An almost convincing scheme for discriminating the preparation basis of quantum ensemble and why it will not work

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$$\sum_n W_{nm} W^*_{nm'} = \delta_{mm'} \equiv W^\dagger W = \mathbb{1}$$

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- After the measurement the quantum system lose all the information about the state $|\psi\rangle$.

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- For example, we start with a system in state $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$ and an ancillary system in state $|0\rangle$.
- The joint state of the system and ancilla is $|\psi\rangle |0\rangle$.

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$$U\left(|\psi\rangle\,|0\rangle \right) = \left(\sqrt{p_0}\alpha\,|0\rangle + \sqrt{1-p_0}\beta\,|1\rangle\right)|0\rangle + \left(\sqrt{1-p_0}\alpha\,|0\rangle + \sqrt{p_0}\beta\,|1\rangle\right)|1\rangle\,.$$

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$$M_+ = \begin{pmatrix} \sqrt{p_0} & 0 \\ 0 & \sqrt{1-p_0} \end{pmatrix}, \quad M_- = \begin{pmatrix} \sqrt{1-p_0} & 0 \\ 0 & \sqrt{p_0} \end{pmatrix}.$$

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$$|\psi\rangle \rightarrow \begin{cases} M_+ |\psi\rangle \text{ with } p_+ = \langle\psi| M_+^\dagger M_+ |\psi\rangle \\ M_- |\psi\rangle \text{ with } p_- = \langle\psi| M_-^\dagger M_- |\psi\rangle \end{cases}$$

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- Since, $p_+ + p_- = 1$ for all the states $|\psi\rangle$ we have $M_+^\dagger M_+ + M_-^\dagger M_- = \mathbb{1}$
- The expectation value of the operator σ_z is proportional to the probabilities p_+ and p_-, i.e.,

$$\langle\sigma_z\rangle \propto p_+ - p_-$$

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$$M_+ M_- = M_- M_+ = \sqrt{p_0(1-p_0)}\mathbb{1}$$

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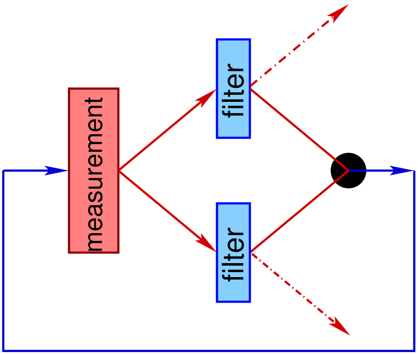

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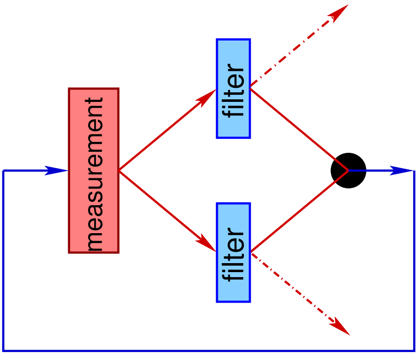
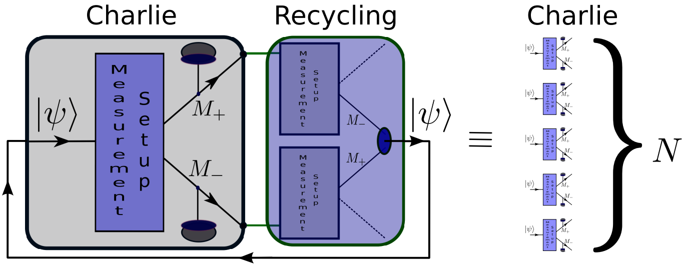


$$p_{rev} = 2p_0(1 - p_0)$$

Unsharp measurements



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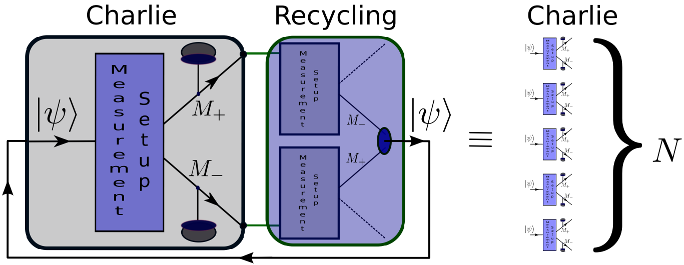

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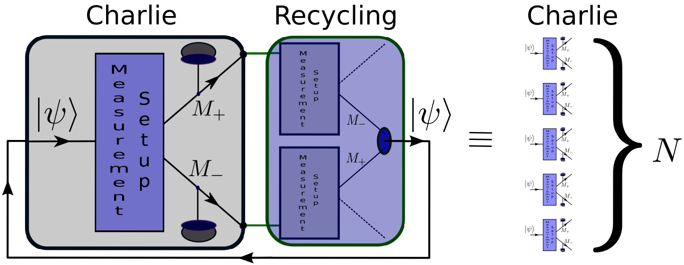

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$$n = (N_+ - N_-)/N$$

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For finite N the value of n will fall on a Gaussian curve centered around the expected value of n with a width proportional to $1/\sqrt{N}$

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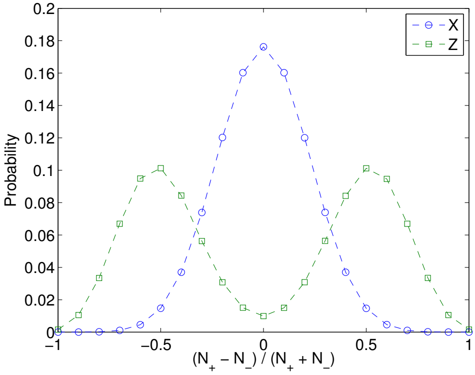

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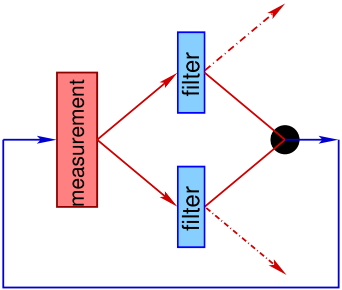

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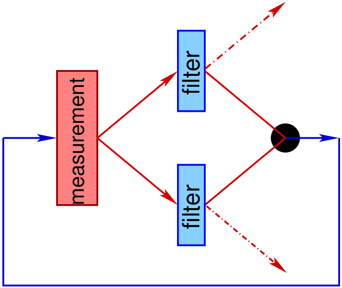

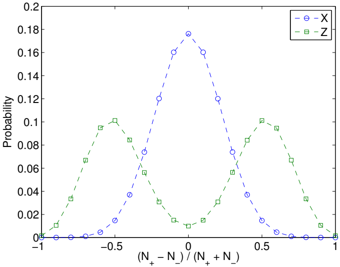

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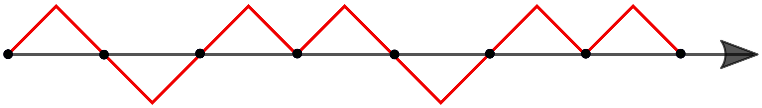

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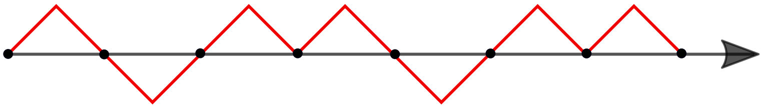

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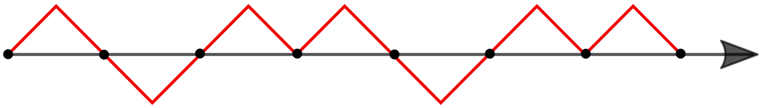

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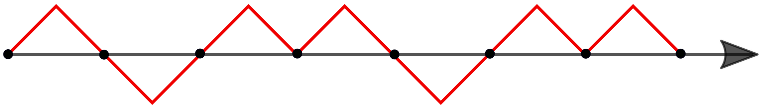

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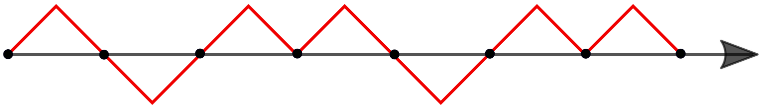

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- Thus, the trajectories which have $N_+ = N_-$ dominates the statistics

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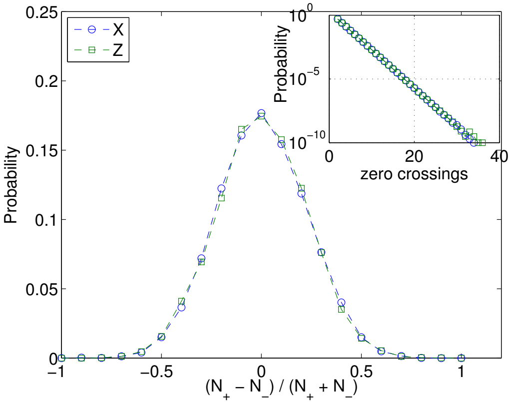

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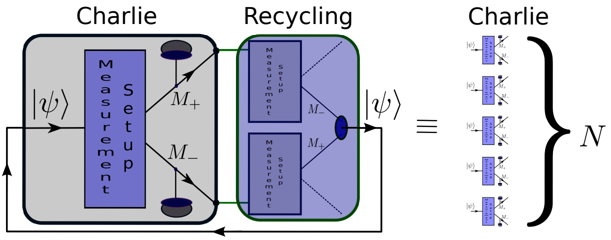

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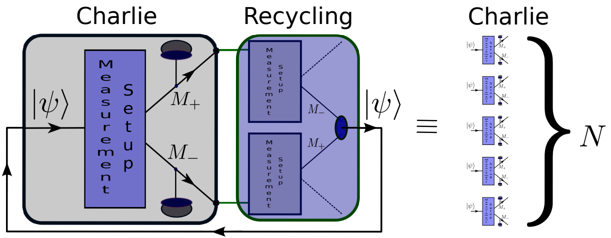

- However, the two ways of preparing the states result in entirely different outcomes

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Even though the systems and the states were identical in both the cases.

Reference

"How measurement reversal could erroneously suggest the capability to discriminate the preparation basis of a quantum ensemble"
Sandeep K. Goyal, Rajeev Singh, and Sibasish Ghosh
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