Open Quantum Systems and Quantum Information in Relativistic and Sub-atomic Systems

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- Here we make use of ideas of quantum information and open quantum systems to study aspects of Unruh effect, neutrino oscillations and correlated neutral mesons.
- We will talk about the Unruh effect as well as correlated neutral mesons which are copiously produced at the high energy frontier experiments, for example, the Large Hadron Collider (LHC) at CERN, from the perspective of open quantum systems .
- Further, we will discuss some aspects of neutrino oscillations from the perspective of quantum information.

Unruh Effect as a Quantum Noise Channel: Motivation

- Make use of the tools of quantum information theory to shed light on the Unruh effect. A modal qubit appears as if subjected to quantum noise that degrades quantum information, as observed in the accelerated reference frame [S. Omkar, SB, R. Srikanth, A. K. Alok (2015)].
- Study various facets of quantum correlations, such as, Bell inequality violations, entanglement, teleportation and measurement-induced disturbance under the effect.
- Unruh effect experienced by a mode of a free Dirac field, as seen by a relativistically accelerated observer, is treated as a noise channel, which we term the "Unruh channel".
- Characterize this channel by providing its operator-sum representation. Compare and contrast this channel from conventional noise due to environmental decoherence.

Unruh effect

The relativistic effect named after Unruh [Davies (1975), Unruh (1976), Takagi (1986)] predicts that the Minkowski vacuum as seen by an observer accelerating with constant proper acceleration a will appear as a warm gas at the *Unruh temperature*:

$$\tau = \frac{\hbar a}{2\pi k_B c},$$

where c is the speed of light in vacuum, and k_B is Boltzmann's constant.

- The Unruh effect produces a decoherence-like effect, earning it the moniker 'Unruh channel'.
- The Unruh effect is usually studied by examining the Minkowski (flat) spacetime in terms of Rindler coordinates [Rindler (1966)].
- The Rindler transformation divides spacetime into two causally disconnected wedges, such that, a uniformily accelerated observer in one wedge is causally separated from the other wedge.

- The fields in question, scalar or Dirac, are quantized and expressed in terms of linear combinations of creation and annihilation operators, for both the Minkowski and Rindler spacetimes.
- Quantization leads to the concepts of particles in either spacetime.
- The annihilation operator of particles in one space time, say for example Minkowski spacetime, can be expressed in terms of creation and annihilation operators of particles in the Rindler spacetime. Thus we see that the Minkowski vacuum is different from the Rindler vacuum and they have different Fock spaces.
- The linear coefficients of the above transformations are the so called Bogoliubov transformations [Bogoliubov (1947), Fulling (1973)], relating the two Fock spaces.
- Consider two observers, Alice (A) and Rob (R) sharing a maximally entangled state of two Dirac field modes (and thus a qubit fermionic Unruh channel), at a point in Minkowski spacetime, of the form

$$|\psi\rangle_{A,R} = \frac{|00\rangle_{A,R} + |11\rangle_{A,R}}{\sqrt{2}},$$

where $|j\rangle$ denote Fock states.

- Let Rob move away from stationary Alice with a uniform proper acceleration a. The effect of constant proper acceleration is described by a Rindler spacetime, which manifests two causally disconnected regions I and II, where region I is accessible to Rob, and separated from region II by an event horizon.
- From Rob's frame the Minkowski vacuum state is a two-mode squeezed state, while the excited state appears as a product state [Alsing et al. (2006)]

$$\begin{aligned} |0\rangle_M &\equiv \cos r|0\rangle_I|0\rangle_{II} + \sin r|1\rangle_I|1\rangle_{II}, \\ |1\rangle_M &\equiv |1\rangle_I|0\rangle_{II}, \end{aligned}$$

  where ω is a Dirac particle frequency while $\cos r = \frac{1}{\sqrt{e^{-\frac{2\pi\omega c}{a}}+1}}$ is one of the Bogoliubov coefficients, connecting the Minkowski and Rindler vacua. It follows that $\cos r \in [\frac{1}{\sqrt{2}}, 1]$ as a ranges from ∞ to 0.
- Tracing out mode II, we obtain the density matrix:

$$\rho'_{A,R} = \frac{1}{2}\left[\cos^2(r)|00\rangle\langle 00| + \cos r(|00\rangle\langle 11| + |11\rangle\langle 00|) + \sin^2(r)|01\rangle\langle 01| + |11\rangle\langle 11|\right],$$

- The 'evolution' of Rob's qubit to a mixed state under the transformation $\mathcal{E}_U : \rho_R \to \rho'_R$ constitutes what we call the Unruh channel for a fermionic qubit.

Quantum Correlations

Quantum correlations are a *many-faceted* entity.

Bell's inequality

- Given a pair of qubits in the state ρ, the elements of correlation matrix T are $T_{mn} = Tr\left[\rho(\sigma_m \otimes \sigma_n)\right]$. If u_i $(i = 1, 2, 3)$ are the eigenvalues of the matrix $T^\dagger T$ then the Bell-CHSH inequality can be written $B(\rho) < 1$ [Horodecki (1995,1996)], where $B(\rho) = \max(u_i + u_j)$ $(i \neq j)$.
- For $\rho'_{A,R}$, $\Gamma^\dagger\Gamma = \mathrm{Diag}(\cos^2 r, \cos^2 r, \cos^4 r)$ and hence $B(\rho) = 2\cos^2 r$.
- In the Figure, the quantity $\frac{B(\rho)}{2}$ (which indicates nonlocality if greater than $\frac{1}{2}$) is plotted as a function of Unruh acceleration. There it is seen that the state perceived by Rob becomes local around $a = 4.6$.

Teleportation Fidelity

- Teleportation provides an operational meaning to entanglement, whenever $F_{\max} > 2/3$, teleportation is possible.
- $F_{\max}$ is computed in terms of the eigenvalues $\{u_i\}$ of $T^\dagger T$.
- $F_{\max} = \frac{1}{2}\left(1 + \frac{1}{3}N(\rho)\right)$ where $N(\rho) = \sqrt{u_1} + \sqrt{u_2} + \sqrt{u_3}$ [Horodecki et. al. (1996)].
- Here $F_{\max} = \frac{1}{2}\left(1 + \frac{1}{3}\left(2\cos r + \cos^2 r\right)\right)$

An inequality involving $B(\rho)$ and $F_{\max}$

$$F_{max} \geq \frac{1}{2}\left(1 + \frac{1}{3}B(\rho)\right) \geq \frac{2}{3} \text{ if } B(\rho) > 1.$$

Concurrence

For a mixed state ρ of two qubits, the concurrence, which is a measure of entanglement, is $C = \max(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0)$. λ_i are the square root of the eigenvalues, in decreasing order, of the matrix $\rho^{\frac{1}{2}}(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)\rho^{\frac{1}{2}}$ where ρ^* is computed in the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ [Wootters (1996)].
For a two-qubit system, concurrence is equivalent to the entanglement of formation which can then be expressed as a monotonic function of concurrence C as
$E_F = -\frac{1+\sqrt{1-C^2}}{2}\log_2(\frac{1+\sqrt{1-C^2}}{2}) - \frac{1-\sqrt{1-C^2}}{2}\log_2(\frac{1-\sqrt{1-C^2}}{2})$.
$C > 0$ asymptotically in Figure 1.

Geometric discord

For the case of two qubits, geometric discord [Dakic et. al. (2010)] is $D_G(\rho) = \frac{1}{3}[\|\vec{x}\|^2 + \|T\|^2 - \lambda_{max}(\vec{x}\vec{x}^\dagger + TT^\dagger)]$ where T is the correlation matrix, $\vec{x}$ is the vector whose components are $x_m = \mathrm{Tr}(\rho(\sigma_m \otimes \mathbb{I}_2))$, and $\lambda_{max}(K)$ is the maximum eigenvalue of the matrix $\vec{x}\vec{x}^\dagger + TT^\dagger$.

Measurement induced disturbance (QMID)

- QMID quantifies the quantumness of the correlation between the quantum bipartite states shared amongst Alice and Rob.
- For the given $\rho'_{A,R}$, if ρ'_A and ρ'_R are the reduced density matrices, then the mutual information that quantifies the correlation between Alice and Rob is

$$I = S(\rho'_A) + S(\rho'_R) - S(\rho'_{A,R}),$$

  $S(.)$ is the von Neumann entropy.
- If $\rho'_A = \sum_i \lambda^i_A \Pi^i_A$ and $\rho'_R = \sum_j \lambda^j_R \Pi^j_R$ denotes the spectral decomposition of ρ'_A and ρ'_R, respectively, then the state $\rho'_{A,R}$ after measuring in joint basis $\{\Pi_A, \Pi_R\}$ is

$$\Pi(\rho'_{A,R}) = \sum_{i,j} (\Pi^i_A \otimes \Pi^j_R) \rho'_{A,R} (\Pi^i_A \otimes \Pi^j_R).$$

- QMID [Luo (2008)] is

$$M(\rho'_{A,R}) = I(\rho'_{A,R}) - I(\Pi(\rho'_{A,R}))$$

  is a measure of quantumness of the correlation.
- In Figure 1, we find that $M > 0$ throughout the range considered, implying that the system remains nonclassical, as expected.

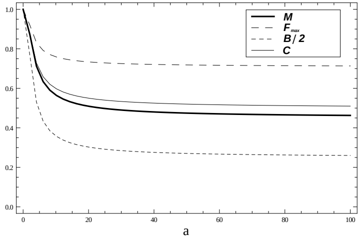


Figure : Degradation of QMID (M), teleportation fidelity ($F_{\max}$), Bell quantity ($B/2$) and concurrence (C) as a function of Unruh acceleration (a). The system becomes local ($B/2 < 1/2$) at $a \approx 4.6$, but stays nonclassical with respect to the other parameters ($C > 0$, $F > \frac{2}{3}$, $M > 0$)).

- Invoke the Choi-Jamiolkowski isomorphism [Choi (1975), Jamiolkowski (1972)] which is a two-way mapping between a quantum state and quantum channel and is an expression of channel-state duality.
- Information is transmitted via a quantum channel and is basically a completely positive map between spaces of operators.
- The Kraus operators for the channel can be obtained by diagonalizing the Choi matrix [Leung (2003), Havel (2003)].
- Once the Kraus operators are obtained, the channel gets completely characterized. We now apply this to the Unruh channel.
- Spectral decomposition gives

$$\rho_U = \sum_{j=0}^{3} |\xi_j\rangle\langle\xi_j|,$$

where $|\xi_j\rangle$ are the eigenvectors normalized to the value of the eigenvalue.

- By Choi's theorem, each $|\xi_j\rangle$ yields a Kraus operator obtained by folding the d^2 (here: 4) entries of the eigenvector into $d \times d$ (2×2) matrix, essentially by taking each sequential d-element segment of $|\xi_j\rangle$, writing it as a column, and then juxtaposing these columns to form the matrix.
- The two eigenvectors corresponding to the two non-vanishing eigenvalues are $|\xi_0\rangle = (\cos r, 0, 0, 1)$, $|\xi_1\rangle = (0, \sin r, 0, 0)$.
- From these, we have the Kraus representation for $\mathcal{E}_U$ as

$$K_1^U = \begin{pmatrix} \cos r & 0 \\ 0 & 1 \end{pmatrix}; \quad K_2^U = \begin{pmatrix} 0 & 0 \\ \sin r & 0 \end{pmatrix},$$

whereby

$$\mathcal{E}_U(\rho) = \sum_{j=1,2} K_j^U \rho \left(K_j^U\right)^\dagger,$$

with the completeness condition

$$\sum_{j=1,2} \left(K_j^U\right)^\dagger K_j^U = \mathbb{I}.$$

- This is formally similar to the operator elements in the Kraus representation of an Amplitude Damping (AD) channel, which models the effect of a zero temperature thermal bath [SB, R. Srikanth (2008); SB, R. Ghosh (2007)].
- This is surprising as the Unruh effect corresponds to a finite temperature and would naively be expected to correspond to the *generalized* AD or SGAD channels, which are finite temperature channels. This is a pointer towards a fundamental difference between the Unruh and the AD channel.
- For an initial pure qubit state $\rho = |0\rangle\langle 0| \cos^2\frac{\theta}{2} + |0\rangle\langle 1| e^{i\phi}\cos\frac{\theta}{2}\sin\frac{\theta}{2} + |1\rangle\langle 0| e^{-i\phi}\cos\frac{\theta}{2}\sin\frac{\theta}{2} + |1\rangle\langle 1|\sin^2\frac{\theta}{2}$, the action of the Unruh channel is

$$\mathcal{E}_U(\rho) = \begin{pmatrix} \cos^2 r \cos^2\frac{\theta}{2} & \cos r e^{i\phi}\cos\frac{\theta}{2}\sin\frac{\theta}{2} \\ \cos r e^{-i\phi}\cos\frac{\theta}{2}\sin\frac{\theta}{2} & \sin^2 r\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} \end{pmatrix}.$$

- For infinite time and acceleration, the asymptotic state is

$$\rho_\infty = \begin{pmatrix} \frac{1}{2}\cos^2\frac{\theta}{2} & \frac{1}{\sqrt{2}}e^{i\phi}\cos\frac{\theta}{2}\sin\frac{\theta}{2} \\ \frac{1}{\sqrt{2}}e^{-i\phi}\cos\frac{\theta}{2}\sin\frac{\theta}{2} & \frac{1}{2}\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} \end{pmatrix},$$

with Bloch vector

$$\hat{n}^\infty(\theta,\phi) \equiv (\hat{x},\hat{y},\hat{z}) = \left(\frac{\cos\phi\sin\theta}{\sqrt{2}}, \frac{\sin\phi\sin\theta}{\sqrt{2}}, -\sin^2\frac{\theta}{2}\right).$$

- This shows that the Bloch sphere gets mapped to the inscribed solid object shown in Figure 2, whose south pole osculates with that of the Bloch sphere, while the north pole (corresponding to initial $\theta = 0$) is located midway between the Bloch sphere center and south pole: $\hat{n}^{\infty}(0, \phi) = (0, 0, 0)$ while $\hat{n}^{\infty}(\pi, \phi) = (0, 0, -1)$. This is thus a kind of an interrupted AD channel.
- By virtue of linearity of the map, it follows that the maximally mixed state maps to the Bloch vector which is the average of the above two, being

$$\hat{n}^{\infty}(\mathbb{I}) = (0, 0, -\frac{1}{2}).$$

- Thus the channel is non-unital, with the new Bloch representation of the initially maximally mixed state being

$$\mathcal{E}_U^{\infty}(\mathbb{I}/2) = \begin{pmatrix} 1/4 & 0 \\ 0 & 3/4 \end{pmatrix}.$$

- The geometry of the contracted, noisy version of the Bloch sphere can be inferred to be

$$R(\theta) = |\hat{n}^{\infty}(\theta, \phi) - \hat{n}^{\infty}(\mathbb{I})| = \frac{\sqrt{3 - \cos 2\theta}}{2\sqrt{2}}.$$

- Since the volume of the Bloch sphere is $V_0 \equiv \frac{4\pi}{3}$, it follows that the volume contraction factor of the Bloch sphere under the relativistic channel is $K \equiv \frac{1}{4}$.
- The eccentricity of the oblate sphere is given by

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{\sqrt{2}},$$

where a and b are the semi-major and semi-minor axis, which are seen from the form of $R(\theta)$ to be $\frac{1}{\sqrt{2}}$ (corresponding to $\theta = \frac{\pi}{2}$) and $\frac{1}{2}$ (corresponding to $\theta = 0, \pi$), respectively.

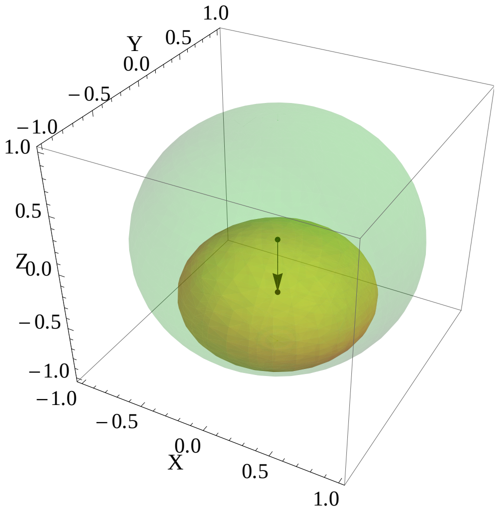


Figure : Under $\mathcal{E}_U$, the Bloch sphere (the outer sphere) is shrunk asymptotically to the inner solid oblate spheroid (eccentricity $e = \frac{1}{\sqrt{2}}$) by a volume factor $\frac{1}{4}$, centered at $(0, 0, -\frac{1}{2})$ (surface described by $\hat{n}^{\infty}(\theta, \phi)$).

- A genuine AD channel may be composed with an Unruh channel, resulting in an AD channel. Consider a state

$$\rho = \begin{pmatrix} \alpha & \beta \\ \beta^* & 1-\alpha \end{pmatrix},$$

with real α.

- The action of Unruh noise yields

$$\rho_U = \mathcal{E}_U(\rho) = \begin{pmatrix} \alpha \cos^2 r & \beta \cos r \\ \beta^* \cos r & 1 - \alpha \cos^2 r \end{pmatrix},$$

while that of an AD channel on state ρ_U above is

$$\mathcal{E}_{UV}(\rho) = \begin{pmatrix} \alpha \cos^2 r(1-\gamma) & \beta \cos r \sqrt{1-\gamma} \\ \beta^* \cos r \sqrt{1-\gamma} & 1 + \alpha(-\cos^2 r + \gamma \cos^2 r) \end{pmatrix}.$$

- Setting $\gamma'' = \gamma \cos^2 r + \sin^2 r$, the Kraus operators of noise $\mathcal{E}_{UV}$ are

$$\begin{pmatrix} \sqrt{1-\gamma''} & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 \\ \sqrt{\gamma''} & 0 \end{pmatrix},$$

which has the form of an AD channel.

- The same observation holds if the order of the AD and Unruh channels are inverted. This closure under composition is a manifestation of the semi-group property of AD channels.

- The origin of the Unruh channel is, as noted, quite different from a conventional noise channel.
- For an AD channel, which is derived using the Born-Markov and rotating-wave approximations, one can invoke the fluctuation-dissipation theorem to require the convergence of the Bloch sphere at initial time to a single fixed point of the evolution asymptotically. The finite contraction factor, for the Unruh channel, thwarts this behavior.
- Another point is that the rank of ρ_U is 2, and not 4. This is reflected in the fact that there are only 2 canonical Kraus operators.
- This entails that the Unruh noise corresponds to only a single Lindblad channel corresponding to a de-excitation process. Now if the environment were a conventional finite-temperature bath, then we should have also the Lindblad excitation channel corresponding to the qubit absorbing a photon from this bath. The lack of the excitation channel here suggests that, from the physical perspective of an inertial detector, the Unruh background interacts as a vacuum, even though Rob views it as a thermal Rindler state in his own reference frame.

- Bloch vector representation of Unruh channel has been developed. This is used to provide a unified, analytical treatment of quantum Fisher and Skew information for a qubit subjected to the Unruh channel [SB, A. K. Alok, S. Omkar (2016)].
- How environmentally induced decoherence modifies the effect of the Unruh channel... [SB, A. K. Alok, S. Omkar, R. Srikanth (2017)].

Foundations of Quantum Mechanics in Neutrinos

- Quantum correlations is a central topic of investigations in the quest for an understanding as well as for the harvesting of the power of quantum mechanics in a plethora of systems provided by nature.
- The foundations of quantum mechanics are usually studied in optical or electronic systems where the interplay between the various measures of quantum correlations is well known.
- Inspired by the recent technical advances in high energy physics experiments, in particular the neutrino oscillation experiments, this quest can now be directed towards neutrinos.

Motivation

- Neutrino system is particularly interesting as the effect of decoherence as compared to other particles widely used in quantum information processing, is minimal.
- Also, the detection efficiency is much higher than that of the corresponding detectors used in optical or electronic systems.
- Thus neutrino system has the potential to provide an alternative platform for testing foundations of quantum mechanics.

- In nature, neutrinos are available in three flavors.
- Owing to their non-zero mass, they oscillate from one flavor to another which has been confirmed by a plethora of experiments, using both natural and "man-made" neutrinos.
- Neutrino oscillations are fundamentally three flavor oscillations. However, in some cases, it can be reduced to effective two flavor oscillations.

Quantum correlations in Neutrinos

- The coherent time evolution of neutrino flavor eigenstates implies that there is a linear superposition between the mass eigenstates which make up a flavour state.
- Neutrino oscillations are related to the multi-mode entanglement of single-particle states which can be expressed in terms of flavor transition probabilities.
- Hence neutrino is an interesting candidate for study of quantum correlations.

Definition of the problem

We are interested in studying various facets of quantum correlations in neutrinos. In particular, we intend to study:

- The interplay between various aspects of quantum correlations such as non-locality, entanglement and weaker measures such as discord.
- To explore relation between neutrino mixing and coherences in the system.

The three flavour states (eigenstates of weak interaction) of neutrinos, ν_e, ν_μ and ν_τ mix via a 3×3 unitary matrix to form the three mass eigenstates (which are the propagation eigenstates) ν_1, ν_2 and ν_3.

Neutrino oscillations occur only if the three corresponding masses, m_1, m_2 and m_3, are non-degenerate.

Of the three mass-squared differences $\Delta_{ij} = m_i^2 - m_j^2$ (where $i, j = 1, 2, 3$ with $i > j$), only two are independent. Oscillation data tells us that $\Delta_{21} \approx 0.03 \times \Delta_{32}$, $\Delta_{31} \approx \Delta_{32}$.

In considering neutrino oscillations, in general, one should use the full three flavour oscillation formulae.

However, in a number of cases, the three flavour formula reduces to an effective two flavour formula, if one or both of the small parameters, Δ_{21}/Δ_{32} and θ_{13}, are set equal to zero.

For example, in long baseline accelerator experiments, both the above parameters can be neglected in doing leading order calculations. Then the problem reduces to that of two flavour mixing of ν_μ and ν_τ to form two mass eigenstates ν_2 and ν_3.

The corresponding oscillations are described by one mixing angle θ ($\equiv \theta_{23}$ in three flavour mixing) and one mass-squared difference Δ ($\equiv \Delta_{32}$ in three flavour mixing).

In the case of two flavour mixing, the relation between the flavour and the mass eigenstates is described by a 2×2 rotation matrix, $U(\theta)$,

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_j \\ \nu_k \end{pmatrix} .$$

Each flavour state is given by a superposition of mass eigenstates,

$$|\nu_\alpha\rangle = \sum_j U_{\alpha j} |\nu_j\rangle \ ,$$

where $\alpha = \mu$ or τ and $j = 2, 3$.
The time evolution of the mass eigenstates $|\nu_j\rangle$ is given by

$$|\nu_j(t)\rangle = e^{-iE_j t} |\nu_j\rangle \ ,$$

where $|\nu_j\rangle$ are the mass states at time $t = 0$.
Thus, we can write

$$|\nu_\alpha(t)\rangle = \sum_j e^{-iE_j t} U_{\alpha j} |\nu_j\rangle \ .$$

The evolving flavour neutrino state $|\nu_\alpha\rangle$ can also be projected on to the flavour basis in the form

$$|\nu_\alpha(t)\rangle = \tilde{U}_{\alpha\alpha}(t) |\nu_\alpha\rangle + \tilde{U}_{\alpha\beta}(t) |\nu_\beta\rangle \ ,$$

where $|\nu_\alpha\rangle$ is the flavour state at time $t = 0$ and $|\tilde{U}_{\alpha\alpha}(t)|^2 + |\tilde{U}_{\alpha\beta}(t)|^2 = 1$.
As all detectable neutrinos are ultra-relativistic, the flavor eigenstates are well defined in the context of quantum mechanics.

We can thus establish the following correspondence, using the occupation number of neutrinos, with two-qubit states [Blasone et al. (2008, 2013, 2015)]

$$|\nu_\alpha\rangle \equiv |1\rangle_\alpha \otimes |0\rangle_\beta \equiv |10\rangle \,, \quad |\nu_\beta\rangle \equiv |0\rangle_\alpha \otimes |1\rangle_\beta \equiv |01\rangle \,.$$

The time evolution of flavor eigenstate can then be written as

$$|\nu_\alpha(t)\rangle = \tilde{U}_{\alpha\alpha}(t)\,|1\rangle_\alpha \otimes |0\rangle_\beta + \tilde{U}_{\alpha\beta}(t)\,|0\rangle_\alpha \otimes |1\rangle_\beta \,,$$

where,

$$\begin{aligned} \tilde{U}_{\alpha\alpha}(t) &= \cos^2\theta e^{-iE_2 t} + \sin^2\theta e^{-iE_3 t} \,, \\ \tilde{U}_{\alpha\beta}(t) &= \sin\theta\cos\theta(e^{-iE_3 t} - e^{-iE_2 t}) \,. \end{aligned}$$

$\Longrightarrow$ The state $|\nu_\alpha(t)\rangle$ has the form of a mode entangled single particle state.

Various measures of quantum correlations can now be determined using the density matrix $\rho_\alpha(t) = |\nu_\alpha(t)\rangle\langle\nu_\alpha(t)|$ as the parameters of the density matrix, mixing angle and mass squared difference, are known [A. K. Alok, SB, Uma Sankar (2014); SB, A. K. Alok, R. Srikanth, B. Heismeyr (2015)].

- The above calculation corresponds to the case when neutrinos travel through vacuum. But the oscillation patterns can be significantly affected if neutrinos travel through a material medium. Therefore matter effect should also be taken care of.

- ν_e interacts with electrons (e^-) present in matter via neutral and charged current interactions, while ν_μ and ν_τ interact only by neutral current interaction. The amplitude corresponding to neutral current interactions are identical for all of the three flavors. Therefore we consider the amplitude corresponding to charged current interaction of ν_e with e^- only.

- Given that equation of motion in mass eigenstate basis is

$$i\frac{d}{dt}\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix} = H\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix}$$

  where

$$H = \begin{bmatrix}E_1 & 0\\ 0 & E_2\end{bmatrix}$$

- We assume that ν is emitted in plane wave state with definite momentum i.e., $E_i^2 = p^2 + m_i^2$ with ultra high relativistic approximation ($p^2 >>> m_i^2$). Then Hamiltonian

$$H = \begin{bmatrix} p + \frac{m_1^2}{2p} & 0 \\ 0 & p + \frac{m_2^2}{2p} \end{bmatrix}$$

- Hamiltonian can also be expressed in terms of mass square difference $\Delta = m_2^2 - m_1^2$ as,

$$H = \begin{bmatrix} p + \frac{m_1^2+m_2^2}{4p} - \frac{\Delta}{4p} & 0 \\ 0 & p + \frac{m_1^2+m_2^2}{4p} + \frac{\Delta}{4p} \end{bmatrix}$$

- Therefore the equation of motion in flavor state basis is given by

$$i\frac{d}{dt}\begin{bmatrix} \nu_e(t) \\ \nu_\mu(t) \end{bmatrix} = \left[p + \frac{m_1^2 + m_2^2}{4p} I + \frac{\Delta}{4p} O^T \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} O\right] \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix}$$

where O is the mixing matrix.

- Neglecting first term and putting p $\approx$ E, the above equation will be,

$$i\frac{d}{dt}\begin{bmatrix}\nu_e(t)\\ \nu_\mu(t)\end{bmatrix} = \frac{\Delta}{4E}\begin{bmatrix}-cos2\theta & sin2\theta\\ sin2\theta & cos2\theta\end{bmatrix}\begin{bmatrix}\nu_e\\ \nu_\mu\end{bmatrix}$$

- The survival and oscillation probabilities take the form

$$P_{ee} = 1 - \sin^2 2\theta \ \sin^2 \frac{\Delta L}{4E\hbar c}$$
$$P_{e\mu} = \sin^2 2\theta \ \sin^2 \frac{\Delta L}{4E\hbar c}$$

- Since ν_e only interacts with matter via charged current interaction, an extra term V (matter density potential) is added to this equation as,

$$i\frac{d}{dt}\begin{bmatrix}\nu_e(t)\\ \nu_\mu(t)\end{bmatrix} = \begin{bmatrix}-\frac{\Delta\cos 2\theta}{4E} + V & \frac{\Delta\sin 2\theta}{4E}\\ \frac{\Delta\sin 2\theta}{4E} & \frac{\Delta\cos 2\theta}{4E}\end{bmatrix}\begin{bmatrix}\nu_e\\ \nu_\mu\end{bmatrix}$$

where $V = \sqrt{2}G_F N_e$ with $G_F \rightarrow$ Fermi constant, $N_e \rightarrow$ electron density.

- As a result of this equation, for constant matter density, Survival and oscillation probabilities become,

$$P_{ee} = 1 - \sin^2 2\theta_m \ \sin^2 \frac{\Delta_m L}{4E\hbar c}$$
$$P_{e\mu} = \sin^2 2\theta_m \ \sin^2 \frac{\Delta_m L}{4E\hbar c}$$

where θ_m and Δ_m are effective mixing angle and mass square difference, respectively, which in terms of vacuum mass square difference Δ and mixing angle θ, are

$$\theta_m = \frac{1}{2}\tan^{-1}\left(\frac{\tan 2\theta}{1 - \frac{2EV}{\Delta\cos 2\theta}}\right)$$
$$\Delta_m = \sqrt{(\Delta\cos 2\theta - 2EV)^2 + \Delta^2\sin^2 2\theta}$$

The resonance condition i.e., $2\text{EV} = \Delta\ \cos 2\theta$, will cause the maximal mixing. This is called $Mikheyev - Smirnov - Wolfenstein$ (MSW) effect.

- To study the effect of CP - violation in neutrino oscillations (for Dirac neutrinos), one has to go through the calculation of three flavor neutrino oscillations. Applying some appropriate approximations, mathematical picture of two flavor oscillation can be reproduced through the three flavor case.

- In three flavor neutrino oscillation
  Propagation states $\rightarrow \{|\nu_1\rangle\,, \;\; |\nu_2\rangle\,, \;\; |\nu_3\rangle\}$;
  Flavor states $\rightarrow \{|\nu_e\rangle\,, \;\; |\nu_\mu\rangle\,, \;\; |\nu_\tau\rangle\}$
- The general state of a neutrino can be expressed in flavor basis as:

$$|\Psi(t)\rangle = \nu_e(t)\,|\nu_e\rangle + \nu_\mu(t)\,|\nu_\mu\rangle + \nu_\tau(t)\,|\nu_\tau\rangle$$

- Same state in propagation basis looks like:

$$|\Psi(t)\rangle = \nu_1(t)\,|\nu_1\rangle + \nu_2(t)\,|\nu_2\rangle + \nu_3(t)\,|\nu_3\rangle$$

- The coefficients in two representations are connected by a *unitary* matrix

$$\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \\ \nu_\tau(t) \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \\ \nu_3(t) \end{pmatrix}.$$

  or,

$$\nu_\alpha(t) = \mathbf{U}\nu_i(t). \tag{1}$$

- A convenient parametrization for $U(\theta_{12}, \theta_{23}, \theta_{13}, \delta)$ is given by the so called Pontecorvo-Maki-Nakagawa-Sakata ($PMNS$) matrix [Progress of Theoretical Physics **28**, 870-880 (1962)]

$$U(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{23}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{13}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, θ_{ij} being the mixing angles and δ the CP violating phase.
- The mass eigenstates evolve as

$$\begin{pmatrix} \nu_1(t) \\ \nu_2(t) \\ \nu_3(t) \end{pmatrix} = \begin{pmatrix} e^{-iE_1 t} & 0 & 0 \\ 0 & e^{-iE_2 t} & 0 \\ 0 & 0 & e^{-iE_3 t} \end{pmatrix} \begin{pmatrix} \nu_1(0) \\ \nu_2(0) \\ \nu_3(0) \end{pmatrix},$$

or,

$$\nu_i(t) = \mathbf{E}\nu_i(0) \tag{2}$$

- From Eqns. (1) and (2), $\nu_\alpha(t) = \mathbf{U}\ \mathbf{E}\mathbf{U}^{-1}\ \nu_\alpha(0) = \mathbf{U}_f\ \nu_\alpha(0)$.

- So the flavor state at time $t = 0$ is connected to the flavor state at time t by

$$\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \\ \nu_\tau(t) \end{pmatrix} = \begin{pmatrix} a(t) & d(t) & g(t) \\ b(t) & e(t) & h(t) \\ c(t) & f(t) & k(t) \end{pmatrix} \begin{pmatrix} \nu_e(0) \\ \nu_\mu(0) \\ \nu_\tau(0) \end{pmatrix}.$$

- Some elements

$$\begin{aligned}
a(t) &= (c_{12}c_{13})^2 \; e^{-iE_1 t} + (s_{12}c_{13})^2 \; e^{-iE_2 t} + s_{13}^2 \; e^{-iE_3 t}, \\
b(t) &= (-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta})c_{12}c_{13} \; e^{-iE_1 t} \\
&\quad + (c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta})s_{12}c_{13} \; e^{-iE_2 t} + c_{13}s_{23}s_{13}e^{i\delta} \; e^{-iE_3 t}, \\
c(t) &= (s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta})(c_{12}c_{13}) \; e^{-iE_1 t} \\
&\quad + (-c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta})(s_{12}c_{13}) \; e^{-iE_2 t} + (c_{13}c_{23})(s_{13}) \; e^{-iE_3 t}, \\
d(t) &= (c_{12}c_{13})(-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{-i\delta}) \; e^{-iE_1 t} \\
&\quad + (s_{12}c_{13})(c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta}) \; e^{-iE_2 t} \\
&\quad + (s_{13}e^{-i\delta})(c_{13}s_{23}) \; e^{-iE_3 t}
\end{aligned}$$

Survival and Transitional Probabilities

Initial state $|\nu_e\rangle$

- $\Psi(t) = a(t)\,|\nu_e\rangle + b(t)\,|\nu_\mu\rangle + c(t)\,|\nu_\tau\rangle$
- Survival probability: $|\,\langle\nu_e|\Psi(t)\rangle\,|^2 = |a(t)|^2$
- Transition Prob. to $\nu_\mu = |\,\langle\nu_\mu|\Psi(t)\rangle\,|^2 = |b(t)|^2$
- Transition Prob. to $\nu_\tau = |\,\langle\nu_\tau|\Psi(t)\rangle\,|^2 = |c(t)|^2$

Initial state $|\nu_\mu\rangle$

- $\Psi(t) = d(t)\,|\nu_e\rangle + e(t)\,|\nu_\mu\rangle + f(t)\,|\nu_\tau\rangle$
- Survival probability: $|\,\langle\nu_\mu|\Psi(t)\rangle\,|^2 = |e(t)|^2$
- Transition Prob. to $\nu_e = |\,\langle\nu_e|\Psi(t)\rangle\,|^2 = |d(t)|^2$
- Transition Prob. to $\nu_\tau = |\,\langle\nu_\tau|\Psi(t)\rangle\,|^2 = |f(t)|^2$

Initial state $|\nu_\tau\rangle$

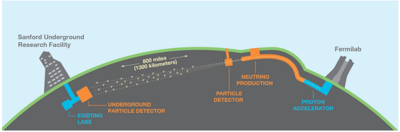
- $\Psi(t) = g(t)\,|\nu_e\rangle + h(t)\,|\nu_\mu\rangle + k(t)\,|\nu_\tau\rangle$
- Survival probability: $|\,\langle\nu_\tau|\Psi(t)\rangle\,|^2 = |k(t)|^2$
- Transition Prob. to $\nu_e = |\,\langle\nu_e|\Psi(t)\rangle\,|^2 = |g(t)|^2$
- Transition Prob. to $\nu_\mu = |\,\langle\nu_\mu|\Psi(t)\rangle\,|^2 = |h(t)|^2$

1. DUNE (Deep Underground Neutrino Experiment): NuMI (Neutrinos at Main Injector) Fermi-Lab, $L = 1300\ km$, $En = 2\ GeV\ to\ 10\ GeV$,

2. NOνA (NuMI Off-Axis ν_e Appearance): $L = 810km$, $En = 1\ GeV\ to\ 4\ GeV$.

3. T2K (Tokai to Kamioka): $L = 295\ km, En = 0.1\ to\ 1 GeV$.

All these experiments use ν_μ source



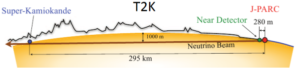


Figure : NOvA (DUNE) and T2K experiments

Non-locality

$$
\begin{aligned}
M(\rho) &= 1 + \left[3 + \cos 4\theta + 2\cos\phi \sin^2 2\theta\right] \sin^2 2\theta \sin^2(\phi/2) \\
&= 1 + 4 P_{sur} P_{osc}.
\end{aligned}
$$

- $\phi = \frac{\Delta t}{2E}$
- $M(\rho)$ is tied up with neutrino mixing.
- In case of no mixing $(\theta = 0)$, $M(\rho) = 1$.

Concurrence

$$
\begin{aligned}
C &= 2\sqrt{\sin^4\theta + \cos^4\theta + 2\cos^2\theta \sin^2\theta \cos\phi)} \sin 2\theta \sin(\phi/2) \\
&= 2\sqrt{P_{sur} P_{osc}}.
\end{aligned}
$$

- In case of no mixing, there is no entanglement.

Geometric discord

$$D_G(\rho) = \frac{2}{3}\sin^2 2\theta \sin^2(\phi/2)\left[3 + \cos 4\theta + 2\cos\phi \sin^2 2\theta\right]$$
$$= \frac{8}{3} P_{sur} P_{osc}.$$

- $D_G(\rho)$ for $\theta = 0$ is 0, a classically allowed value of geometric discord.

Teleportation fidelity

$$F_{\max} = \frac{2}{3} + \frac{1}{3}\sin 2\theta \sin(\phi/2)\sqrt{3 + \cos 4\theta + 2\sin^2 2\theta \cos\phi}$$
$$= \frac{2}{3}(1 + P_{sur} P_{osc}).$$

- In the absence of mixing, $F_{\max} = 2/3$, the classical value of teleportation fidelity.

Quantum correlations in neutrinos

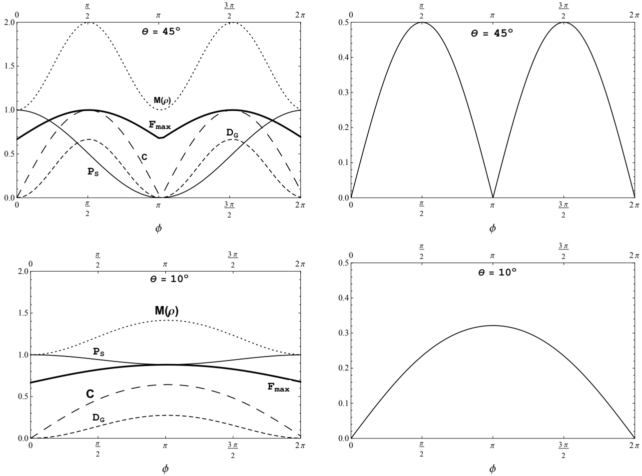


Figure : The left panel of the top and bottom figure depicts $M(\rho)$ (Bell's inequality violation; small-dashed line), F_{max} (teleportation fidelity; thick solid line), C (concurrence, large-dashed line) and D_G (geometric discord; medium-dashed line) with respect to phase ϕ ($\equiv \Delta t/2E$) for the mixing angle $\theta = 45^\circ$ and $\theta = 10^\circ$, respectively. The thin solid line in the left panel of the top and bottom figure, P_S, represents the neutrino survival probability. The right panel of the figure depicts the magnitude of the off-diagonal elements of the density matrix.

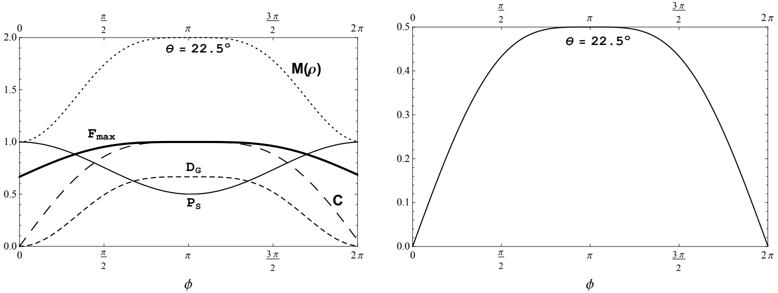


Figure : The left panel of the figure depicts various quantum correlations with respect to phase ϕ for the critical mixing angle $\theta = 22.5^\circ$. The thin solid line in the left panel, P_S, represents the neutrino survival probability. The right panel of the figure depicts the magnitude of the off-diagonal elements of the density matrix.

- Bell's inequality is always violated and hence the evolution of neutrinos is highly non local in nature.
- Teleportation fidelity is always greater than 2/3 thus obeying the usual relation between Bell's inequality violation and teleportation fidelity, as seen in electronic and photonic systems.
- It is quite remarkable that the measurement of neutrino oscillations due to a non zero value of the mixing angle implies quantum correlations.
- The quantum correlations are seen to be very closely tied to the neutrino mixing angle.
- There exists a critical value of the mixing angle $\pi/8$, for which the Bell's inequality violation is maximal over a broad range of the kinematic variable ϕ.
- Also, it is interesting to note that the off diagonal order, introduced here, is gaining prominence in a number of recent studies related to quantum coherence [Girolami (2014), Bromely et.al. (2015), U. Singh et. al. (2015)].

Recent works in neutrinos

- Geometric phase for neutrinos at various man-made facilities, such as the reactor and accelerator neutrino experiments [K Dixit, A K Alok, SB, D Kumar, (2017)].
- Leggett-Garg type inequality violations in the context of three flavor neutrino oscillations studied in the presence of matter and CP violating effects [J A Naikoo, A K Alok, SB, S Uma Sankar, G Guarnieri, B C Hiesmayr, (2017)].

- We study a number of well-established measures of quantum correlations, such as Bell's inequality violations, teleportation fidelity, concurrence and geometric discord, in the correlated $B\bar{B}$ and $K\bar{K}$ systems. We also study an interplay between these measures [A. K. Alok, SB (2013); SB, A. K. Alok, R. MacKenzie (2015)].
- B factories, electron-positron colliders tailor-made to study the production and decay of B mesons, and ϕ factories, which perform the same function for K mesons, provide an ideal testing ground.
- In the case of B factories, the collider energy is tuned to the Υ resonance, so the first stage of the process is $e^+e^- \to \Upsilon$. The Υ then decays into $b\bar{b}$; these form a $B_q\bar{B}_q$ $(q = s, d)$ pair through hadronization. All this happens essentially instantaneously. The B mesons then fly apart and decay on a much longer time scale.
- An important feature of these systems for the study of correlations is the oscillation of the bottom and strangeness flavors, giving rise to $B\bar{B}$ oscillations.
- Another feature about these systems is that they are decaying and thus one needs to study quantum correlations in unstable, decaying $B\bar{B}$ and $K\bar{K}$ systems.

$M\bar{M}$ as an open quantum system

- For the B system, imagine the decay $\Upsilon \rightarrow b\bar{b}$ followed by hadronization into a $B\bar{B}$ pair.
- In the Υ rest frame, the mesons fly off in opposite directions (left and right, say).
- The same considerations apply to the K system, with the Υ replaced by a ϕ meson.
- The flavor-space wave function of the correlated $M\bar{M}$ meson systems ($M = K,\ B_d,\ B_s$) at the initial time $t = 0$ is

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left[\left|M\bar{M}\right\rangle - \left|\bar{M}M\right\rangle\right],$$

  where the first (second) particle in each ket is the one flying off in the left (right) direction and $|M\rangle$ and $\left|\bar{M}\right\rangle$ are flavor eigenstates.
- Thus, the initial state of the neutral meson system is a singlet (maximally entangled) state.

- The state of the two-particle decaying system at time t is

$$\rho(t) = \frac{1}{4}\begin{pmatrix} a_- & 0 & 0 & -a_- \\ 0 & a_+ & -a_+ & 0 \\ 0 & -a_+ & a_+ & 0 \\ -a_- & 0 & 0 & a_- \end{pmatrix},$$

  where $a_\pm = 1 \pm e^{-2\lambda t}$.

- The density matrix depends on only one parameter (in addition to time), the decoherence parameter λ, which describes the interaction of the mesons with the environment.

Non-locality

$$M(\rho) = (1 + e^{-4\lambda t}).$$

Concurrence

$$C = e^{-2\lambda t}.$$

- Entanglement of formation is

$$E_F = -\frac{1+\sqrt{1-C^2}}{2}\log_2\left(\frac{1+\sqrt{1-C^2}}{2}\right) - \frac{1-\sqrt{1-C^2}}{2}\log_2\left(\frac{1-\sqrt{1-C^2}}{2}\right).$$

Geometric discord

$$D_G(\rho) = M(\rho)/3.$$

Teleportation fidelity

$$F_{\mathrm{max}} = \frac{1}{12}\left[6 + 2e^{-2\lambda t} + \sqrt{2}\sqrt{\alpha - \sqrt{\beta}} + \sqrt{2}\sqrt{\alpha + \sqrt{\beta}}\right],$$

where

$$\alpha = 1 + \cosh(4\lambda t) - \sinh(4\lambda t), \quad \beta = 3 - 2\alpha + \cosh(8\lambda t) - \sinh(8\lambda t).$$

- To take into account the effect of decay in the systems under study, the various correlations are modified by the probability of survival of the pair of particles up to that time, which can be shown to be $e^{-2\Gamma t}$, where Γ is the meson decay width.
- For the K meson, $\Gamma = \frac{1}{2}(\Gamma_S + \Gamma_L)$ (where Γ_S and Γ_L are the decay widths of short and long neutral kaon states, respectively); its value is $5.59 \times 10^9\,\mathrm{s}^{-1}$ [K. A. Olive et al. (PDG 2014)].
- The decay widths for B_d and B_s mesons are $6.58 \times 10^{11}\,\mathrm{s}^{-1}$ and $6.61 \times 10^{11}\,\mathrm{s}^{-1}$, respectively [Y. Amhis et al. (HFAG 2012)].
- In the case of the K meson system, λ has been obtained by the KLOE collaboration by studying the interference between the initially entangled kaons and the decay product in the channel $\phi \to K_S K_L \to \pi^+\pi^-\pi^+\pi^-$ [F. Ambrosino et al. (KLOE Collaboration 2006)]. The value of λ is at most $1.58 \times 10^9\,\mathrm{s}^{-1}$ at 3σ.

- In the case of B_d meson systems, the standard method of determining the decoherence parameter uses time-integrated dilepton events. The value of λ for B_d mesons is determined by the measurement of R_d, the ratio of the total same-sign to opposite-sign dilepton rates in the decays of coherent $B_d - \bar{B}_d$ coming from the $\Upsilon(4S)$ decays; the upper bound is $2.82 \times 10^{11}\,\mathrm{s}^{-1}$ at 3σ [R. A. Bertlmann and W. Grimus (2001)].
- In [A. K. Alok, SB, S. U. Sankar (2015)], a number of methods were suggested to determine the decoherence parameter in the B meson systems.
- For B_s mesons, to the best of our knowledge, there is no experimental information about λ so we will take it to be zero in what follows.

Quantum correlations in neutral mesons

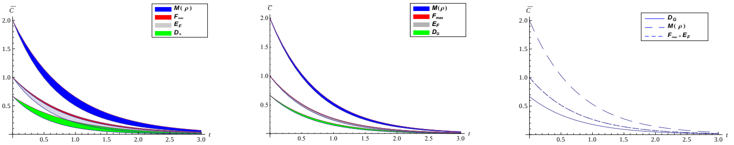


Figure : Average correlation measures, i.e., the various measures modulated by the exponential factor $e^{-2\Gamma t}$, as a function of time t. The left, middle and right panels correspond to the correlations of a $K\bar{K}$, $B_d\bar{B}_d$ and $B_s\bar{B}_s$ pair created at $t = 0$, respectively. The four correlation measures are (top to bottom): $M(\rho)$ (Bell's inequality; blue band), $F_{\max}$ (teleportation fidelity; red band), E_F (entanglement of formation; grey band) and D_G (geometric discord; green band). For $K\bar{K}$ pairs, left panel, time is in units of 10^{-10} seconds whereas for the $B_d\bar{B}_d$ and $B_s\bar{B}_s$ pairs, time is in units of 10^{-12} seconds (in all cases, the approximate lifetime of the particles). In the left and middle panels, the bands represent the effect of decoherence corresponding to a 3σ upper bound on the decoherence parameter λ. The right panel has no such bands because there is currently no experimental evidence for decoherence in the case of B_s mesons; for this case, $F_{\max} = E_F$.

- On average, Bell's inequality in these correlated-meson systems is violated for about half of the meson lifetime.
- We find that the quantum correlations here can be *nontrivially different* from their stable counterparts. This is made explicit by the interplay between Bell's inequality violation and teleportation fidelity.
- One *particularly surprising result* is that teleportation fidelity does not exceed the classical threshold of 2/3 for all Bell's inequality violations.

Conclusions

- We applied ideas of quantum information and open quantum systems to study some aspects of Unruh effect, neutrino oscillations and correlated neutral mesons.
- Unruh effect as well as correlated neutral mesons were examined from the perspective of open quantum systems .
- We also tried to understand neutrino oscillations from the perspective of quantum information.