmatrix}$: Nambu field ($\uparrow,\downarrow$: pseudospins describing atomic hyperfine states)
- Δ : superfluid order parameter
- U : tunable s-wave pairing interaction → $\dfrac{4\pi a_s}{m}=-\dfrac{U}{1-U\sum_{\mathbf{p}}\gamma_{\mathbf{p}}^2/(2\varepsilon_{\mathbf{p}})}$
- $\rho_j(\mathbf{q})=\sum_{\mathbf{p}}\gamma_{\mathbf{p}}\Psi^{\dagger}_{\mathbf{p}+\mathbf{q}/2}\tau_j\Psi_{\mathbf{p}-\mathbf{q}/2}$: generalized density operator

$$\gamma_{\mathbf{p}}=\frac{1}{\sqrt{1+(p/p_c)^2}}$$

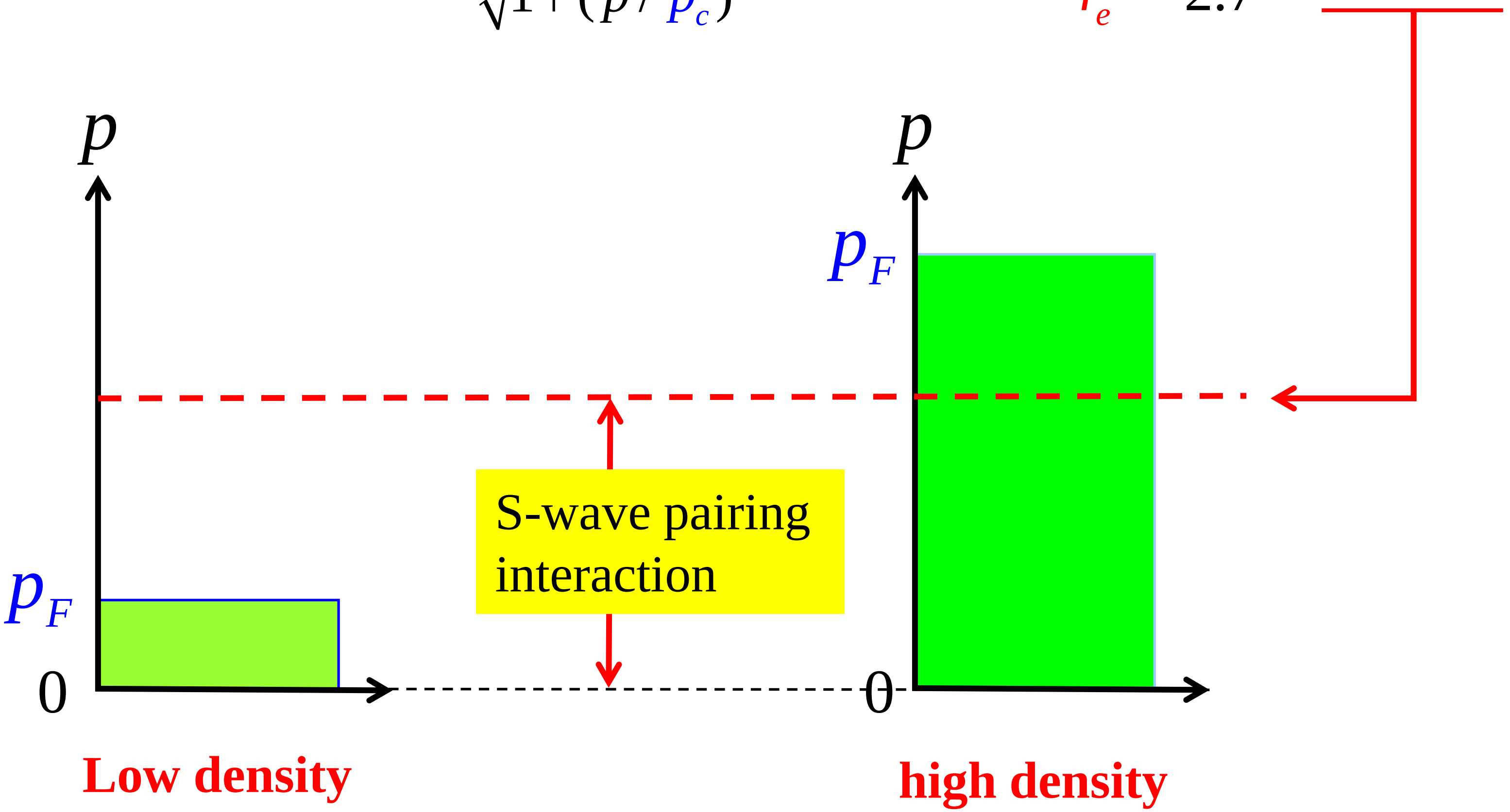
←

$$\boxed{p_c\cong\frac{2}{r_e}=\frac{2}{2.7\,\mathrm{fm}}}$$

key effect of the non-zero effective range r_e

$$H_{\rm int} = -U \sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} \gamma_{\mathbf{p}}\gamma_{\mathbf{p}'} c^{\dagger}_{\mathbf{p}+\mathbf{q}/2,\uparrow} c^{\dagger}_{-\mathbf{p}+\mathbf{q}/2,\downarrow} c_{-\mathbf{p}'+\mathbf{q}/2,\downarrow} c_{\mathbf{p}'+\mathbf{q}/2,\uparrow}$$

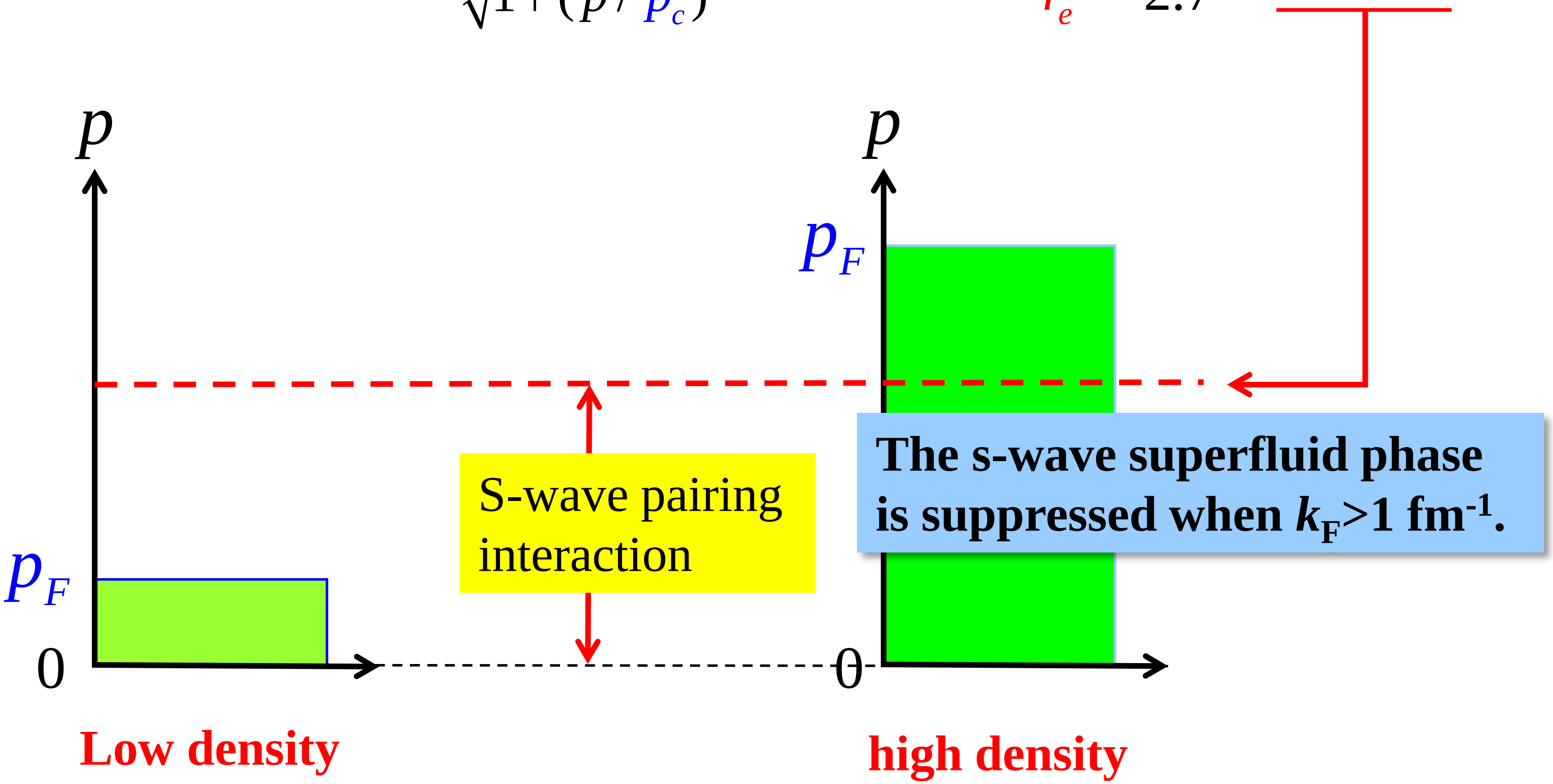
$$\gamma_{\mathbf{p}} = \frac{1}{\sqrt{1+(p/p_c)^2}} \quad \Rightarrow \quad p_c \cong \frac{2}{r_e} = \frac{2}{2.7} = 0.74\,\mathrm{fm}^{-1}$$



key effect of the non-zero effective range r_e

$$H_{\text{int}} = -U \sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} \gamma_{\mathbf{p}} \gamma_{\mathbf{p}'} c^{\dagger}_{\mathbf{p}+\mathbf{q}/2,\uparrow} c^{\dagger}_{-\mathbf{p}+\mathbf{q}/2,\downarrow} c_{-\mathbf{p}'+\mathbf{q}/2,\downarrow} c_{\mathbf{p}'+\mathbf{q}/2,\uparrow}$$

$$\gamma_{\mathbf{p}} = \frac{1}{\sqrt{1+(p/p_c)^2}} \quad \Rightarrow \quad p_c \cong \frac{2}{r_e} = \frac{2}{2.7} = 0.74\,\text{fm}^{-1}$$



S-wave pairing interaction

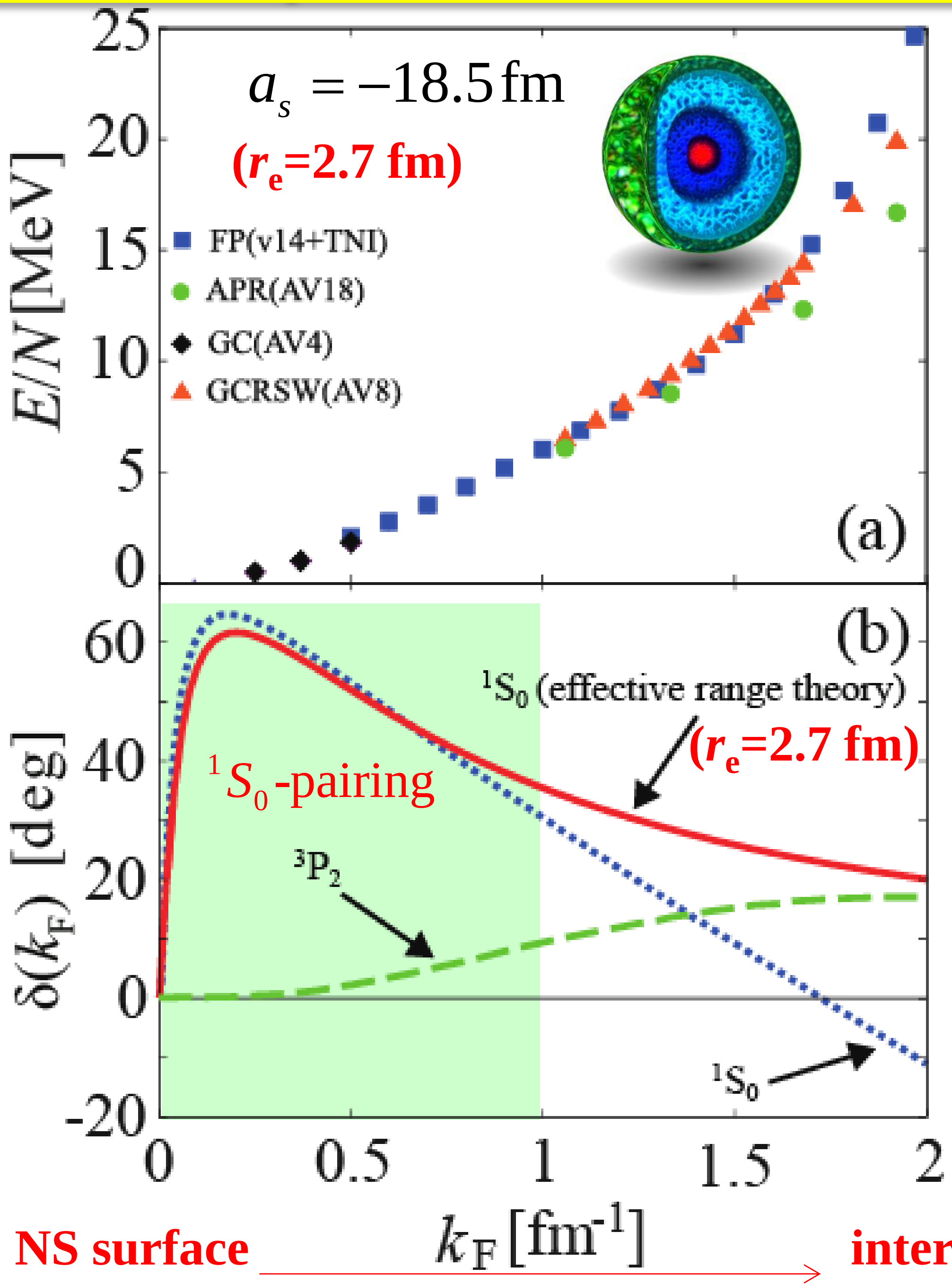
The s-wave superfluid phase is suppressed when k_F>1 fm^{-1}.

Low density

high density

Comparison with previous "neutron star calculations"

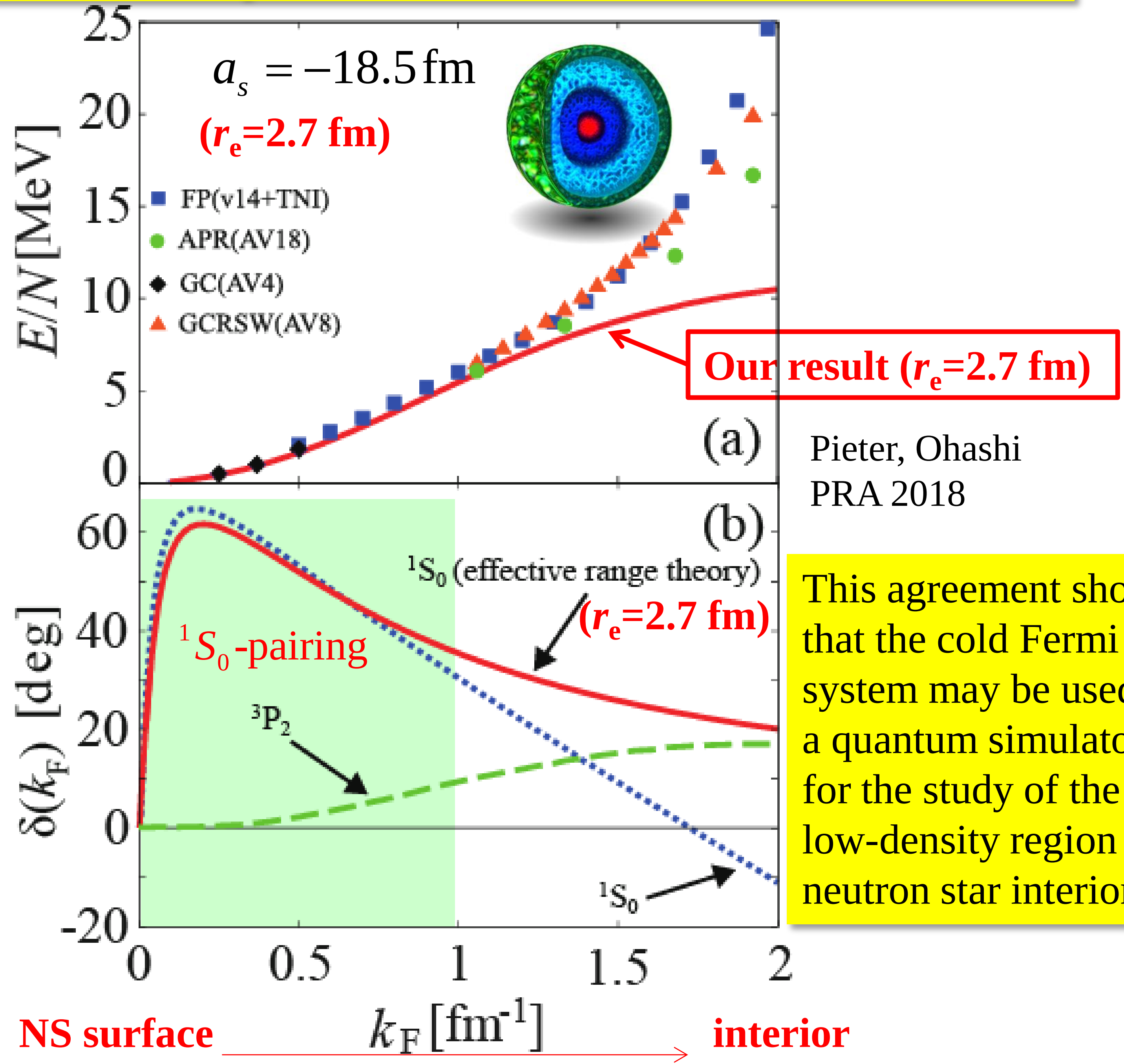
EoS



NS surface → **interior**

Comparison with previous "neutron star calculations"

EoS

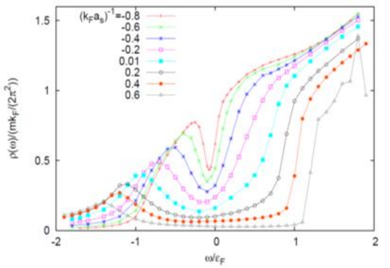


Pieter, Ohashi
PRA 2018

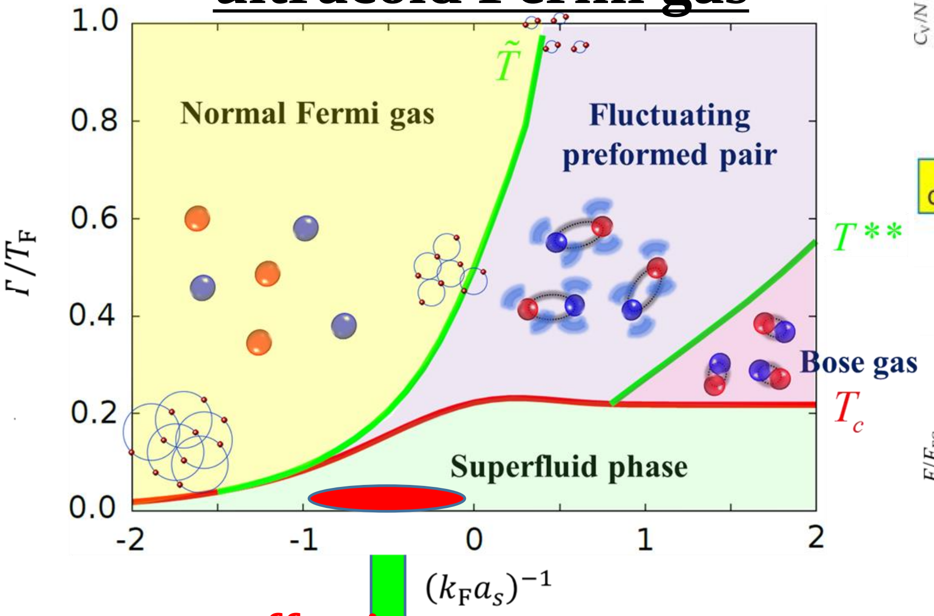
This agreement shows that the cold Fermi gas system may be used as a quantum simulator for the study of the low-density region of neutron star interior.

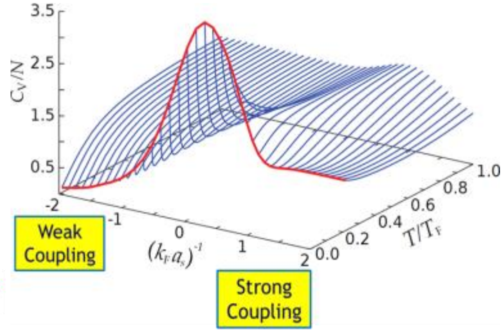
Summary

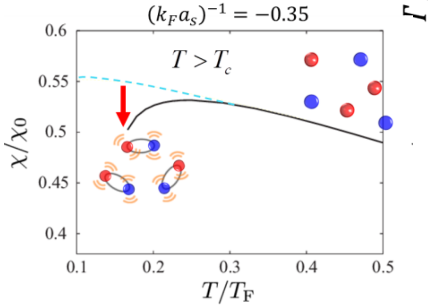
We have discussed strong coupling properties of an ultracold Fermi gas in the BCS-BEC crossover region. We also showed that this atomic system may be used as a quantum simulator for the study of neutron-star physics.

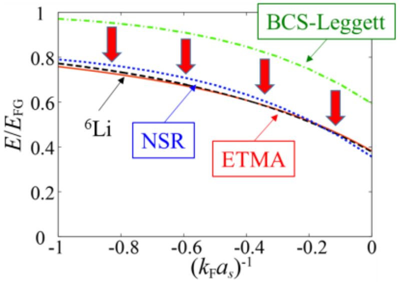


ultracold Fermi gas

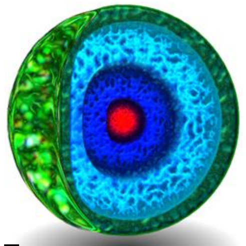








+ effective range



Neutron star