ISNFQC18

Efficient measurement of high-dimensional quantum states

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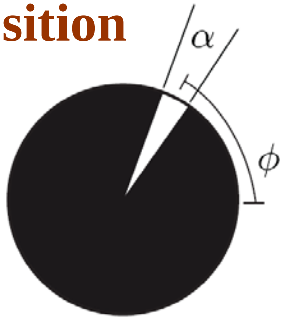
Outline

Efficient measurement of states of light

- in the orbital angular momentum (OAM) basis.
- in the transverse momentum basis.

Orbital angular momentum (OAM) of light

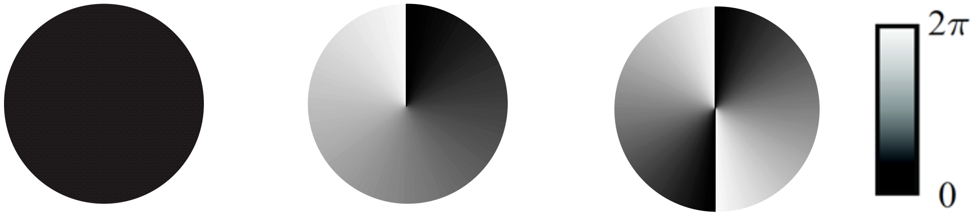
Angular position



$$\psi(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi} \ ,$$

$$\alpha_l = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} \psi(\phi) e^{il\phi} \, d\phi \ ,$$

Orbital Angular momentum $\langle \boldsymbol{\phi} | \boldsymbol{l} \rangle = e^{-il\phi}$



l=0 l=1 l=2

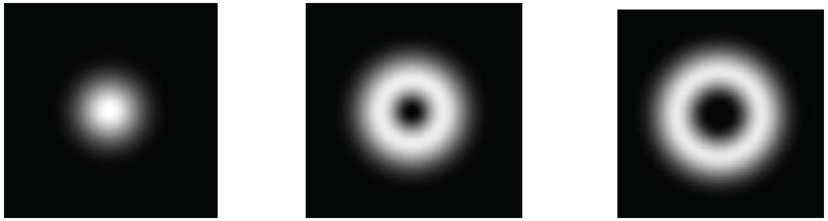
Barnett and Pegg, PRA **41**, 3427 (1990)
Franke-Arnold et al., New J. Phys. **6**, 103 (2004)
Forbes, Alonso, and Siegman J. Phys. A **36**, 707 (2003)

Laguerre-Gaussian modes are solutions to paraxial Helmholtz Equation

$$\psi_{pl}(\rho,\phi,z) = \frac{C}{(1+z^2/z_R^2)^{1}/2} \exp\left[i(2p+l+1)\tan^{-1}\left(\frac{z}{z_R}\right)\right] \left[\frac{\rho/2}{w(z)}\right]^{l} L_p^l\left[\frac{2\rho^2}{w^2(z)}\right]$$

$$\times \exp\left[-\frac{\rho^2}{w^2(z)}\right] \exp\left[-\frac{ik^2\rho^2 z}{2(z^2+z_R^2)}\right] e^{-il\phi}$$

$$\frac{J_z}{W} = \frac{\iint \rho d\rho d\phi (\boldsymbol{\rho} \times \langle \mathbf{E} \times \mathbf{B} \rangle)_z}{c \iint \rho d\rho d\phi \langle \mathbf{E} \times \mathbf{B} \rangle_z} = \frac{\hbar l}{\hbar \omega}$$



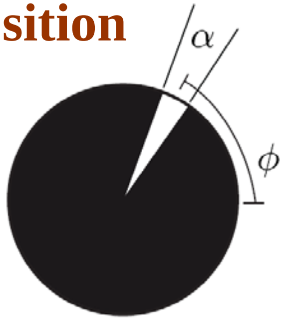
l=0 l=1 l=2

Orbital angular momentum per photon: $\hbar l$

Allen et al., PRA **45**, 8185 (1992)

Orbital angular momentum (OAM) of light

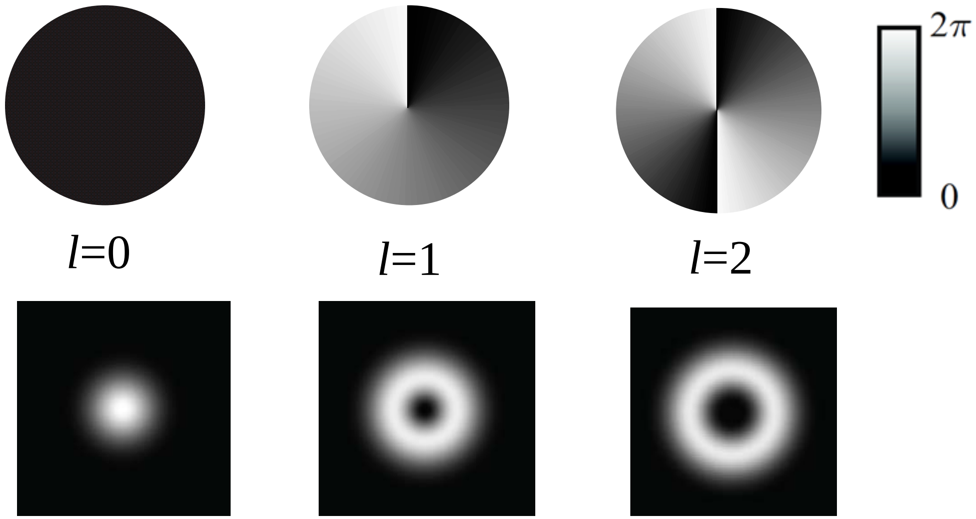
Angular position



$$\psi(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi} \ ,$$

$$\alpha_l = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} \psi(\phi) e^{il\phi} \, d\phi \ ,$$

Orbital Angular momentum $\langle \boldsymbol{\phi} | \boldsymbol{l} \rangle = e^{-il\phi}$



OAM provides an infinite dimensional discrete basis

Benefits:	(i) higher allowed error rate in cryptography	Phys. Rev. Lett. 88, 127902 (2002).
	(ii) higher transmission bandwidth	Phys. Rev. Lett. 90, 167906 (2003).
	(iii) Supersensitive angle measurements	Phys. Rev. A 83, 053829 (2011).
	(iv) Fundamental tests of quantum mechanics	Phys. Rev. Lett. 85, 4418–4421 (2000).

States in OAM basis : $\langle \boldsymbol{\phi} | \boldsymbol{l} \rangle = e^{-il\phi}$

State in the OAM basis (classical)

State in the OAM basis (quantum)

$$\psi(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi} \quad \longleftrightarrow \quad |\psi\rangle = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l |l\rangle$$

Pure States

$$W(\phi_1, \phi_2) = \langle \psi(\phi_1)\psi^*(\phi_2) \rangle_e \quad \longleftrightarrow \quad \rho = \langle\, |\psi\rangle\langle\psi|\, \rangle_e$$

$$= \frac{1}{2\pi} \sum_{l_1, l_2=-\infty}^{\infty} \langle \alpha_{l_1} \alpha_{l_2}^* \rangle_e \, e^{-i(l_1\phi_1 - l_2\phi_2)} \qquad = \frac{1}{2\pi} \sum_{l_1, l_2=-\infty}^{\infty} \langle \alpha_{l_1} \alpha_{l_2}^* \rangle_e \, |l_1\rangle\langle l_2|$$

Mixed States

When different OAM eigenmodes are uncorrelated. $\langle \alpha_{l_1} \alpha_{l_2}^* \rangle_e = S_{l_1} \delta_{l_1, l_2}$

$$W(\phi_1, \phi_2) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l \, e^{-il(\phi_1 - \phi_2)} \quad \longleftrightarrow \quad \rho = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l |l\rangle\langle l|$$

Diagonal Mixed States

$$W(\Delta\phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l \, e^{-il\Delta\phi} \Rightarrow \qquad S_l = \int_{-\pi}^{\pi} W(\Delta\phi) e^{il\Delta\phi} d\phi$$

Angular Wiener-Khintchine theorem

A K Jha, G S Agarwal, R W Boyd, PRA **84**, 063847 (2011)

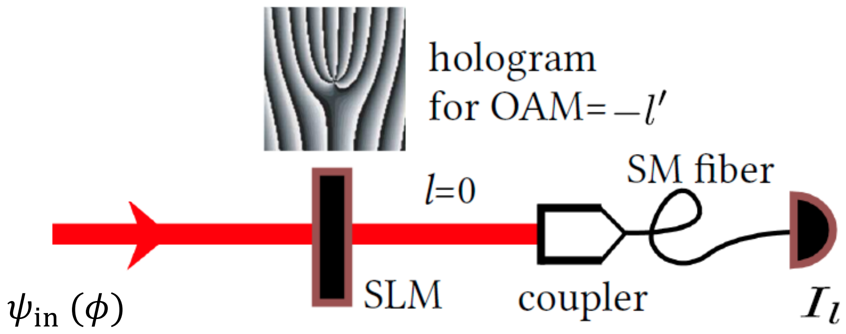
Angular correlation function

OAM spectrum

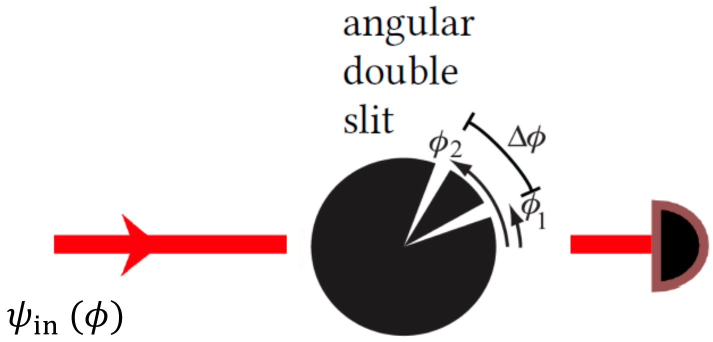
Aim: Measure the angular correlation function $W(\phi_1, \phi_2)$

For diagonal states it yields the OAM spectrum

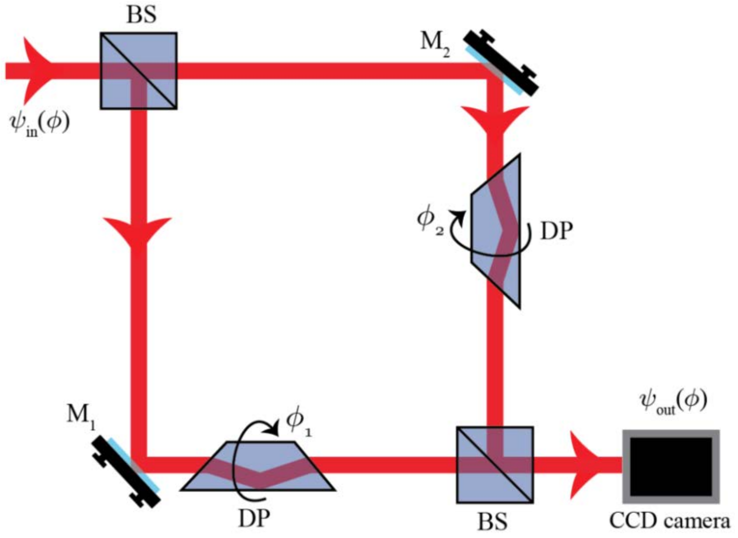
Existing methods for measuring OAM spectrum of Light



A Mair *et al*. Nature 412 **313** (2001)
N R Heckenberg *et al*. Opt. Lett. **17**, 221 (1992)



A K Jha, G S Agarwal, R W Boyd, PRA **84**, 063847 (2011)
M Malik *et al*., PRA **86**, 063806 (2012).



H D L Pires *et al*., Opt. Lett., **35**, 889 (2010)
H D L Pires *et al*., Phys Rev Lett **104**, 020505 (2010)

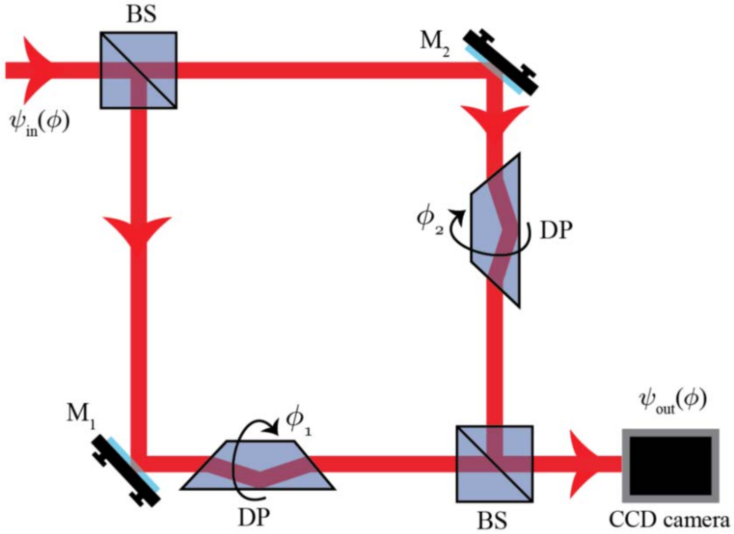
Existing methods for measuring OAM spectrum of Light

State in the OAM basis

$$\psi_{\text{in}}\,(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi}$$

Uncorrelated eigenmodes: $\langle \alpha_{l_1} \alpha^*_{l_2} \rangle_e = S_{l_1} \delta_{l_1,l_2}$

$$W(\Delta\phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l\, e^{-il\Delta\phi} \Rightarrow S_l = \int_{-\pi}^{\pi} W(\Delta\phi) e^{il\Delta\phi} d\phi$$



H D L Pires *et al.*, Opt. Lett., **35**, 889 (2010)
H D L Pires *et al.*, Phys Rev Lett **104**, 020505 (2010)

$$\psi_{\text{out}}\,(\phi) = \sqrt{k_1} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il(\phi+\phi_1)+i\omega t_1} + \sqrt{k_2} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il(\phi+\phi_2)+i\omega t_2}$$

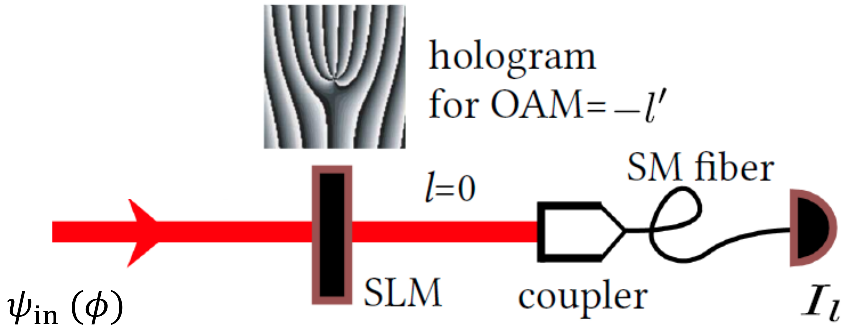
$$I_{\text{out}}(\phi) = \langle\, \psi_{\text{out}}(\phi) \psi^*{}_{\text{out}}(\phi) \rangle_e = \frac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2} W(\Delta\phi) \cos\delta$$

$$\boxed{\delta \equiv \omega(t_1 - t_2)}$$

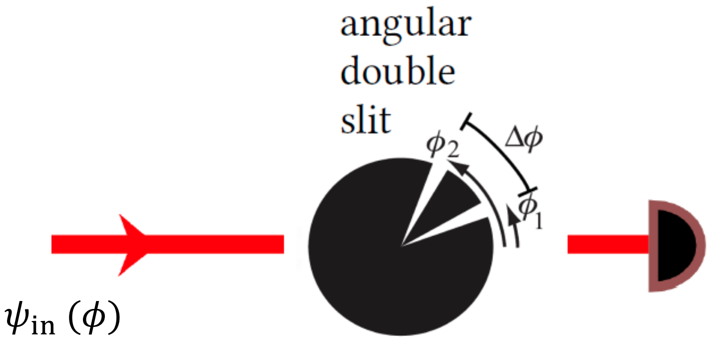
$$\text{Visibility: V} = \frac{[I_{\text{out}}(\phi)]_{max} - [I_{\text{out}}(\phi)]_{min}}{[I_{\text{out}}(\phi)]_{max} + [I_{\text{out}}(\phi)]_{min}} = \frac{4\pi\sqrt{k_1 k_2}}{k_1 + k_2} W(\Delta\phi) \;\; \propto W(\Delta\phi)$$

- By measuring V, $W(\Delta\phi)$ can be measured
- From $W(\Delta\phi)$, S_l can be computed

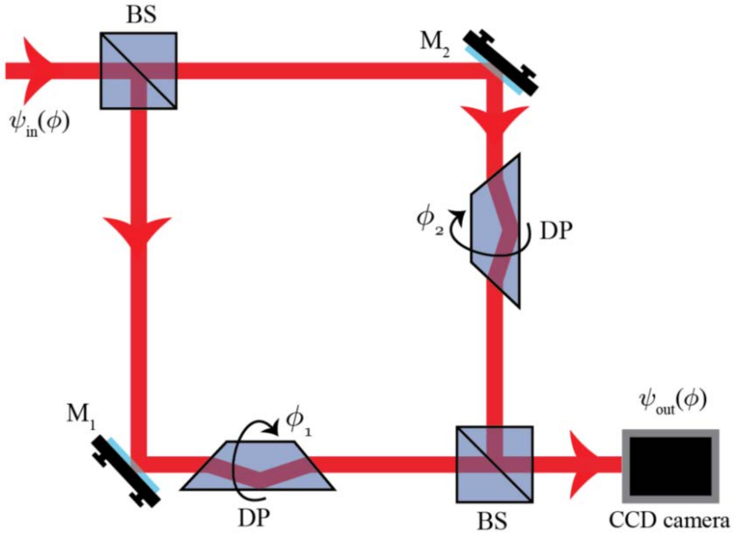
Existing methods for measuring OAM spectrum of Light



A Mair *et al*. Nature 412 **313** (2001)
N R Heckenberg *et al*. Opt. Lett. **17**, 221 (1992)



A K Jha, G S Agarwal, R W Boyd, PRA **84**, 063847 (2011)
M Malik *et al*., PRA **86**, 063806 (2012).



H D L Pires *et al*., Opt. Lett., **35**, 889 (2010)
H D L Pires *et al*., Phys Rev Lett **104**, 020505 (2010)

$$V = \frac{4\pi\sqrt{k_1 k_2}}{k_1 + k_2} W(\Delta\phi)$$

Limitations:

- Efficiency/purity issues
- Too much loss
- Stringent alignment requirements
- Sensitive to background noise and other experimental parameters

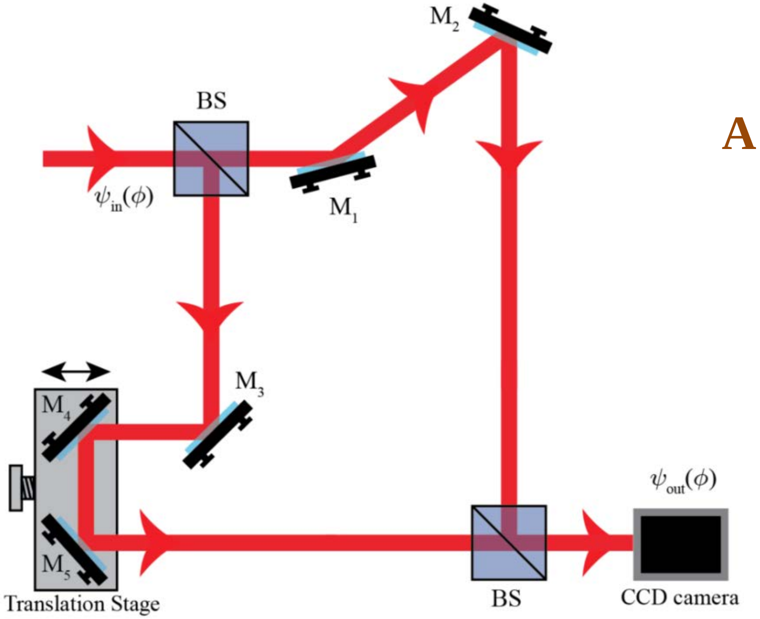
Measuring Orbital Angular Momentum of Light (A new scheme)

State in the OAM basis

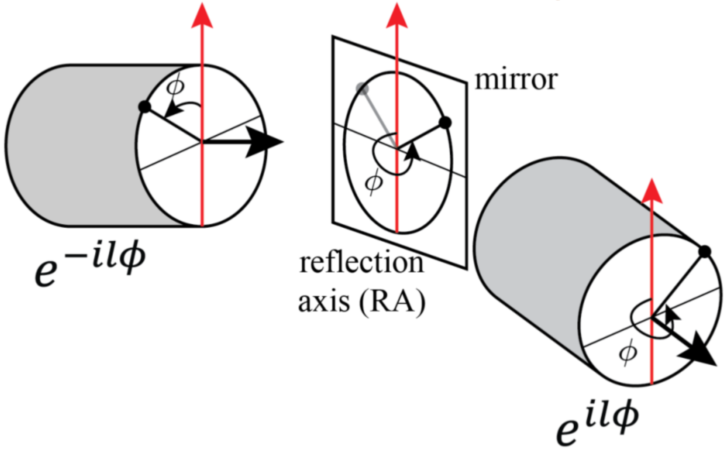
$$\psi_{\text{in}}\,(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi}$$

Uncorrelated eigenmodes:

$$W(\Delta\phi) \; = \frac{1}{2\pi} \sum_{l\,=-\infty}^{\infty} S_l \; e^{-il\Delta\phi} \Rightarrow \; S_l = \int_{-\pi}^{\pi} W(\Delta\phi) e^{il\Delta\phi} d\phi$$



A reflection flips the wave-front along the reflection axis



$$\psi_{\text{out}}\,(\phi) = \sqrt{k_1} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi + i\omega t_1}$$

$$+ \sqrt{k_2} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{+il\phi + i\omega t_2}$$

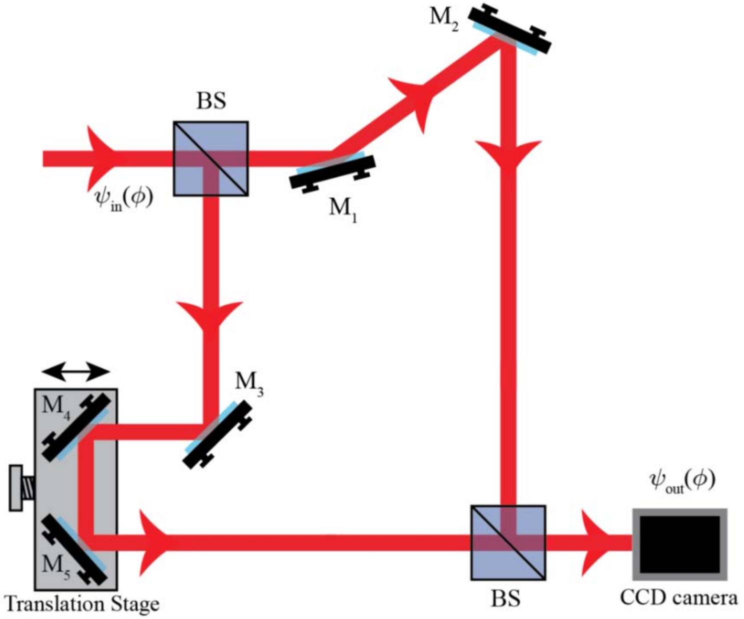
Measuring Orbital Angular Momentum of Light (A new scheme)

State in the OAM basis

$$\psi_{\text{in}}(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi}$$

Uncorrelated eigenmodes:

$$W(\Delta\phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l\, e^{-il\Delta\phi} \Rightarrow S_l = \int_{-\pi}^{\pi} W(\Delta\phi) e^{il\Delta\phi} d\phi$$



$$\psi_{\text{out}}(\phi) = \sqrt{k_1} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi + i\omega t_1} + \sqrt{k_2} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{+il\phi + i\omega t_2}$$

$$I_{\text{out}}(\phi) = \langle \psi_{\text{out}}(\phi) \psi^*{}_{\text{out}}(\phi) \rangle_e = \frac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2} W(2\phi) \cos\delta$$

$$\boxed{\delta \equiv \omega(t_1 - t_2)}$$

- $W(2\phi)$ gets encoded in the interferogram.
  So, a single-shot measurement of $I_{\text{out}}(\phi)$ yields $W(2\phi)$
- From $W(\Delta\phi)$, S_l can be computed, in a single shot manner.
- Still sensitive to background noise and other experimental parameters

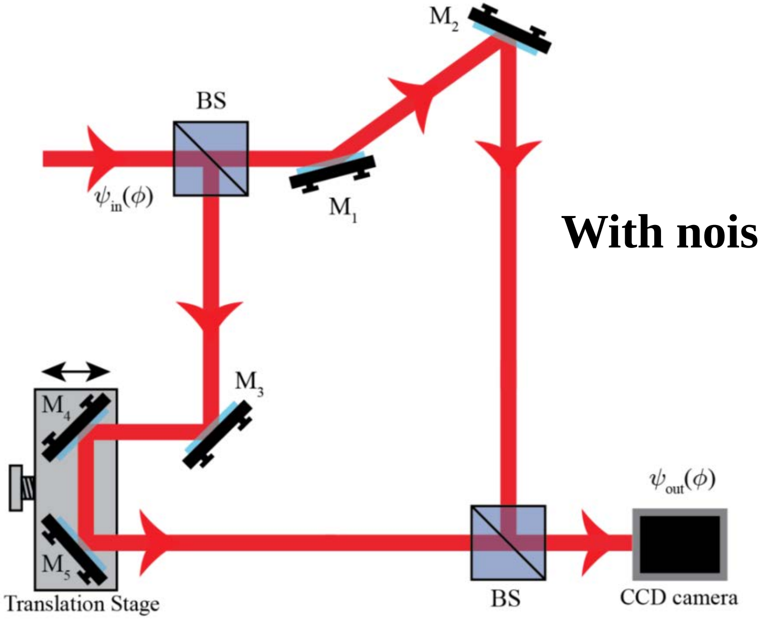
Measuring OAM spectrum of Light (in a noise-insensitive manner)

State in the OAM basis

$$\psi_{\text{in}}(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi}$$

Uncorrelated eigenmodes:

$$W(\Delta\phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l\, e^{-il\Delta\phi} \Rightarrow S_l = \int_{-\pi}^{\pi} W(\Delta\phi) e^{il\Delta\phi} d\phi$$



$$I_{\text{out}}(\phi) = \frac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2} W(2\phi) \cos\delta$$

With noise: $I^{\delta}{}_{\text{out}}(\phi) = I^{\delta}{}_{n}(\phi) + \frac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2} W(2\phi) \cos\delta$

$$\Delta I_{\text{out}}(\phi) \equiv I^{\delta_c}{}_{\text{out}}(\phi) - I^{\delta_d}{}_{\text{out}}(\phi)$$

$$\Delta I_{\text{out}}(\phi) = \Delta\, I\,_{n}(\phi) + 2\sqrt{k_1 k_2} W(2\phi)\,(\cos\delta_c - \cos\delta_d)$$

If shot-to-shot noise is the same: $\Delta\, I\,_{n}(\phi) = 0$

Then: $\Delta I_{\text{out}}(\phi) = 2\sqrt{k_1 k_2}\,(\cos\delta_c - \cos\delta_d) W(2\phi) \quad \propto W(2\phi)$

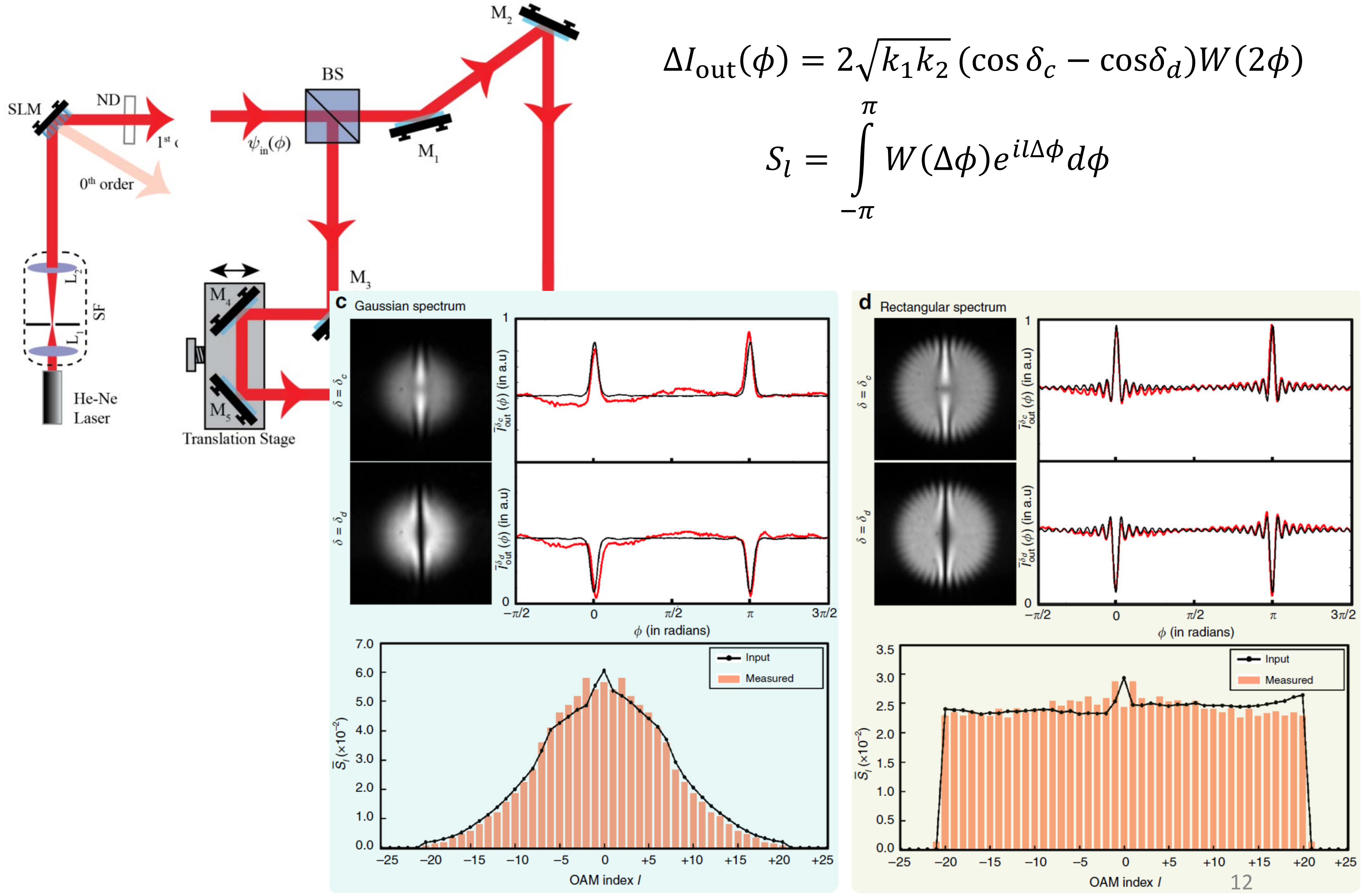
- $\Delta I_{\text{out}}(\phi)$ has the same functional form as $W(2\phi)$.
- So by measuring $\Delta I_{\text{out}}(\phi)$ the spectrum S_l can be obtained in a single-shot as well as in a noise-insensitive manner

G. Kulkarni, R. Sahu, O. S. Magana-Loaiza, R. W. Boyd, and A. K. Jha; **Nature Communications, 8, 1054 (2017)**

Experimental measurement of OAM spectrum of Light (classical)

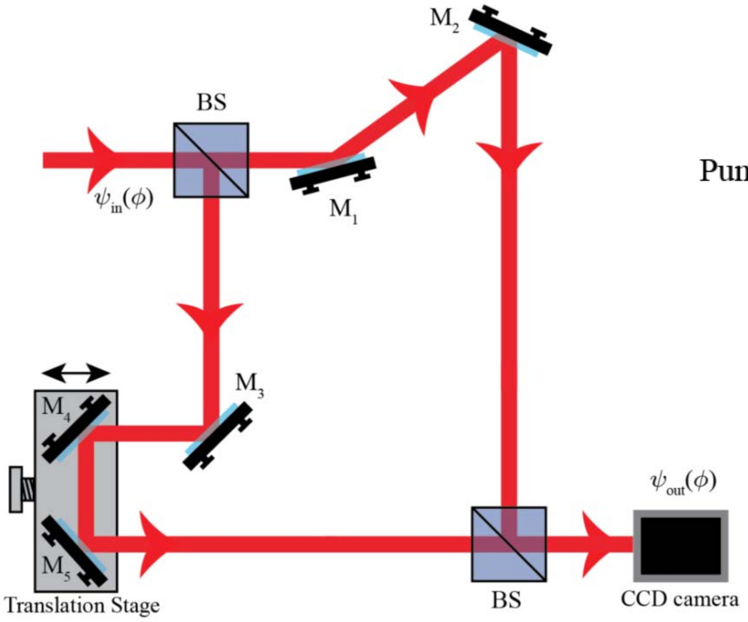
$$\Delta I_{\mathrm{out}}(\phi) = 2\sqrt{k_1 k_2}\,(\cos\delta_c - \cos\delta_d)W(2\phi)$$

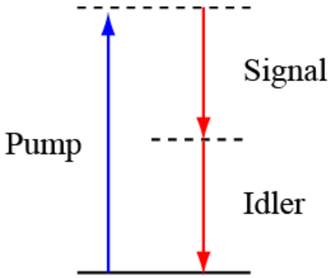
$$S_l = \int_{-\pi}^{\pi} W(\Delta\phi)e^{il\Delta\phi}d\phi$$



G. Kulkarni, R. Sahu, O. S. Magana-Loaiza, R. W. Boyd, and A. K. Jha; **Nature Communications, 8, 1054 (2017)**

Measuring Orbital Angular Momentum of Light (Quantum)





$$|\psi\rangle = \sum_{l=-\infty}^{\infty} \sqrt{S_l}\ |l\rangle_s |-l\rangle_i$$

OAM-Entangled State:

- S_l is called the angular Schmidt spectrum
- Very important to have an accurate measurement of S_l
- The current methods involve coincidence measurements, which is very difficult.

Nature 412 **313** (2001)
Phys Rev A **76**, 042302 (2007)
Phys Rev Lett **104**, 020505 (2010)
New J Phys **14**, 073046 (2012)

Angular coherence function of the signal photon is

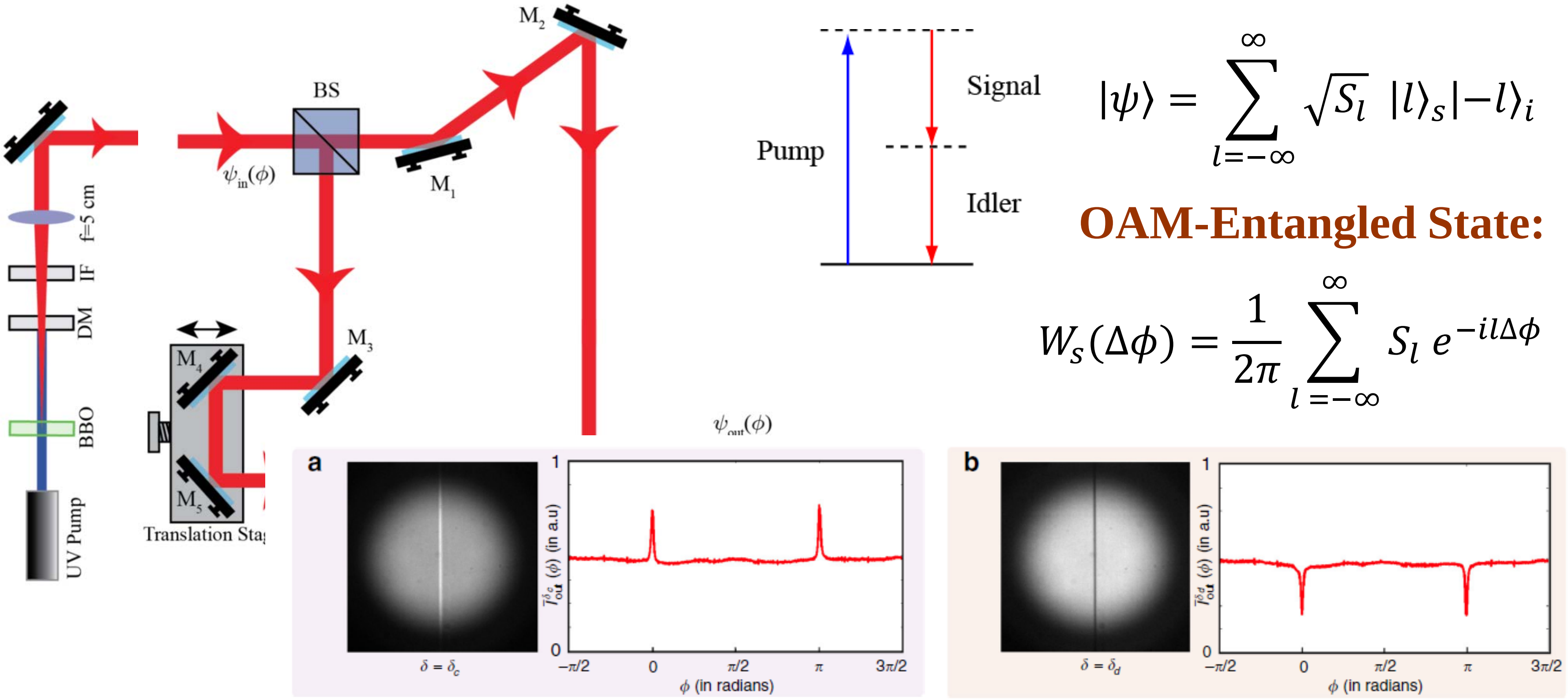
$$W_s(\phi_1, \phi_2)\ \rightarrow W_s(\Delta\phi)\ = \frac{1}{2\pi} \sum_{l\ =-\infty}^{\infty} S_l\ e^{-il\Delta\phi}$$

- The OAM spectrum of signal photon is same as the angular Schmidt spectrum of the entangled state

A K Jha, G S Agarwal, R W Boyd, PRA **84**, 063847 (2011)

G. Kulkarni, R. Sahu, O. S. Magana-Loaiza, R. W. Boyd, and A. K. Jha; **Nature Communications, 8, 1054 (2017)**

Measuring Orbital Angular Momentum of Light (Quantum)



$$|\psi\rangle = \sum_{l=-\infty}^{\infty} \sqrt{S_l}\ |l\rangle_s |{-l}\rangle_i$$

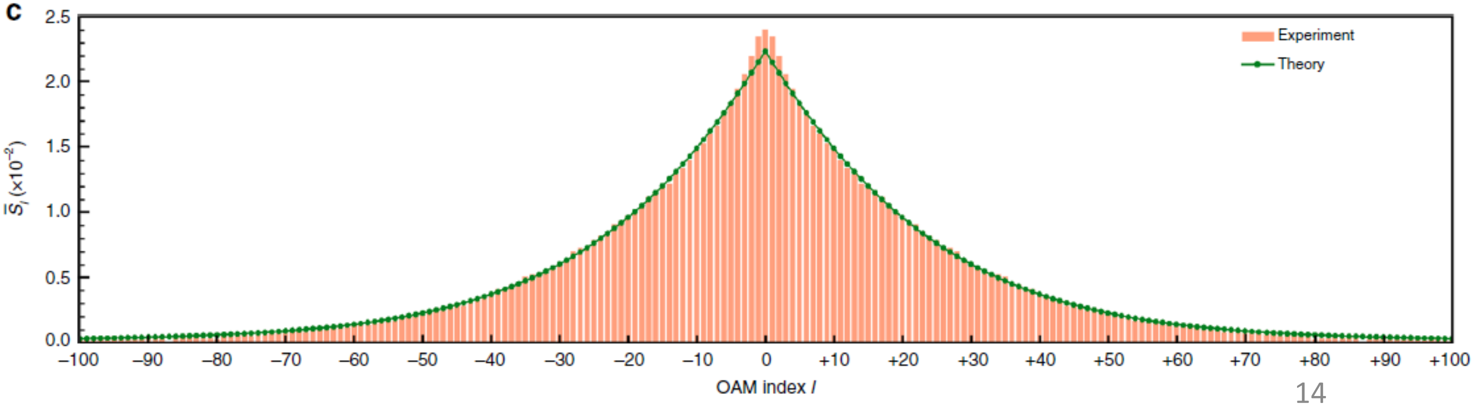
OAM-Entangled State:

$$W_s(\Delta\phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l\, e^{-il\Delta\phi}$$

Schmidt Number

$$K = \frac{1}{\sum_l {S_l}^2}$$

$K = 82.1$



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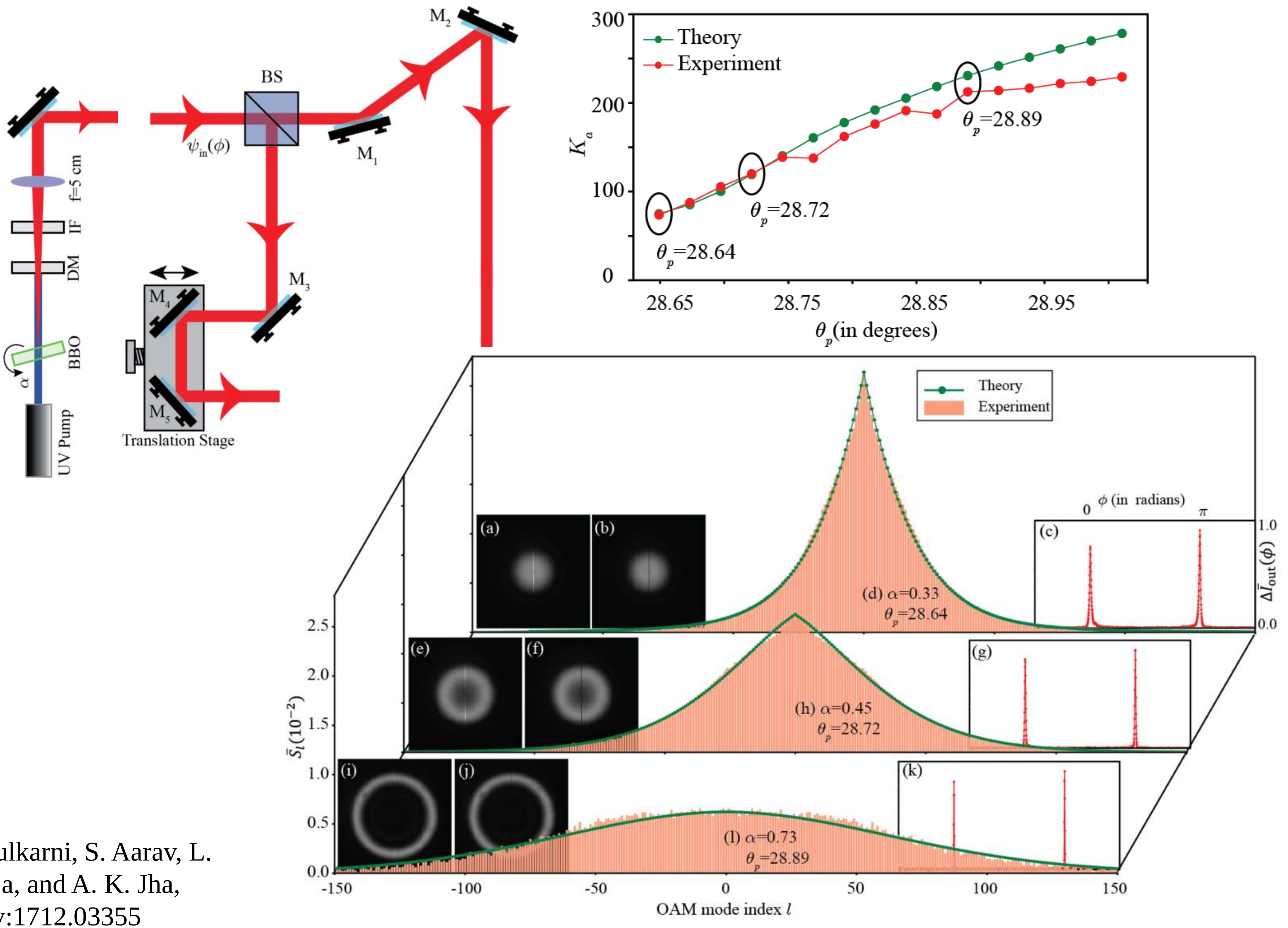
Measuring Orbital Angular Momentum of Light (Quantum)



G. Kulkarni, S. Aarav, L. Taneja, and A. K. Jha, Arxiv:1712.03355

States in transverse momentum basis: $\langle \boldsymbol{\rho} | \boldsymbol{q} \rangle = e^{i\,\boldsymbol{q}\cdot\boldsymbol{\rho}}$

State in the transverse momentum basis

$$V(\boldsymbol{\rho}, z) = \int_{-\infty}^{\infty} a(\boldsymbol{q}) e^{i\,\boldsymbol{q}\cdot\boldsymbol{\rho}}\, e^{-\frac{iq^2 z}{2k_0 z}} d\boldsymbol{q}$$

When the eigenmodes are uncorrelated. $\langle a^*(\boldsymbol{q}_1) a\,(\boldsymbol{q}_2)\rangle_e = I(\boldsymbol{q}_1)\delta(\boldsymbol{q}_1 - \boldsymbol{q}_2)$ **Diagonal Mixed States**

$$W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \langle V^*(\boldsymbol{\rho}_1, z) V\,(\boldsymbol{\rho}_2, z)\rangle_e$$

$$\rightarrow W(\boldsymbol{\Delta\rho}\,) = \int_{-\infty}^{\infty} I(\boldsymbol{q}) e^{-i\,\boldsymbol{q}\cdot\boldsymbol{\Delta\rho}}\, d\boldsymbol{q}$$

Spatial Wiener-Khintchine theorem

Spatial correlation function

Spectral Intensity

- Such partially coherent fields have propagation-invariant spatial correlation function
- The correlation function is the Fourier transform of the spectral intensity
- Partially coherence fields are extremely important for imaging through scattering, etc.

B. Redding, M. A. Choma, and H. Cao, Nature Photonics 6, 355 (2012).

Aim: Measure the spatial correlation function $W(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2)$

For diagonal states it yields the spectral intensity

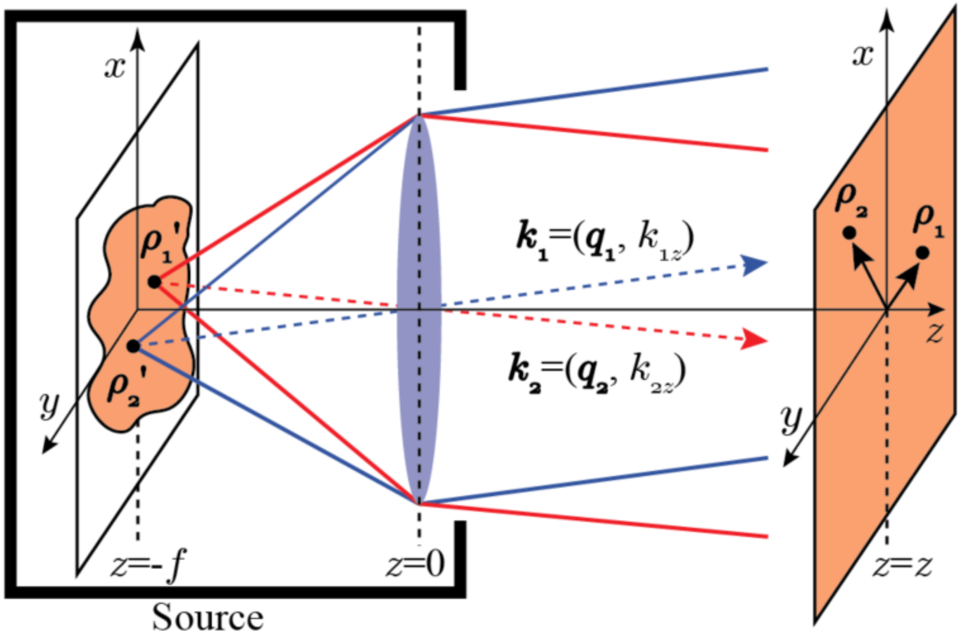
Efficient generation of spatially partially coherent field

How to produce partially coherent field?

- Most light sources (sunlight, light bulbs, etc.)
- Take a spatially coherent field and introduce randomness to it.

Phys. Rev. A **43**, 7079 (1991).
Opt. Express **13**, 9629 (2005).
Opt. Lett. **38**, 3452 (2013).
Opt. Lett **39**, 769 (2014)

- Start from a planar incoherent source



$$\langle V^*(\boldsymbol{\rho'}_1)V(\boldsymbol{\rho'}_2)\rangle_e = I_s(\boldsymbol{\rho'}_1)\delta(\boldsymbol{\rho'}_1-\boldsymbol{\rho'}_2)$$

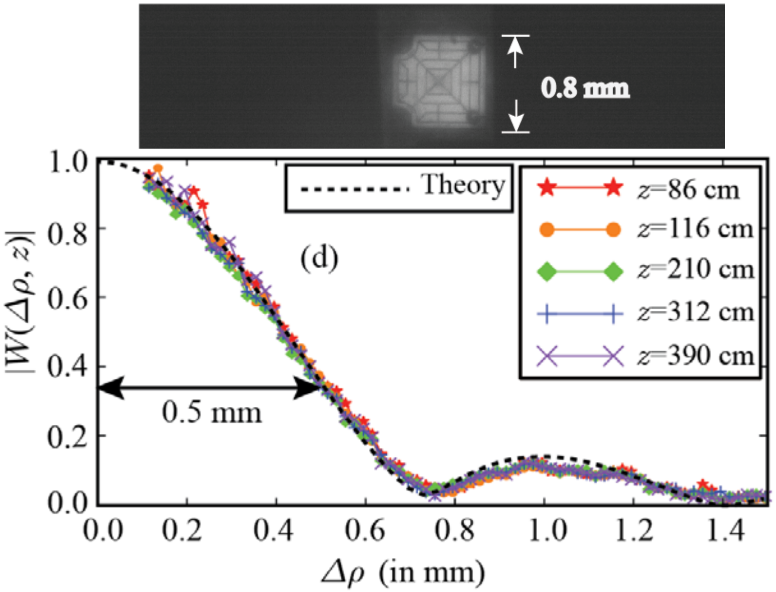
$$\langle a^*(\boldsymbol{q}_1)a\,(\boldsymbol{q}_2)\rangle_e = I(\boldsymbol{q}_1)\delta(\boldsymbol{q}_1-\boldsymbol{q}_2)$$

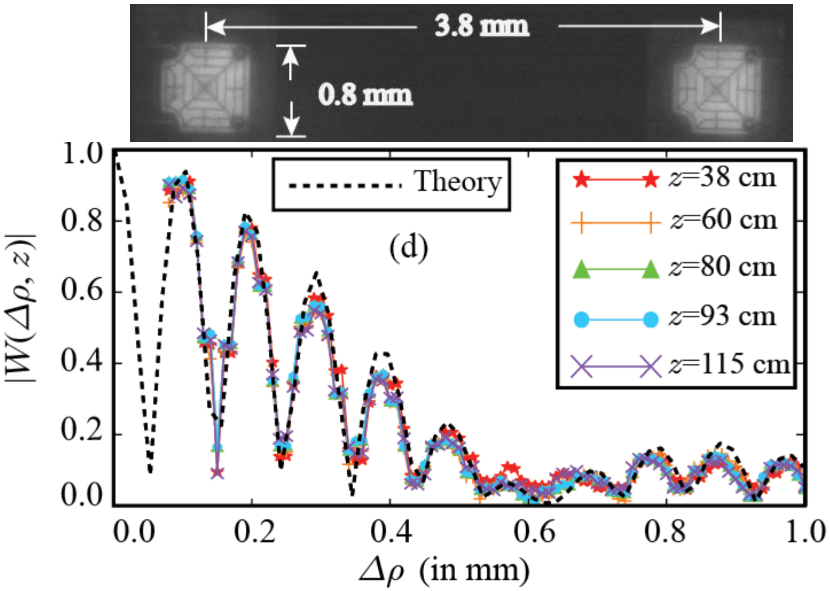
Diagonal Mixed States

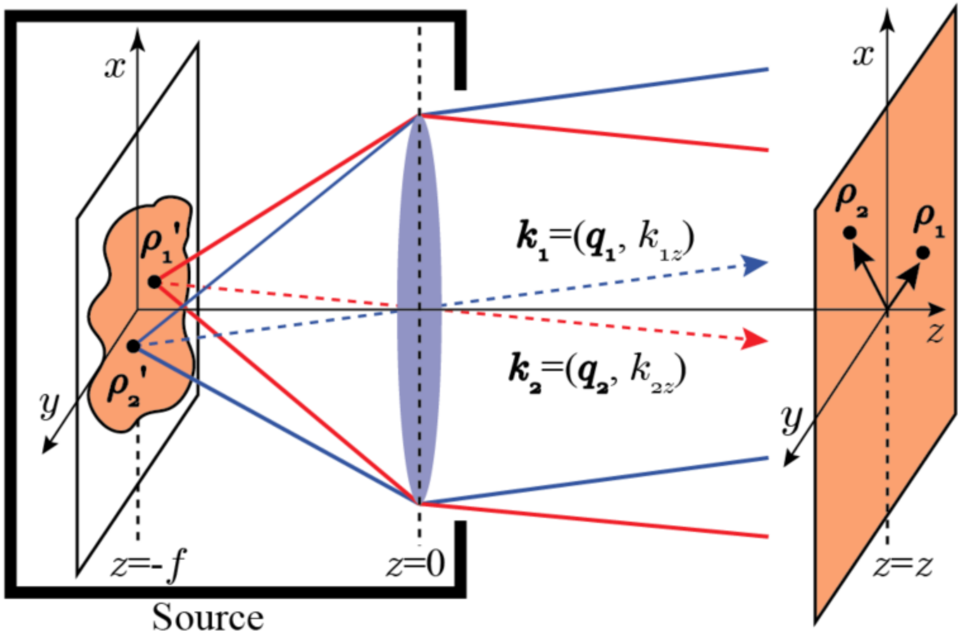
$$W(\boldsymbol{\Delta\rho}\,) = \int_{-\infty}^{\infty} I(\boldsymbol{q})e^{-i\,\boldsymbol{q}\cdot\boldsymbol{\Delta\rho}}\,d\boldsymbol{q}$$

S. Aarav, A. Bhattacharjee, H. Wanare, and A. K. Jha, PRA **96**, 033815 (2017)

Efficient generation of spatially partially coherent field







How do we measure the spatial correlation function?

$$\langle V^*(\boldsymbol{\rho'}_1)V(\boldsymbol{\rho'}_2)\rangle_e = I_s(\boldsymbol{\rho'}_1)\delta(\boldsymbol{\rho'}_1 - \boldsymbol{\rho'}_2)$$

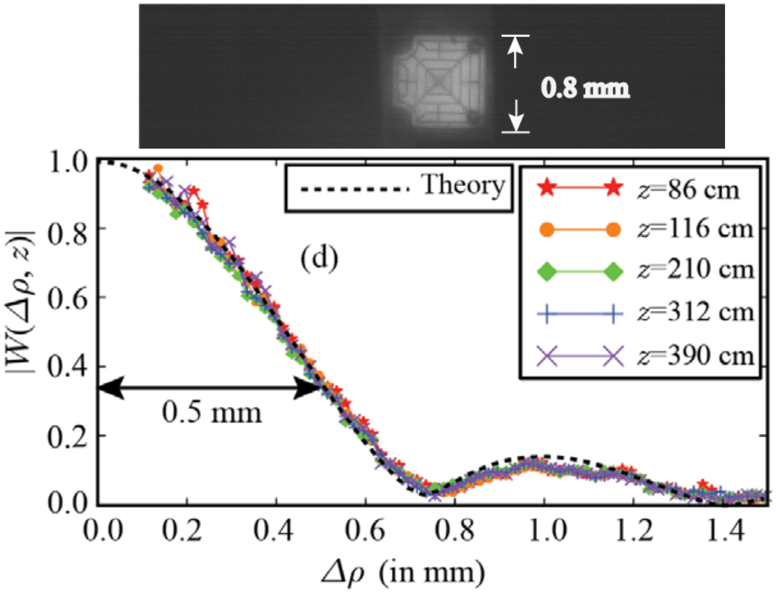
$$\langle a^*(\boldsymbol{q}_1)a\,(\boldsymbol{q}_2)\rangle_e = I(\boldsymbol{q}_1)\delta(\boldsymbol{q}_1 - \boldsymbol{q}_2)$$

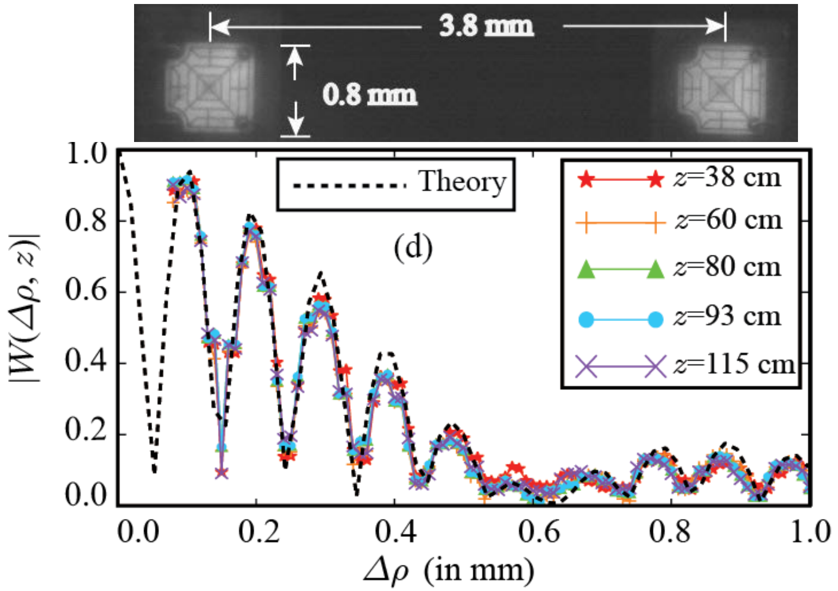
Diagonal Mixed States

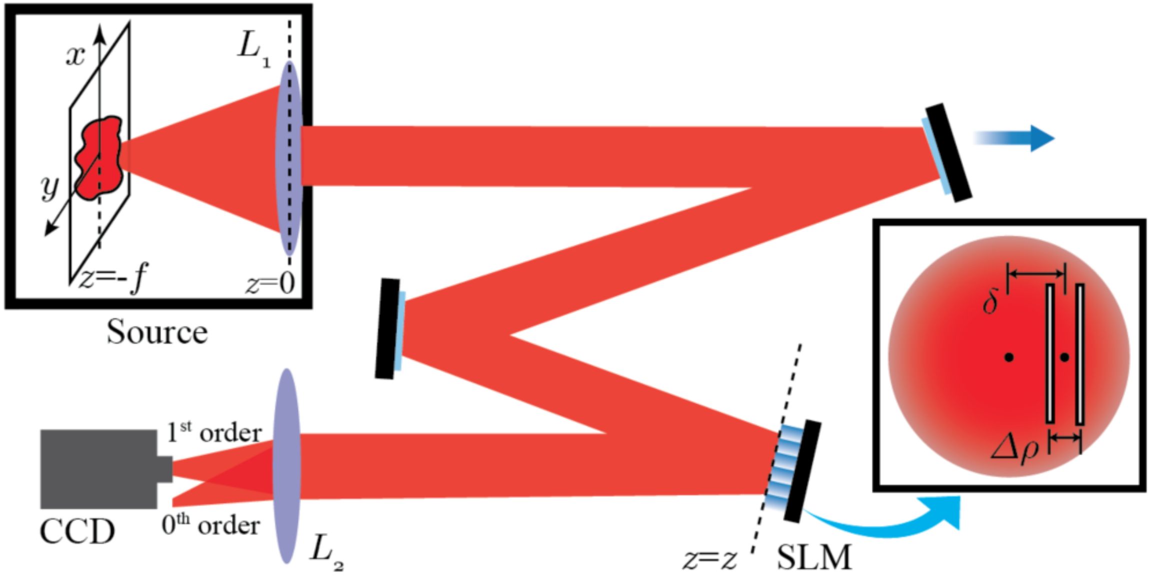
$$W(\boldsymbol{\Delta\rho}) = \int_{-\infty}^{\infty} I(\boldsymbol{q})e^{-i\,\boldsymbol{q}\cdot\boldsymbol{\Delta\rho}}\,d\boldsymbol{q}$$

S. Aarav, A. Bhattacharjee, H. Wanare, and A. K. Jha, PRA **96**, 033815 (2017)

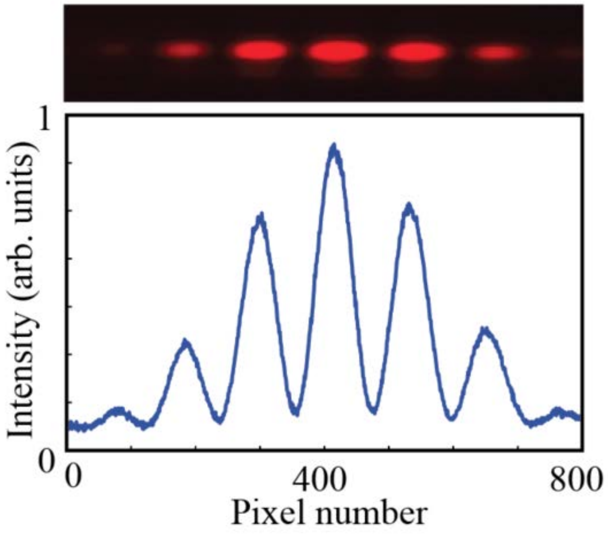
Efficient generation of spatially partially coherent field







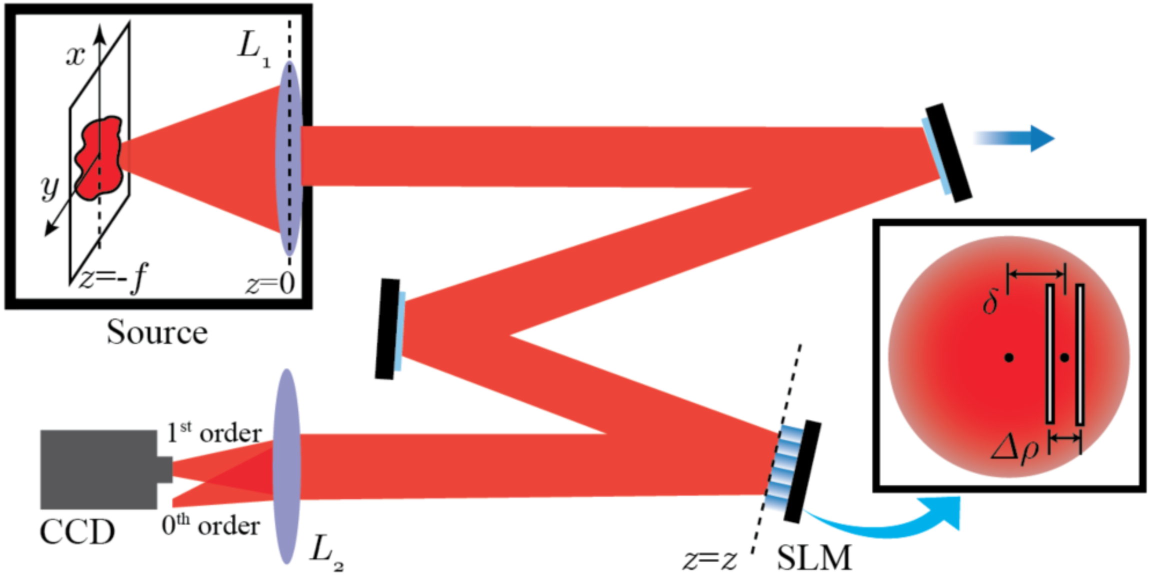
How do we measure the spatial correlation function?



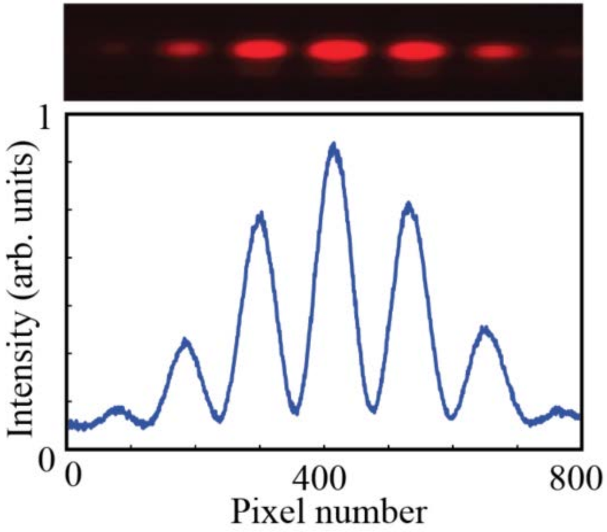
S. Aarav, A. Bhattacharjee, H. Wanare, and A. K. Jha, PRA **96**, 033815 (2017)

Conventional methods for measuring spatially partially coherent field

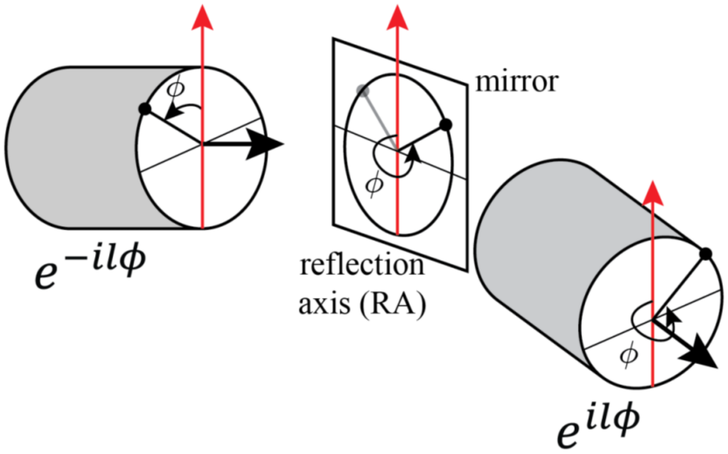
This is not a very efficient method for measuring spatial correlation functions.



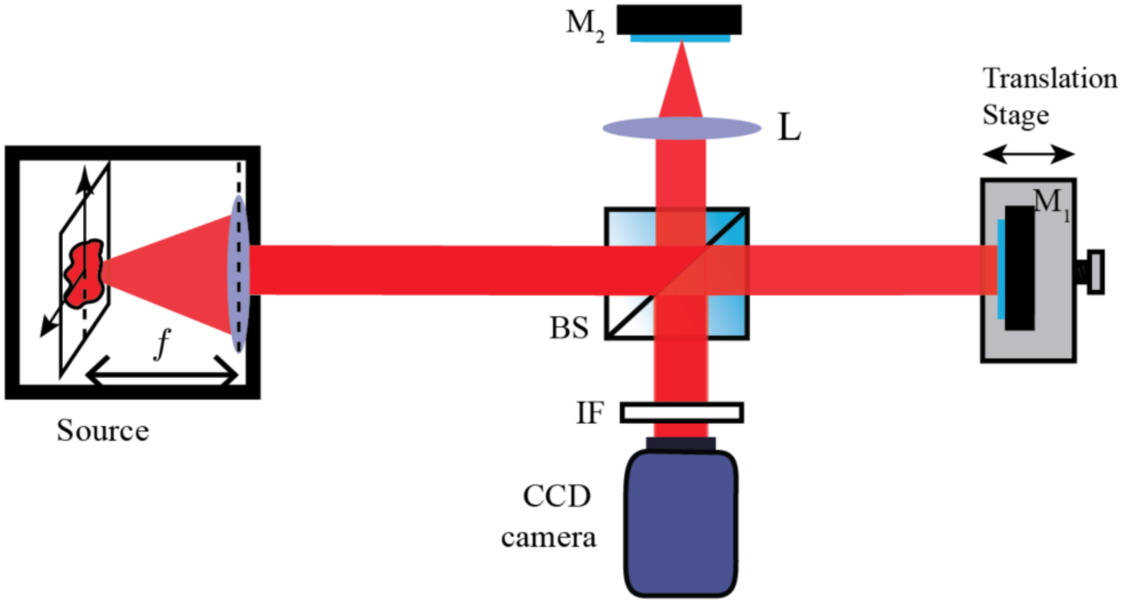
How do we measure the spatial correlation function?



Single-shot technique for measuring the spatial correlation function



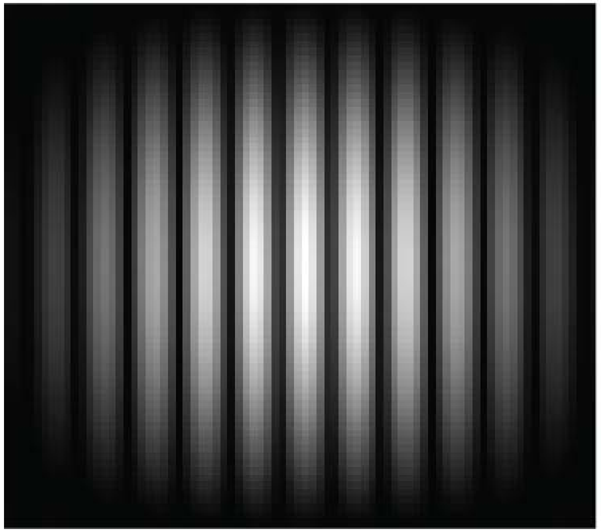
A reflection flips the wave-front along the reflection axis



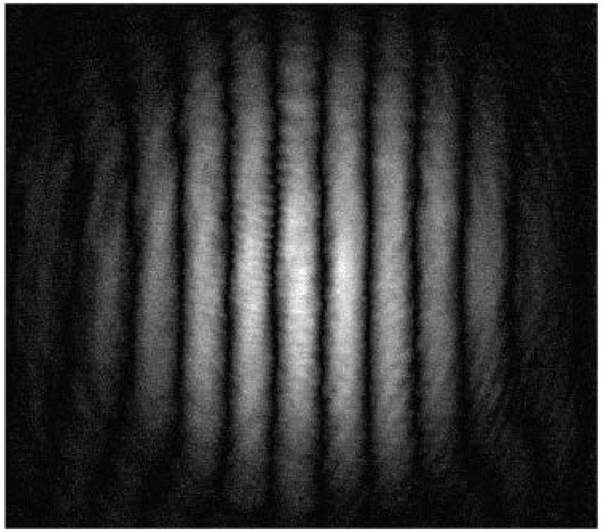
A converging lens flips the wavefront in both x- and y-directions.

A. Bhattacharjee, S. Aarav, and A. K. Jha, Arxiv:1712.04517

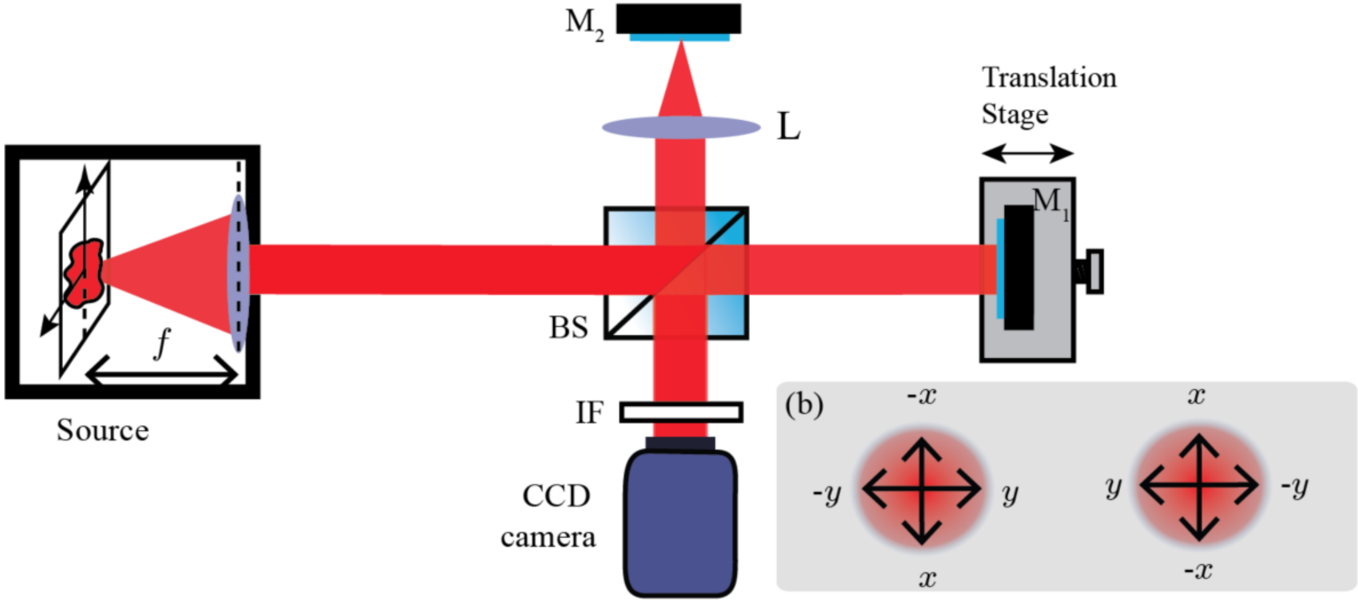
Single-shot technique for measuring the spatial correlation function



Theory



Experiment



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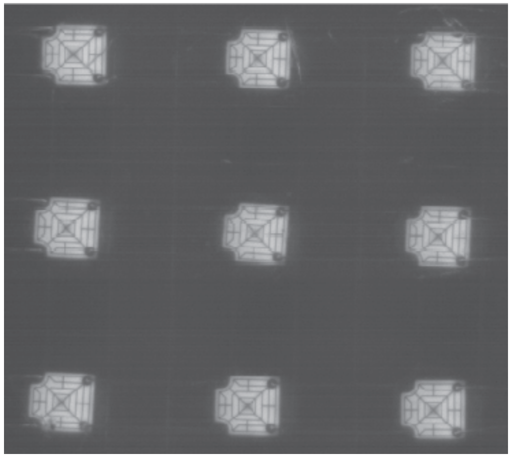
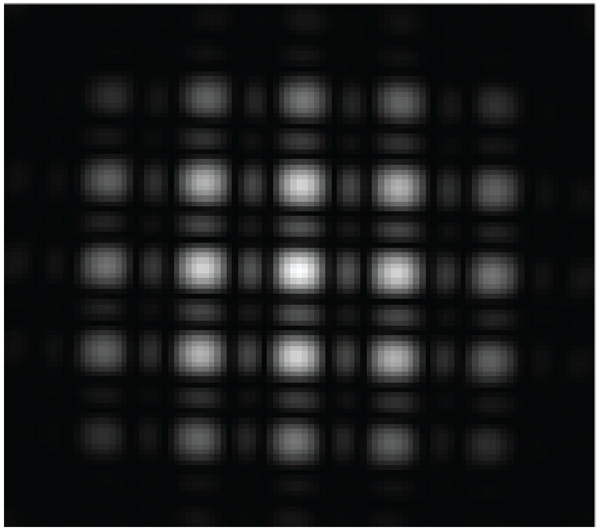
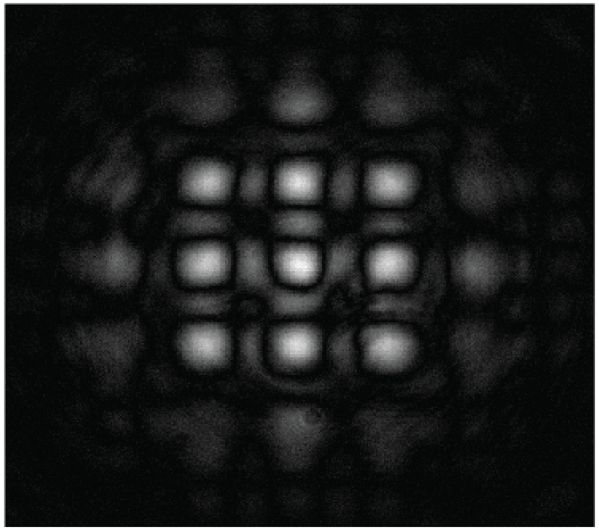
Single-shot technique for measuring the spatial correlation function

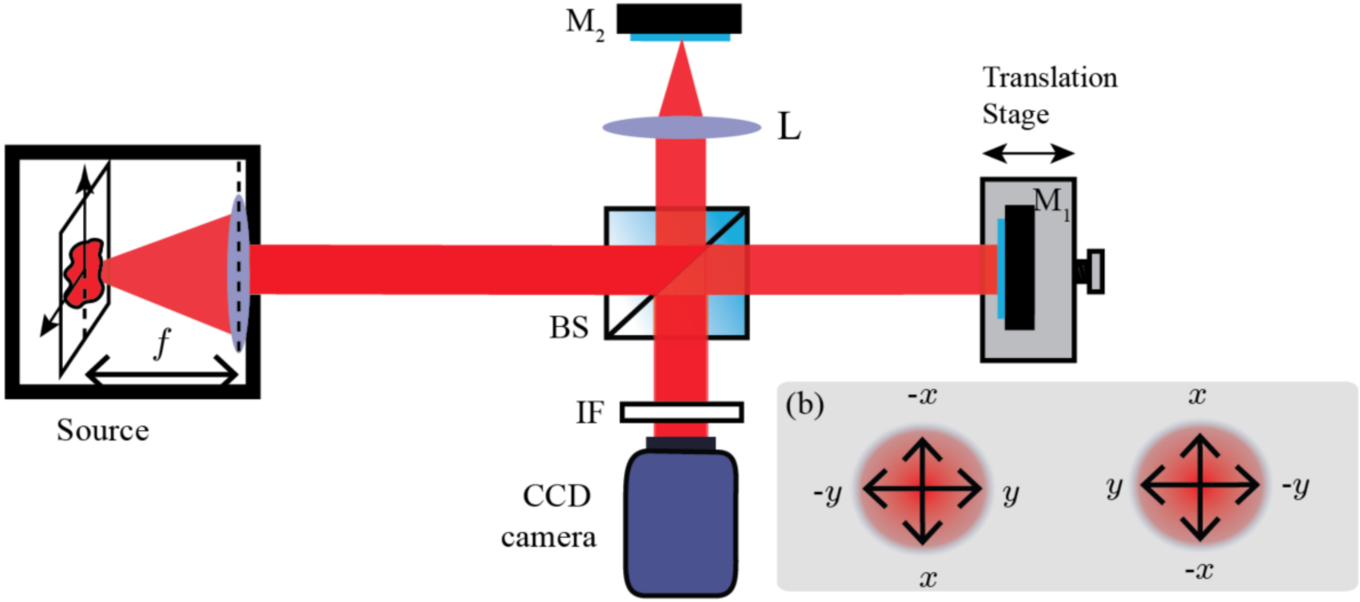




Theory



Experiment



A converging lens flips the wavefront in both x- and y-directions.

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Conclusions

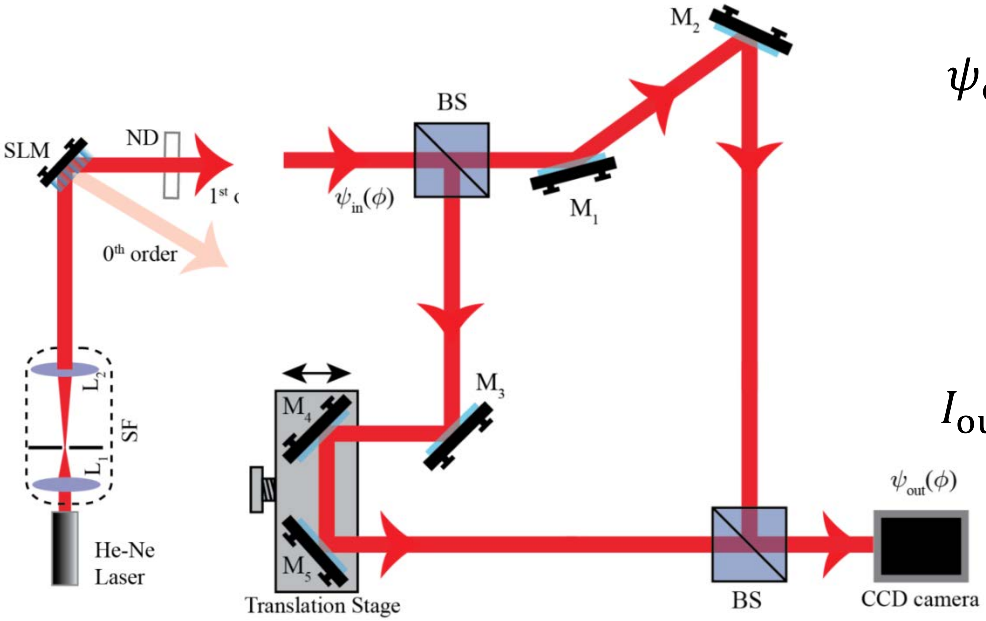
- Demonstrated a single-shot technique for measuring the angular correlation function.
- For diagonal mixed states, the angular correlation function yields the OAM spectrum through a Fourier transform.
- The technique can be used for measuring the angular Schmidt spectrum of OAM-entangled states in a single-shot manner without requiring coincidence detection.
- Extended the technique for measuring the spatial correlation function in a single-shot manner.

Acknowledgements

- Initiation grant from IIT Kanpur
- Research grant from SERB, DST

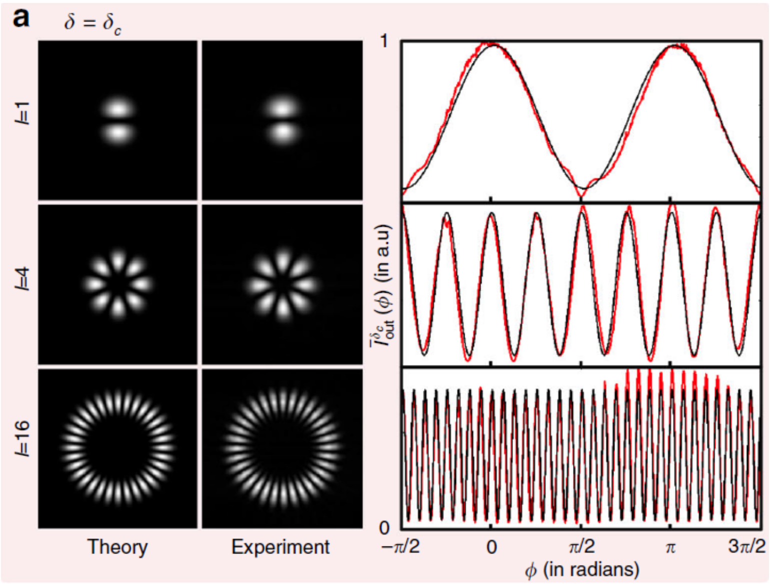
Thank you for your attention

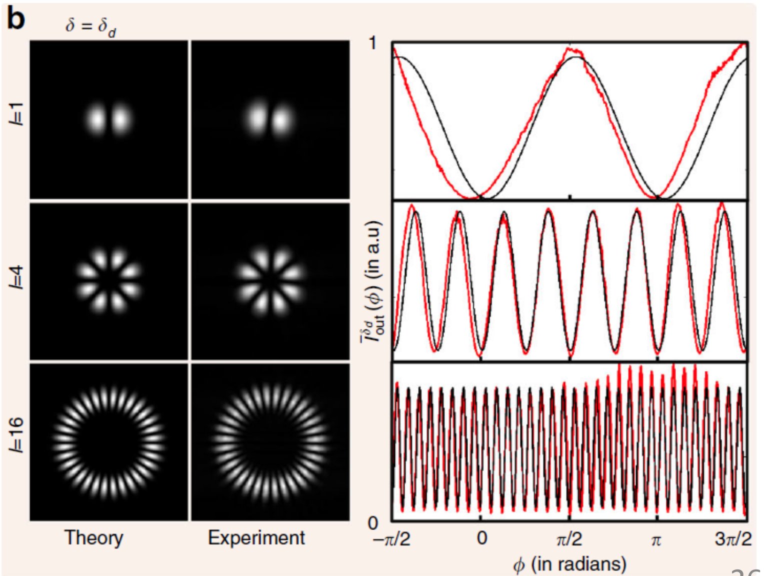
Experimental measurement of OAM spectrum of Light (classical)



$$\psi_{\mathrm{out}}\,(\phi) = \sqrt{k_1}\frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty}\alpha_l e^{-il\phi+i\omega t_1} + \sqrt{k_2}\frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty}\alpha_l e^{+il\phi+i\omega t_2}$$

$$I_{\mathrm{out}}(\phi) = \frac{k_1+k_2}{2\pi} + 2\sqrt{k_1k_2}W(2\phi)\cos\delta$$





G. Kulkarni, R. Sahu, O. S. Magana-Loaiza, R. W. Boyd, and A. K. Jha; **Nature Communications, 8, 1054 (2017)**