Quantum processes and correlations with no definite causal order

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Based on some joint work with
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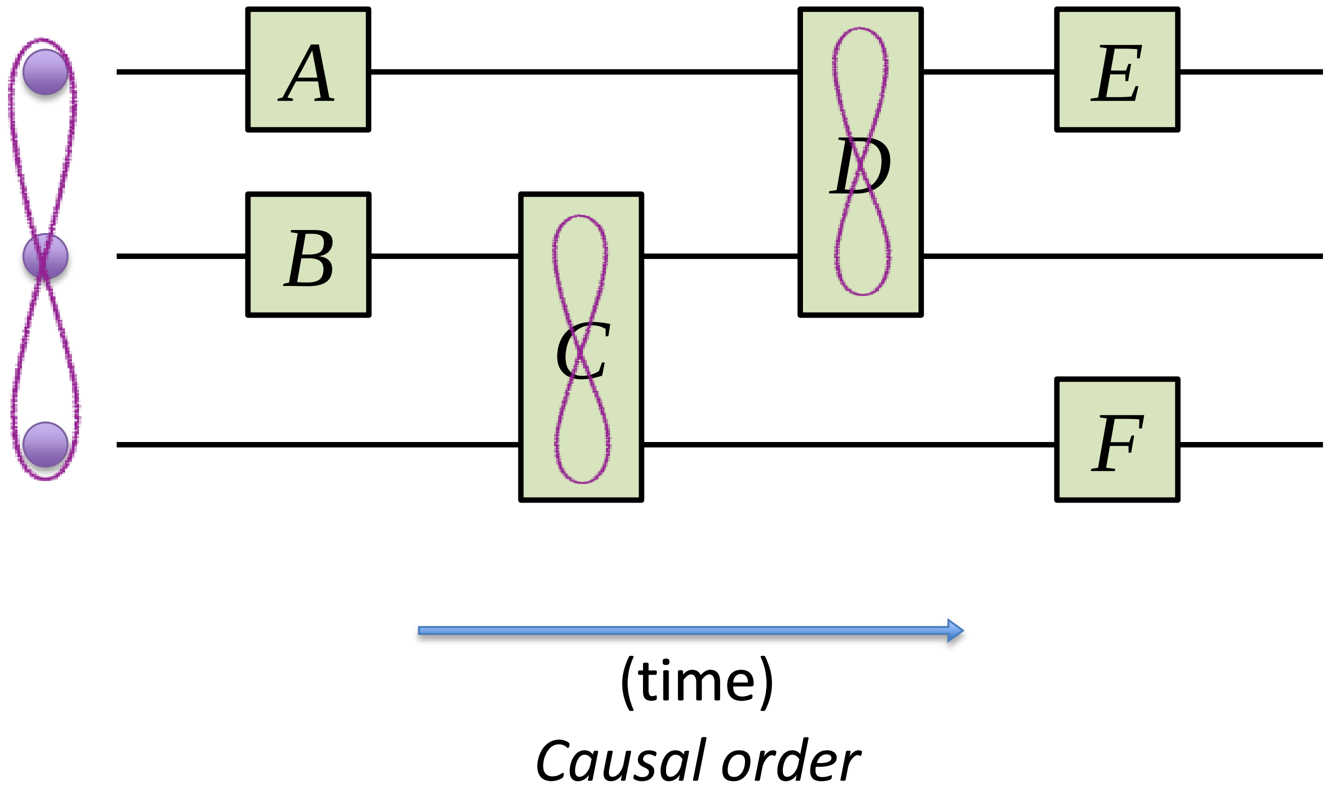
Causal order

Quantum circuit model:

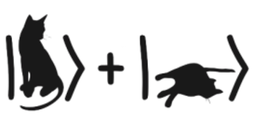


Causal order

Quantum circuit model:

- Typically assumes a definite causal order
- …but does it have to be the case?

|alive> + |dead>



"|A causes B> + |B causes A>"?

Is that something that can be seen/verified in the lab?

New phenomena, **new resource for new applications**?

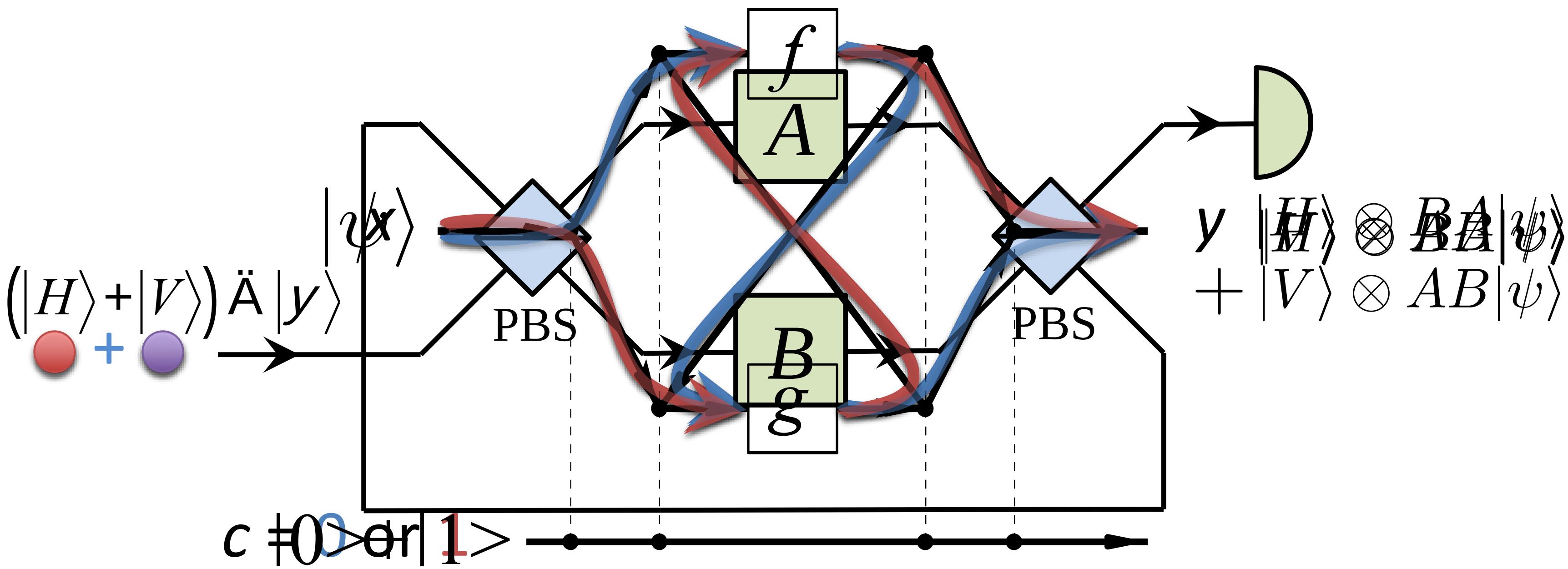
Outline

- Superposing causal orders: the “Quantum Switch”

- The framework of “locally quantum” processes
  - Causally separable vs causally nonseparable processes
  - Violation of causal inequalities
  - Analogy with entanglement and Bell nonlocality

- Definition of characterisation of “noncausal resources” in multipartite scenarios

Superposing causal orders



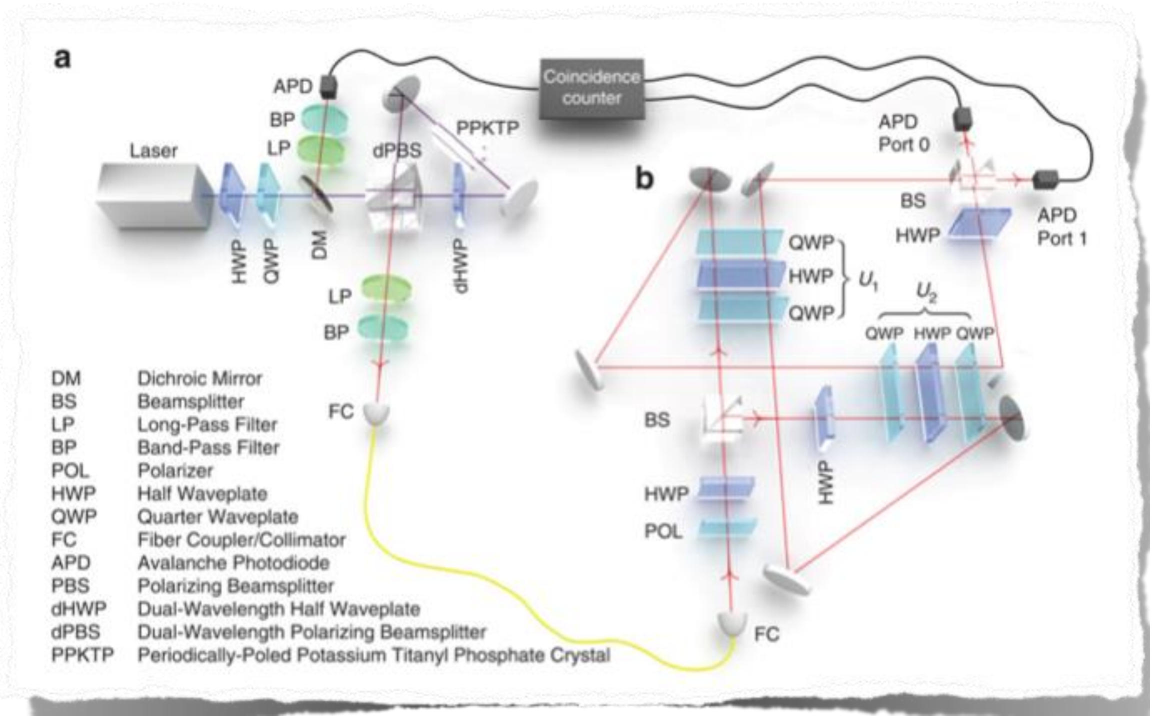
Classical switch:

- If $c = 0$, apply f then g: $y = g \circ f(x)$
- If $c = 1$, apply g then f: $y = f \circ g(x)$

The “Quantum Switch”

[Theory: Chiribella *et al.*, PRA 2013; Araújo *et al.*, PRL 2014;
Experiments: Procopio *et al.*, Nat. Commun. 2015;
Rubino *et al.*, Sci. Adv. 2017]

Superposing causal orders



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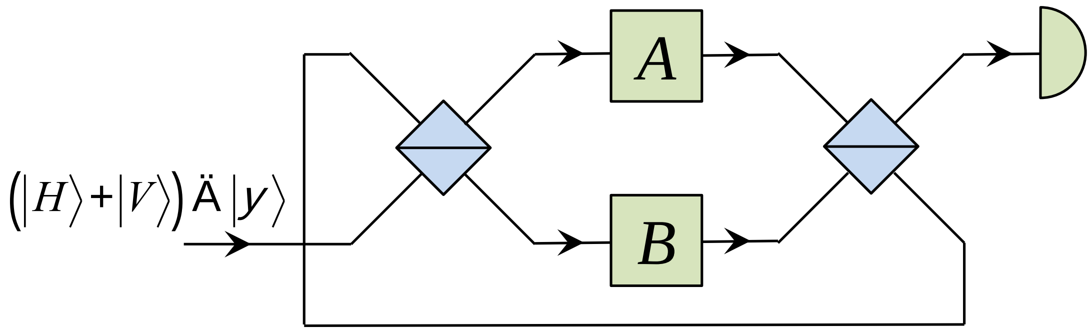
Superposing causal orders

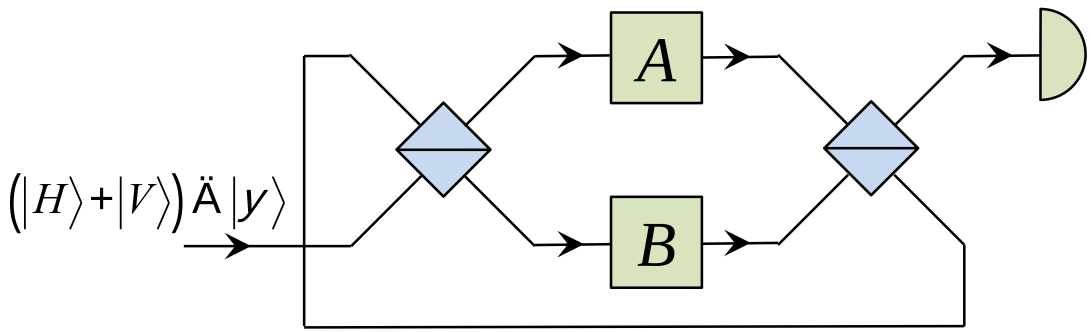


The "Quantum Switch" *does **not** fit* in the standard framework of (causally ordered) quantum circuits

- **A new resource!**

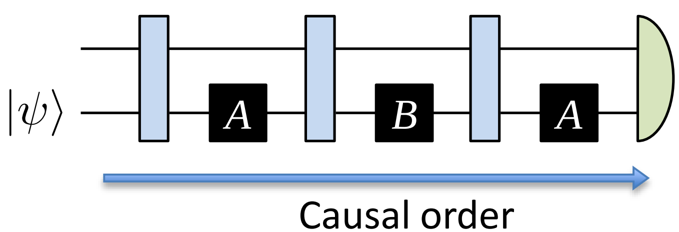
New Frontiers in Quantum Correlations

The Quantum Switch: a new resource



Task: Given A and B (a single copy),
determine whether they commute or anti-commute

- **Cannot** be done in a standard causally ordered quantum circuit

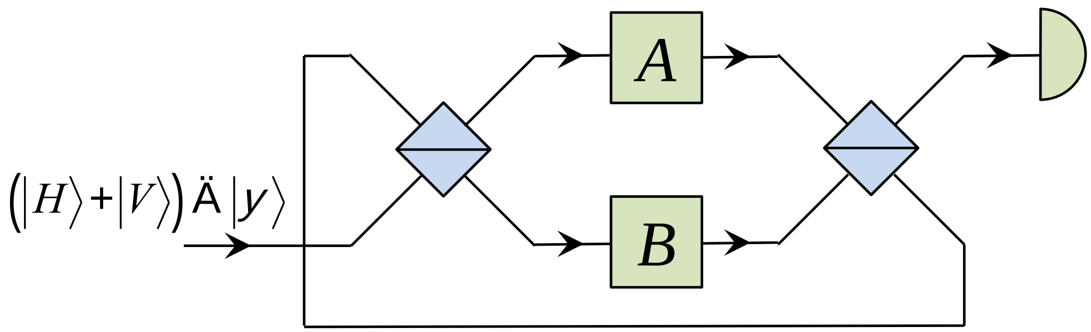


- Can be done **in a single shot** using the quantum switch

(by measuring the photon polarization at the output in the ±45° basis)

[Chiribella, PRA 2012; Procopio *et al.*, Nat. Commun. 2015]

The Quantum Switch: a new resource



- New tasks made possible: e.g. classification problem (commuting vs anti-commuting)

  [Chiribella, PRA 2012]

- Generalization to an *N*-partite classification problem: polynomial advantage

  [Araújo *et al.*, PRL 2014]

- Advantage in communication complexity; can be exponential!

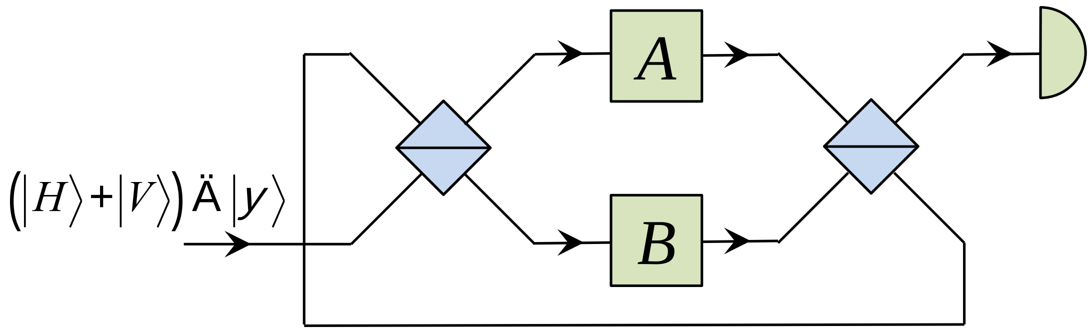
  [Feix *et al.*, PRA 2015; Allard Guerin *et al.*, PRL 2016]

- Enhanced communication

  [Ebler *et al.*, arXiv:1711.10165]

- …?

Superposing causal orders



The “Quantum Switch” *does **not** fit* in the standard framework of (causally ordered) quantum circuits

- **A new resource!**
- We need a new framework, need to change our viewpoint

Outline

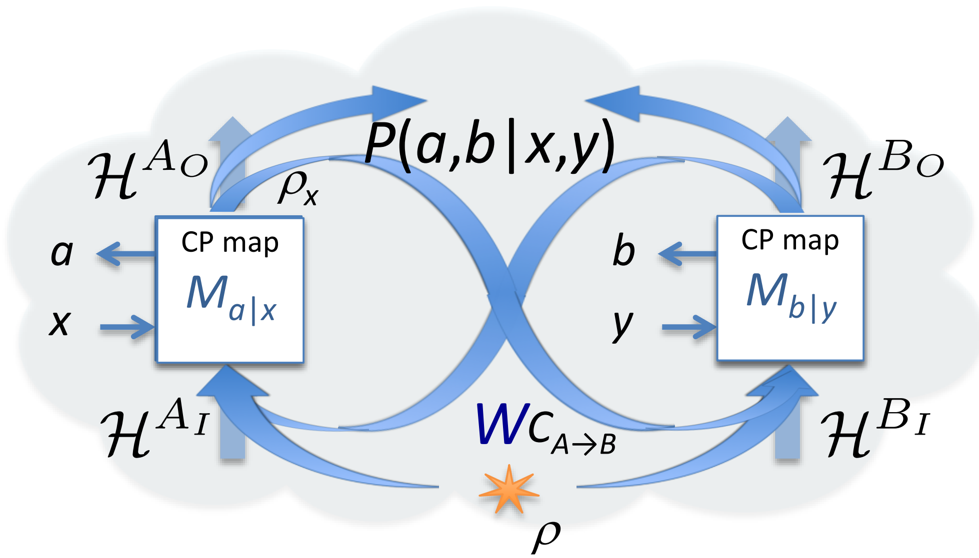
- Superposing causal orders: the "Quantum Switch"

[Oreshkov, Costa, Brukner, Nat. Commun. 2012]

- The framework of "locally quantum" processes
  - Causally separable vs causally nonseparable processes
  - Violation of causal inequalities
  - Analogy with entanglement and Bell nonlocality

- Definition of characterisation of "noncausal resources" in multipartite scenarios

Locally quantum processes



- Assuming "local quantum mechanics" only: CP maps $M_{a|x}$, $M_{b|y}$
  - ➢ Correlations are bilinear functions of Alice and Bob's CP maps

$$P(a,b|x,y) = \mathrm{Tr}[\,(M_{a|x} \otimes M_{b|y}) \bullet W\,]$$

the ***"process matrix"***

$$W \in A_I \otimes A_O \otimes B_I \otimes B_O$$

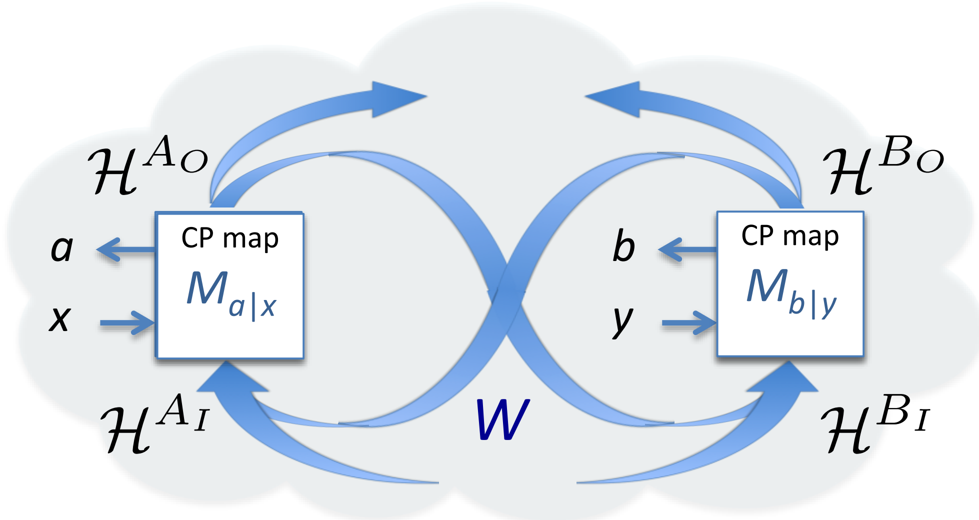
For a quantum state ρ / channel $C_{A\to B}$:

$P(a,b|x,y) = \mathrm{Tr}[(E_{a|x} \otimes E_{b|y}) \bullet \rho]$

$P(b|x,y) = \mathrm{Tr}[(O_{a|x} \otimes I_{b|y}) \bullet C_{A\to B}]$

[Oreshkov, Costa, Brukner, Nat. Commun. 2012]

Locally quantum processes



- Some processes are compatible with a definite causal order
  - E.g. channel *A* → *B* : $W^{A<B} = W^{A_I A_O B_I} \otimes \mathbb{1}^{B_O}$

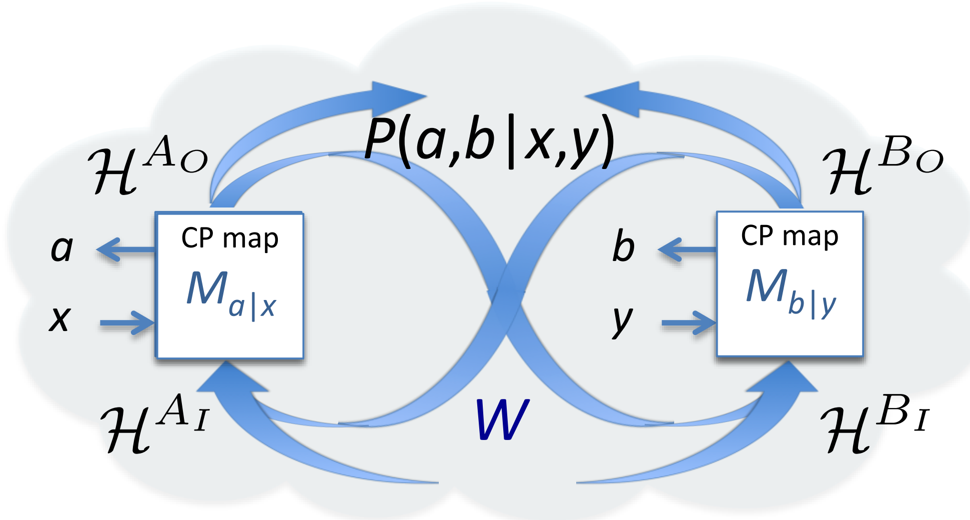
[Gutoski & Watrous, STOC 2006; Chiribella, D'Ariano, Perinotti, PRA 2009]

- **"Causally separable processes":**

$$W^{sep} = qW^{A\prec B} + (1-q)W^{B\prec A}$$
$$= q\,W^{A_I A_O B_I} \otimes \mathbb{1}^{B_O} + (1-q)\,W^{A_I B_I B_O} \otimes \mathbb{1}^{A_O}$$

[Oreshkov, Costa, Brukner, Nat. Commun. 2012]

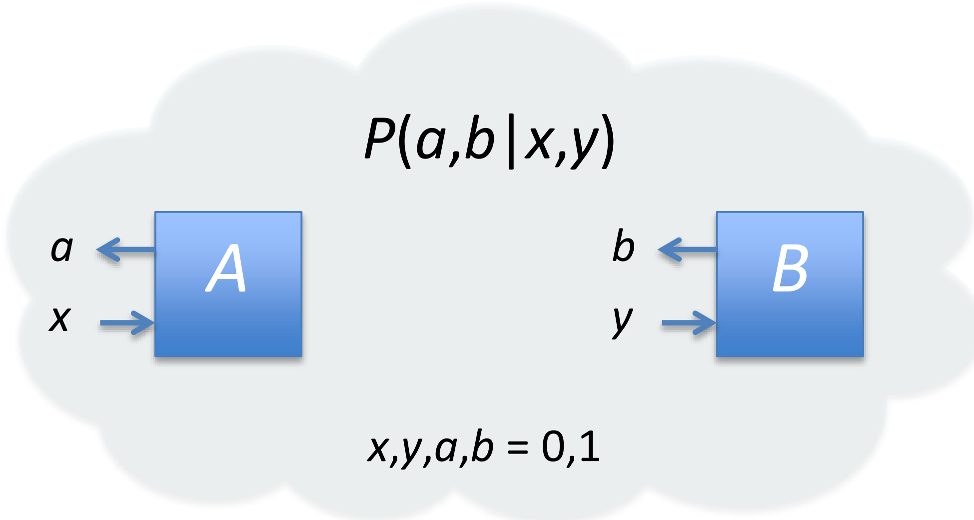
Locally quantum processes



- "Causally separable processes": $W^{sep} = qW^{A\prec B} + (1-q)W^{B\prec A}$
- **"Causally nonseparable processes":** $W^{nsep} \neq qW^{A\prec B} + (1-q)W^{B\prec A}$
  - Processes that are **incompatible with a definite causal order**
  - May generate **correlations *P*(*a*,*b*|*x*,*y*) with no definite causal order**, which **violate "causal inequalities"**

[Oreshkov, Costa, Brukner, Nat. Commun. 2012]

A "causal game"



"Guess you neighbour's input" game: we want $a = y$, $b = x$

Assuming a definite causal order, e.g. A < B:

- Alice can only make a random guess for Bob's input: $P(a=y) = ½$

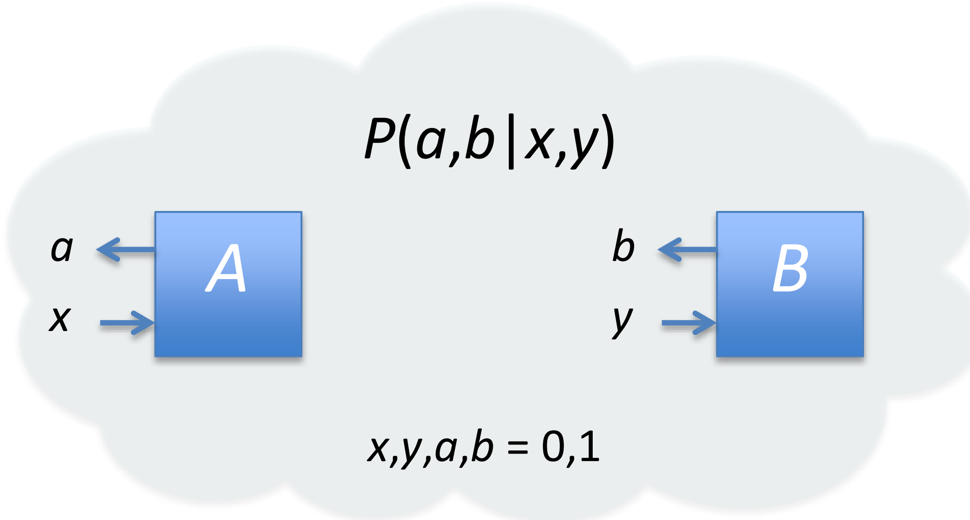
  (while Bob can correctly guess Alice's input: $P(b=x) \leq 1$)

- $p_{succ} = P(a=y,b=x) \leq ½$ → *a "causal inequality"*

Satisfied by all "causal correlations",
of the form $P = q\, P^{A<B} + (1-q)\, P^{B<A}$

[CB *et al.*, NJP 2016]

A “causal game”



“Guess you neighbour’s input” game: we want $a = y$, $b = x$

$$p_{succ} = \mathrm{P}(a{=}y, b{=}x) \;\leq\; ½$$

- Can be violated by process matrix correlations

$$P(a,b|x,y) = \mathrm{Tr}[\,(M_{a|x} \otimes M_{b|y}) \bullet W\,]$$

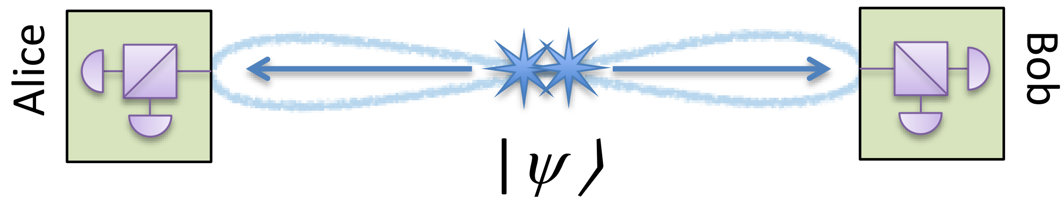
“Noncausal correlations”

[CB *et al.*, NJP 2016]

Analogy with entanglement and Bell nonlocality

Device-dependent

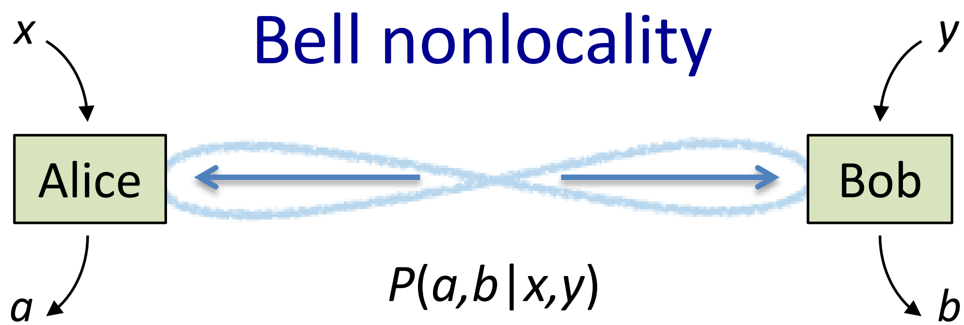
Entanglement



Detected by entanglement witnesses

Device-independent

Bell nonlocality



Violate Bell inequalities

Analogy with entanglement and Bell nonlocality

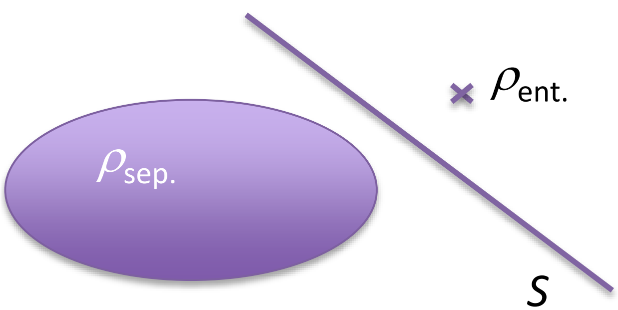
Quantum states *(density matrices ρ)*	⇔	Quantum processes *(process matrices W)*
Entanglement	⇔	Causal Nonseparability
Nonlocal correlations *violating Bell inequalities*	⇔	"Noncausal" correlations *violating "causal inequalities"*

Analogy with entanglement and Bell nonlocality

Quantum states *(density matrices ρ)*	Quantum processes *(process matrices W)*
Entanglement	Causal Nonseparability
Nonlocal correlations *violating Bell inequalities*	"Noncausal" correlations *violating "causal inequalities"*

Can be detected by an *entanglement witness S* :

$\mathrm{Tr}[\, S \,.\, \rho_{\text{ent.}}] < 0$ and $\mathrm{Tr}[\, S \,.\, \rho_{\text{sep.}}] \geq 0$ for all $\rho_{\text{sep.}}$

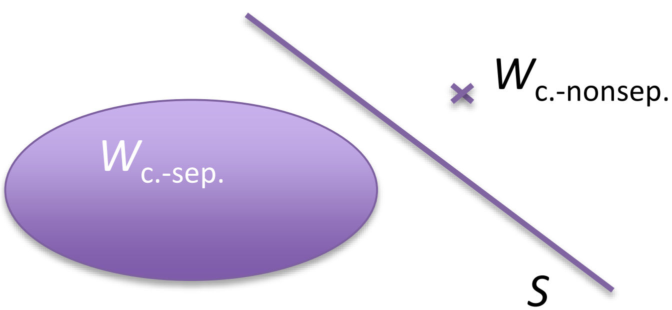


Analogy with entanglement and Bell nonlocality

Quantum states *(density matrices ρ)*	Quantum processes *(process matrices W)*
Entanglement	Causal Nonseparability
Nonlocal correlations *violating Bell inequalities*	"Noncausal" correlations *violating "causal inequalities"*

Can be detected by ***a causal witness*** S :

$\mathrm{Tr}[\, S \,.\, W_{\text{c.-nonsep.}}] < 0$ and $\mathrm{Tr}[\, S \,.\, W_{\text{c.-sep.}}] \geq 0$ for all $W_{\text{c.-sep.}}$

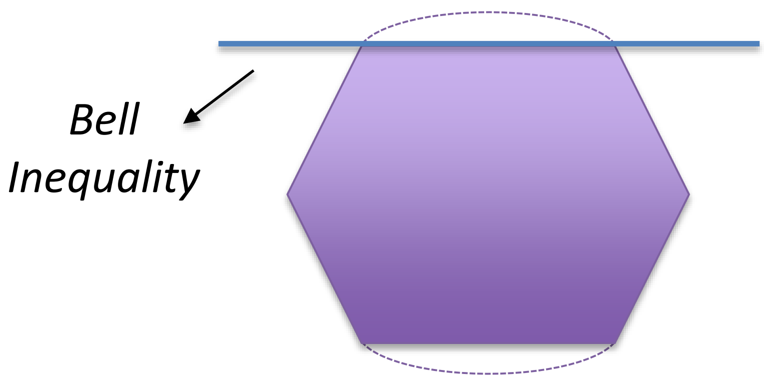


[Araújo, CB *et al.*, NJP 2015; CB, Sci. Rep. 2016]

Analogy with entanglement and Bell nonlocality

Quantum states *(density matrices ρ)*	⟺	Quantum processes *(process matrices W)*
Entanglement	⟺	Causal Nonseparability
Nonlocal correlations *violating Bell inequalities*	⟺	"Noncausal" correlations *violating "causal inequalities"*

Can be characterized geometrically:
local correlations form a convex polytope, the "local polytope"



Analogy with entanglement and Bell nonlocality

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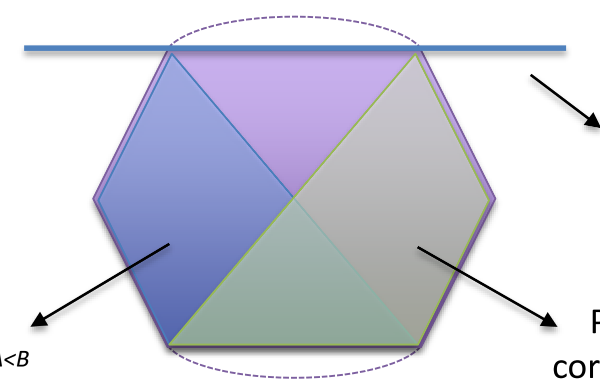
Can be characterized geometrically:
causal correlations form a convex polytope, the **"causal polytope"**

$$P_{causal} = q\, P^{A<B} + (1\text{-}q)\, P^{B<A}$$



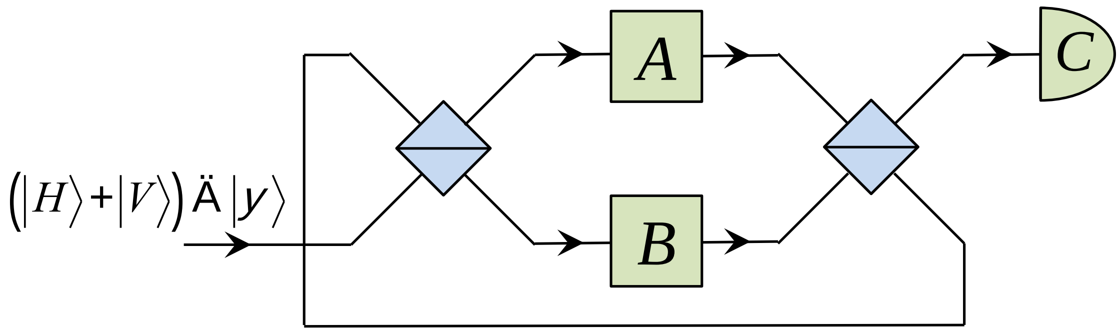
Causal Inequality

Polytope of correlations $P^{A<B}$

Polytope of correlations $P^{B<A}$

[CB *et al.*, NJP 2016]

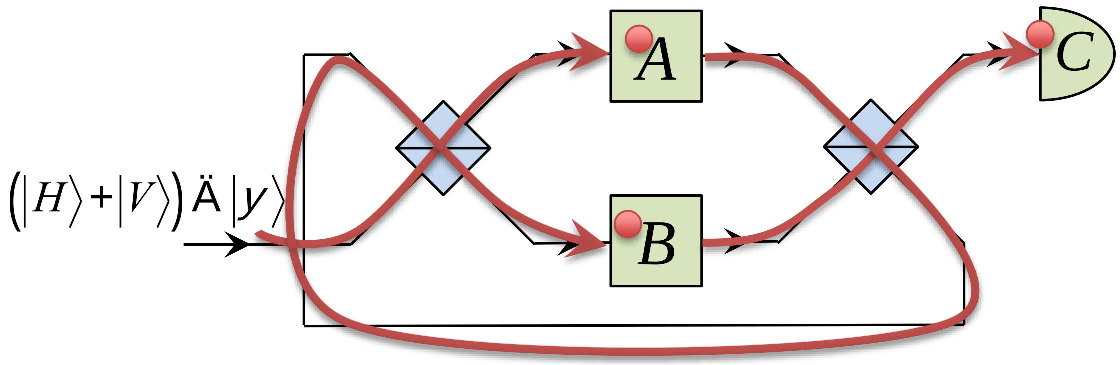
The "Quantum Switch" as a quantum process



$$(|H\rangle + |V\rangle) \otimes |\psi\rangle \to |H\rangle \otimes BA|\psi\rangle + |V\rangle \otimes AB|\psi\rangle$$

- Tracing out the control qubit makes the process an (uninteresting) random mixture of 2 causally ordered processes
  - ➢We should keep it! And give it to a 3rd party, *C*

The "Quantum Switch" as a quantum process



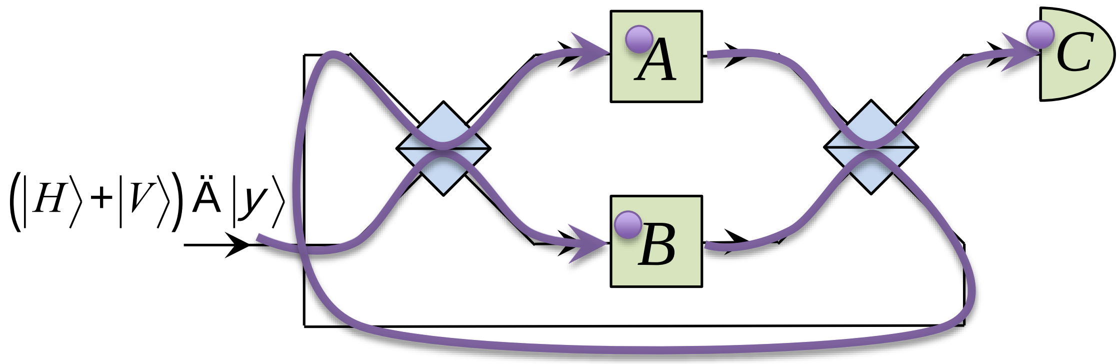
$$(|H\rangle + |V\rangle) \otimes |\psi\rangle \to |H\rangle \otimes BA|\psi\rangle + |V\rangle \otimes AB|\psi\rangle$$

- For the order *A*<*B*: $|\psi\rangle^{A_I} \, |\mathbb{1}\rangle\rangle^{A_O B_I} \, |\mathbb{1}\rangle\rangle^{B_O C_I} \, |H\rangle^{C_I'}$

$|\mathbb{1}\rangle\rangle = |00\rangle + |11\rangle$
(identity channel)

[M. Araújo, CB *et al.*, NJP 2015; O. Oreshkov, C. Giarmatzi, NJP 2016]

The "Quantum Switch" as a quantum process

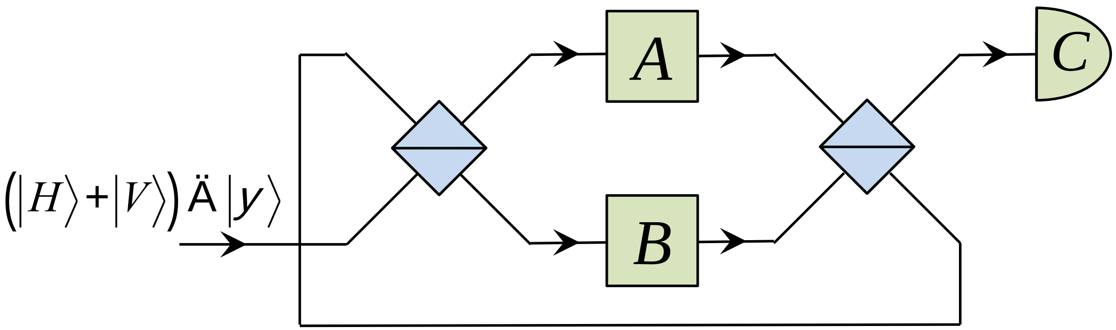


$$(|H\rangle + |V\rangle) \otimes |\psi\rangle \rightarrow |H\rangle \otimes BA|\psi\rangle + |V\rangle \otimes AB|\psi\rangle$$

- For the order *A*<*B*: $|\psi\rangle^{A_I}\, |\mathbb{1}\rangle\rangle^{A_O B_I}\, |\mathbb{1}\rangle\rangle^{B_O C_I}\, |H\rangle^{C_I'}$
- For the order *B*<*A*: $|\psi\rangle^{B_I}\, |\mathbb{1}\rangle\rangle^{B_O A_I}\, |\mathbb{1}\rangle\rangle^{A_O C_I}\, |V\rangle^{C_I'}$

[M. Araújo, CB *et al.*, NJP 2015; O. Oreshkov, C. Giarmatzi, NJP 2016]

The “Quantum Switch” as a quantum process



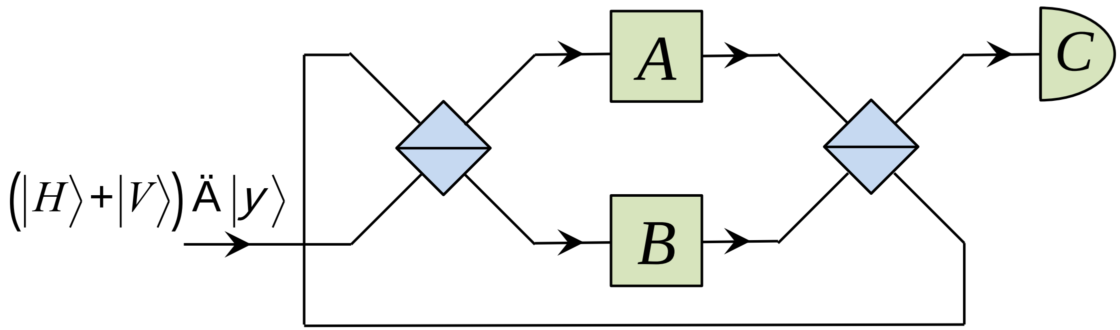
$$(|H\rangle + |V\rangle) \otimes |\psi\rangle \to |H\rangle \otimes BA|\psi\rangle + |V\rangle \otimes AB|\psi\rangle$$

$$\begin{aligned} |w\rangle = \quad & |\psi\rangle^{A_I} \, |\mathbb{1}\rangle\!\rangle^{A_O B_I} \, |\mathbb{1}\rangle\!\rangle^{B_O C_I} \, |H\rangle^{C_I'} \\ + \, & |\psi\rangle^{B_I} \, |\mathbb{1}\rangle\!\rangle^{B_O A_I} \, |\mathbb{1}\rangle\!\rangle^{A_O C_I} \, |V\rangle^{C_I'} \end{aligned}$$

$$W = |w\rangle\langle w|$$

[M. Araújo, CB *et al.*, NJP 2015; O. Oreshkov, C. Giarmatzi, NJP 2016]

The "Quantum Switch" as a quantum process



$$|w\rangle = |\psi\rangle^{A_I} |\mathbb{1}\rangle\rangle^{A_O B_I} |\mathbb{1}\rangle\rangle^{B_O C_I} |H\rangle^{C'_I} + |\psi\rangle^{B_I} |\mathbb{1}\rangle\rangle^{B_O A_I} |\mathbb{1}\rangle\rangle^{A_O C_I} |V\rangle^{C'_I}$$

$$W = |w\rangle\langle w| \neq qW^{A\prec B\prec C} + (1-q)W^{B\prec A\prec C}$$

➢ ***Causally nonseparable***

(→ a causal witness can be constructed and measured experimentally)

[Rubino *et al.*, Sci. Adv. 2017]

- Nevertheless, the quantum switch *cannot violate any causal inequality*

[M. Araújo, CB *et al.*, NJP 2015; O. Oreshkov, C. Giarmatzi, NJP 2016]

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Defining multipartite (non)causal correlations

- (Recall bipartite case:)

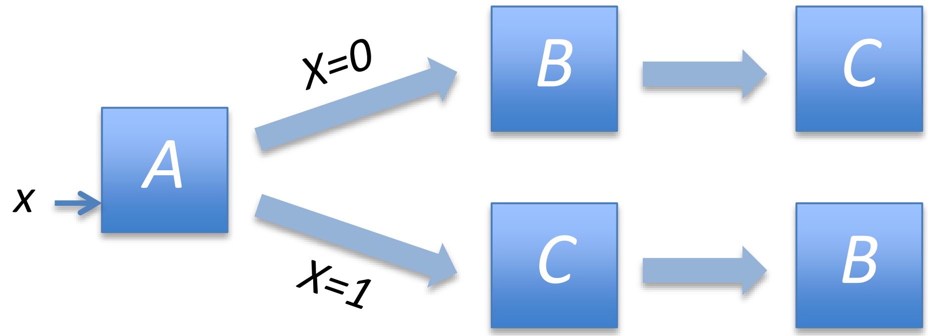
$$P(a,b|x,y) \;=\; q\; P^{A<B}(a,b|x,y) \;+\; (1-q)\; P^{B<A}(a,b|x,y)$$

- Naïve generalisation:

$$P(\vec{a}|\vec{x}) = \sum_{\substack{\pi:\text{permutation} \\ \text{of } \{1,\dots,N\}}} q_\pi \; P^{A_{\pi(1)}<A_{\pi(2)}<\dots<A_{\pi(N)}}(\vec{a}|\vec{x})$$

$$(\text{with } q_\pi \geq 0,\; \sum_\pi q_\pi = 1)$$

➢ Not enough: we want to allow for **adaptive order**



Defining multipartite (non)causal correlations

- (Recall bipartite case:)

$$P(a,b|x,y) \;=\; q\; P^{A<B}(a,b|x,y) \;+\; (1-q)\; P^{B<A}(a,b|x,y)$$

- Even allowing for an adaptive order, there will always be a party coming first (which party this is could be probabilistic)
  - ➢ Recursive definition:
    - Any single-partite probability distribution is causal
    - For $N \geq 2$, a correlation P is **causal iff**

$$P(\vec{a}|\vec{x}) = \sum_{\substack{k\in\mathcal{N} \\ = \{1,\ldots,N\}}} q_k\; P_k(a_k|x_k)\; \underbrace{P_{k,x_k,a_k}(\vec{a}_{\mathcal{N}\backslash k}|\vec{x}_{\mathcal{N}\backslash k})}_{(N\text{-}1)\text{-partite causal correlation}}$$

(N-1)-partite **causal** correlation

[Oreshkov & Giarmatzi, NJP 2016; Abbott *et al.*, PRA 2016]

Characterizing multipartite causal correlations

- Multipartite causal correlations form a convex polytope
  - Fully characterised in the simplest tripartite case [Abbott *et al.*, PRA 2016]
- Vertices correspond to deterministic causal strategies, possibly with an adaptive causal order
- Facets then define causal inequalities for multipartite causal correlations

[Oreshkov & Giarmatzi, NJP 2016; Abbott *et al.*, PRA 2016]

Multipartite causally (non)separable processes

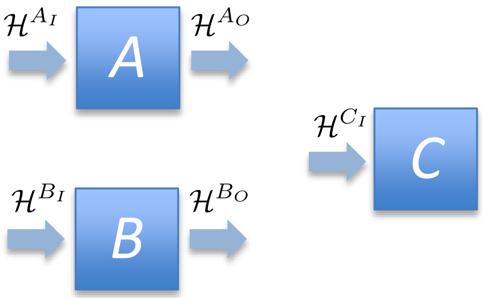
- (Recall bipartite case:)

$$W^{sep} = q\, W^{A<B} + (1-q)\, W^{B<A}$$

- In the particular tripartite scenario of the quantum switch, where one party (*C*) has no outgoing system:

$$W^{sep} = q\, W^{A<B<C} + (1-q)\, W^{B<A<C}$$

[Araújo *et al.*, NJP 2015]



- Simultaneously, [Oreshkov & Giarmatzi, NJP 2016] considered the fully general multipartite case, and gave another definition for causally (non)separable processes, inspired by the previous definition of causal correlations

Oreshkov & Giarmatzi's causal (non)separability

[Oreshkov & Giarmatzi, NJP 2016]

- Recall recursive definition for causal correlations:
  - Any single-partite probability distribution is causal
  - For $N \geq 2$, P causal iff $P(\vec{a}|\vec{x}) = \sum_{k \in \mathcal{N}} q_k \; P_k(a_k|x_k) \; \underbrace{P_{k,x_k,a_k}(\vec{a}_{\mathcal{N}\setminus k}|\vec{x}_{\mathcal{N}\setminus k})}$

    (N-1)-partite causal correlation

- Oreshkov & Giarmatzi's causal separability (OG-CS):
  - Any single-partite process is causally separable
  - For $N \geq 2$, W is **causally separable iff** $W = \sum_k q_k \; \underbrace{W_k}$

    Valid process compatible with party A_k first, such that the (N-1)-partite conditional process

    $$W_{M_k} := \mathrm{Tr}_k[M_k \otimes \mathbb{1}^{\mathcal{N}\setminus k} \cdot W]$$

    is **causally separable** for all CP maps M_k

Oreshkov & Giarmatzi's causal (non)separability

[Oreshkov & Giarmatzi, NJP 2016]

- Oreshkov & Giarmatzi's causal separability (OG-CS):
  - Any single-partite process is causally separable
  - For $N \geq 2$, W is **causally separable iff** $W = \sum_k q_k \underbrace{W_k}$

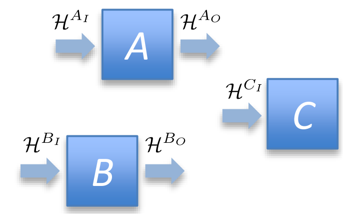
    Valid process compatible with party A_k first, such that the (N-1)-partite conditional process W_{Mk} is **c.-sep.** for all M_k

- Oreshkov & Giarmatzi's "extensible causal separability" (OG-ECS):
  - W is **extensibly causally separable iff** $W \otimes \rho$ is causally separable for all ρ

- OG-CS ≠ OG-ECS: "activation of non-causality"

2 definitions of causal (non)separability

- In the particular tripartite case where *C* has no outgoing system:



$$W^{sep} = q\, W^{A<B<C} + (1-q)\, W^{B<A<C}$$

[Araújo *et al.*, NJP 2015]

vs OG-CS / OG-ECS

[Oreshkov & Giarmatzi, NJP 2016]

- Are the 2 definitions equivalent?

➢ **Araújo *et al.*'s definition ⇔ OG-ECS**

⇔ (crossed out) **OG-CS**

[Wechs *et al.*, in prep.]

Is the most natural definition!

Characterising multipartite causal (non)separability

- (Recall bipartite case:) $W^{sep} = q\, W^{A<B} + (1-q)\, W^{B<A}$

- Tripartite case: [Oreshkov & Giarmatzi, NJP 2016; Wechs *et al.*, in prep.]

Valid process compatible with *A* first (up to norm.)

$$W^{sep} = \underbrace{W^{A}}_{} + \underbrace{W^{B}}_{} + \underbrace{W^{C}}_{}$$

$$= \tilde{W}^{ABC} + \tilde{W}^{ACB} + \tilde{W}^{BAC} + \tilde{W}^{BCA} + \tilde{W}^{CAB} + \tilde{W}^{CBA}$$

Not necessarily a valid process;
But such that for any CP map M_A the conditional process $\tilde{W}^{ABC}_{M_A} = \mathrm{Tr}_A[M_A \otimes \mathbb{1}^{BC} \cdot \tilde{W}^{ABC}]$
is a valid bipartite process compatible with *B* first

- ➢ Not just a convex combination of processes!
  - ➢ Allows for adaptive causal order

Characterising multipartite causal (non)separability

- Tripartite case: [Oreshkov & Giarmatzi, NJP 2016]

$$W^{sep} = \underbrace{W^A}_{} + \underbrace{W^B}_{} + \underbrace{W^C}_{}$$
$$= \tilde{W}^{ABC} + \tilde{W}^{ACB} + \tilde{W}^{BAC} + \tilde{W}^{BCA} + \tilde{W}^{CAB} + \tilde{W}^{CBA}$$

- Generalisation to 4 parties and more?

[O. Oreshkov (private communication); Wechs *et al.*, in prep.]

➢ Sufficient condition:

Valid process compatible with *A* first

$$W^{sep} = W^A + W^B + W^C + W^D$$
$$= \tilde{W}^{AB} + \tilde{W}^{AC} + \tilde{W}^{AD} + \ldots + \ldots + \ldots$$
$$= \tilde{W}^{ABCD} + \tilde{W}^{ABDC} + \tilde{W}^{ACBD} + \tilde{W}^{ACDB} + \tilde{W}^{ADBC} + \tilde{W}^{ADCB} + \ldots + \ldots + \ldots$$

For any CP map M_A, conditional process $\tilde{W}^{AB}_{M_A}$ is valid, compatible with *B* first

For any CP maps M_A, M_B, conditional process $\tilde{W}^{ABCD}_{M_A \otimes M_B}$ is valid, compatible with *C* first

Conclusion – Outlook

- Quantum theory allows for processes with no definite causal order:
  "Causally nonseparable processes"
- The "process matrix formalism" appears to be well suited to analyse such situations beyond causally ordered quantum circuits
- Rich analogy with entanglement and Bell nonlocality, to be exploited further
- A concrete example: the quantum switch
  - Can be realised experimentally; one can verify its causal nonseparability
  - But it does not violate any causal inequality;
    still an open question, whether any physical process can
- Extension of the framework to multipartite scenarios
  - Also to "genuinely multipartite non-classical correlations"

[Abbott *et al.*, Quantum **1**, 39 (2017)]

Conclusion – Outlook

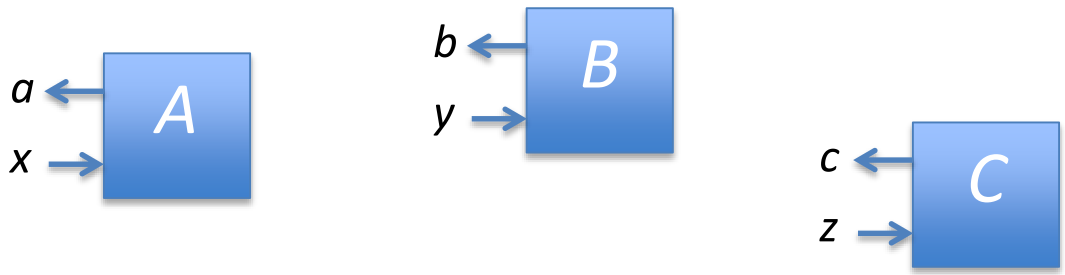
- Quantum theory allows for processes with no definite causal order:
  "Causally nonseparable processes"
- Need to properly characterise what can and cannot be done with QM
- New applications made possible; new applications to be discovered…

→ A new resource for QIP

Understanding precisely how quantum processes
defy the classical notion of causality should help us discover new applications

Thank you for your attention

Example: the simplest tripartite scenario



- 3 parties
- Binary inputs *x*,*y*,z = 0,1
- Fixed outputs *a*,*b*,c = 0 for inputs *x*,*y*,z = 0;
  Binary outputs *a*,*b*,c = 0,1 for inputs *x*,*y*,z = 1

➢ Correlation space is 19-dimensional

➢ Causal polytope has 680 vertices
(488 compatible with a fixed order, 192 requiring a dynamical order),

13 074 facets, defining 305 inequivalent families of causal ineqs (incl. 3 trivial ones)

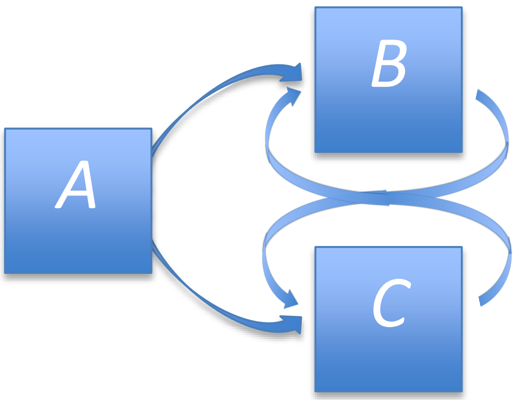
➢ All nontrivial causal inequalities can be violated by *W* correlations
(all except 18 by *classical processes*; algebraic max obtained for 65 families)

[Baumeler &Wolf, ISIT 2014]

[Abbott *et al.*, PRA 2016]

Tripartite (non)causal correlations

Consider $P(abc|xyz) = P(a|x)\ \underbrace{P_{x,a}(bc|yz)}_{\text{noncausal}}$

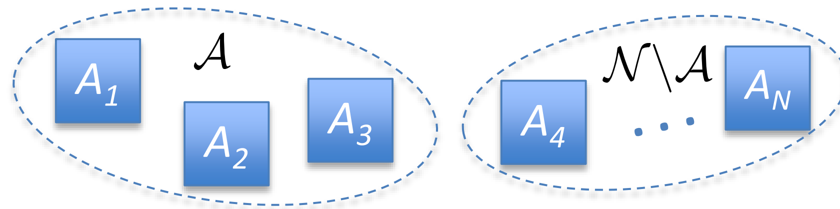


- Such a correlation is compatible with the causal order $A < (B,C)$: there is some “partial”, effectively “bipartite causal” order
  - The noncausality of *P* only concerns *A* and *C*, it is not really a tripartite phenomenon

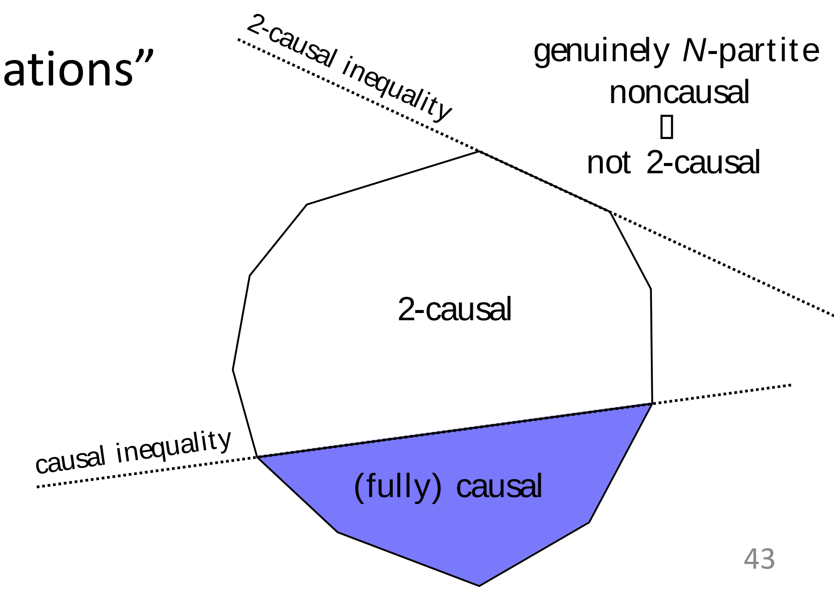
“Genuinely *N*-partite noncausal correlations”

- “Genuinely *N*-partite noncausal correlations”:
  no subset of parties can have a definite causal relation to any other subset
  - ➢ Correlations that **cannot** be decomposed as

$$P(\vec{a}|\vec{x}) = \sum_{\emptyset \subsetneq \mathcal{A} \subsetneq \mathcal{P}} q_{\mathcal{A}} \; P_{\mathcal{A}}(\vec{a}_{\mathcal{A}}|\vec{x}_{\mathcal{A}}) \; P_{\vec{x}_{\mathcal{A}},\vec{a}_{\mathcal{A}}}(\vec{a}_{\mathcal{N}\backslash\mathcal{A}}|\vec{x}_{\mathcal{N}\backslash\mathcal{A}})$$

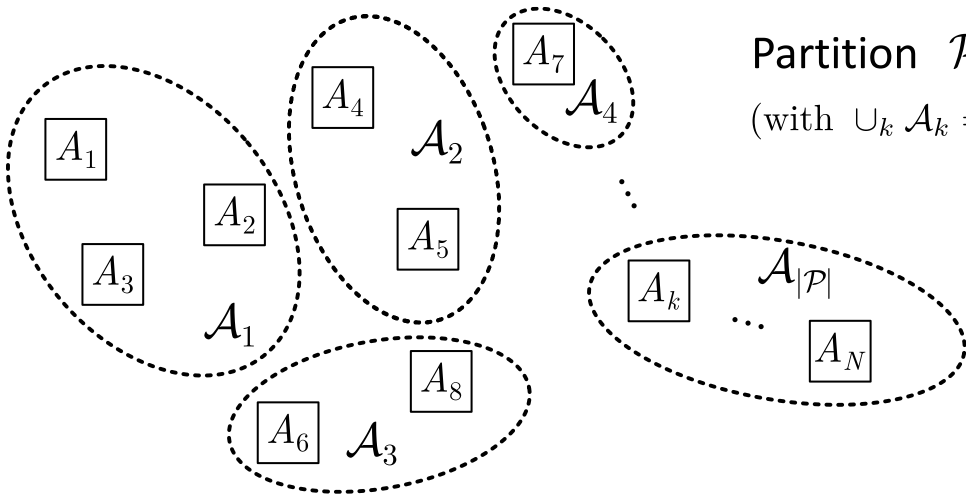


- Non-“genuinely *N*-partite noncausal correlations” form a convex polytope
  - ➢ In the simplest (“lazy”) tripartite case:
  - ▪ Dim. = 19, 1 520 vertices, 21 154 facets, 480 nonequivalent families of inequalities (incl. 3 trivial ones), only 2 nontrivial ones common with the “just-causal” polytope



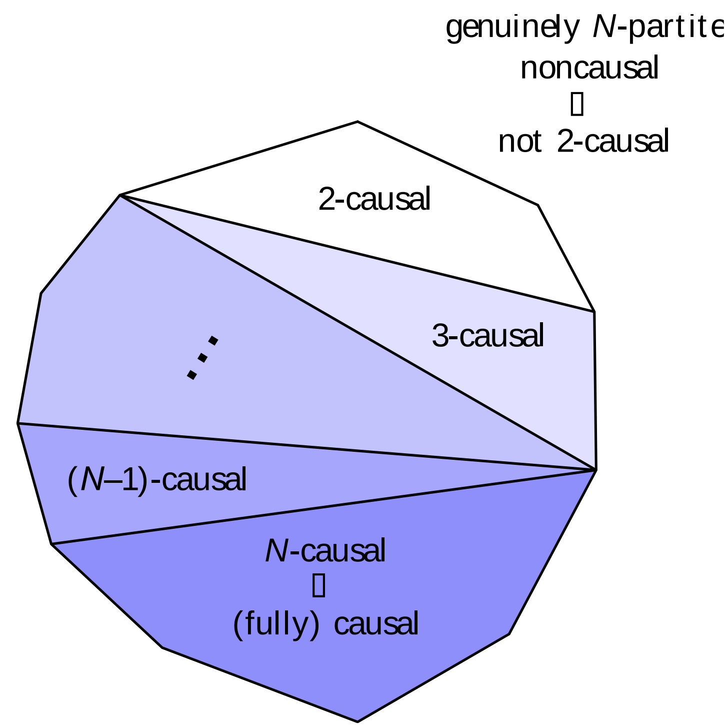
[Abbott *et al.*, Quantum **1**, 39 (2017)]

Refining the definition



Partition $\mathcal{P} = \{\mathcal{A}_1, \ldots, \mathcal{A}_M\}$

(with $\cup_k \mathcal{A}_k = \{1, \ldots, N\}$, $\mathcal{A}_i \cap \mathcal{A}_j = \emptyset$ for $i \neq j$)



- Depending on the number or size of the groups, defines a hierarchy of correlations, with e.g. *M*-causal ⇒ *M'*-causal if *M* ≥ *M'*

[Abbott *et al.*, Quantum **1**, 39 (2017)]