# *Entanglement generation in accelerated quantum walks*

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# *Discrete-time quantum walks - Motivation*

- Superposition and interference : Evolution (dynamics) exploits superposition and interference aspects of quantum mechanics entangling the Hilbert space involved in the dynamics ( particle and position space)
  Explores multiple possible paths simultaneously with amplitude corresponding to different paths interfering

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- Experimentally implemented and control over the dynamics demonstrated
  NMR system, ion traps, photons in optical waveguide, neutral atoms on optical lattice.......

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- Quantum algorithms and other quantum information processing tasks: quantum memory, quantum state transfer

  (a) Quantum Information Processing, Volume 13, Issue 6, Pages 1313-1329 (2014) & (b) European Physics Letters 110, 10005 (2015)

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- Simulation of photosynthesis and synthesis of topological insulators

  (a) Journal of Chemical Physics 129, 174106 (2008) & (b) arXiv:1502.00436

  Environment-Assisted Quantum Walks in Photosynthetic Energy Transfer

# *Discrete-time quantum walk*

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- Walk is defined on the Hilbert space $\mathcal{H} = \mathcal{H}_c \otimes \mathcal{H}_p$

  $\mathcal{H}_c$ (particle) is spanned by $|\uparrow\rangle$ and $|\downarrow\rangle$
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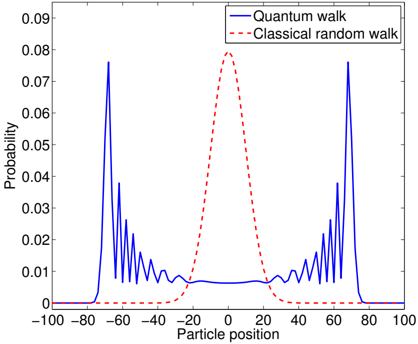
  - Coin operation - Hadamard operation : $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
  - Conditional unitary shift operation $S$:

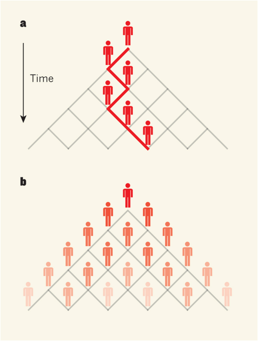
$$S = \sum_{j\in\mathbb{Z}} \left[ |\uparrow\rangle\langle\uparrow| \otimes |j-1\rangle\langle j| + |\downarrow\rangle\langle\downarrow| \otimes |j+1\rangle\langle j| \right]$$

state $|\uparrow\rangle$ moves to the left and state $|\downarrow\rangle$ moves to the right

# *Hadamard walk*

- Each step of QW (Hadamard walk) : $W = S(H \otimes \mathbb{1})$




100 step of CRW and QW $[S(H \otimes \mathbb{1})]^{100}$ on a particle with initial state $\frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle)$

- G. V. Riazanov (1958), R. Feynman (1986)
- K.R. Parthasarathy, Journal of applied probability 25, 151-166 (1988)
- Y. Aharonov, L. Davidovich and N. Zugury, Phys. Rev. A, 48, 1687 (1993)
- Use of word Quantum ~~random~~ walk

# $QW$ using generalized quantum coin operation

- Hadamard walk :
  $|\Psi_{in}\rangle = |\uparrow\rangle \otimes |j=0\rangle \rightarrow$ peak to left
  $|\Psi_{in}\rangle = |\downarrow\rangle \otimes |j=0\rangle \rightarrow$ peak to right

# *QW using generalized quantum coin operation*

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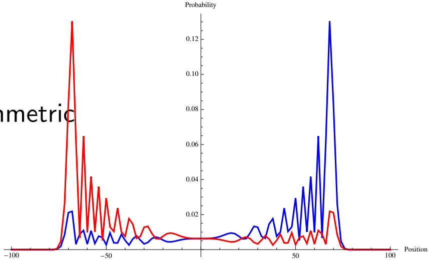
  $|\Psi_{in}\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow\rangle \pm i|\downarrow\rangle\Big] \otimes |j=0\rangle \rightarrow$ symmetric



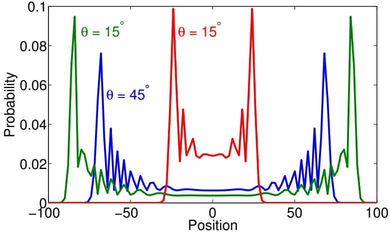

- SU(2) operation :

$$B_{\xi,\theta,\zeta} \equiv \begin{bmatrix} e^{i\xi}\cos(\theta) & e^{i\zeta}\sin(\theta) \\ -e^{-i\zeta}\sin(\theta) & e^{-i\xi}\cos(\theta) \end{bmatrix}$$

- Each step of generalized QW :

  $W_{\xi,\theta,\zeta} = S(B_{\xi,\theta,\zeta} \otimes \mathbb{1})$

  $\left(W_{\xi,\theta,\zeta}\right)^t|\Psi_{in}\rangle$ implements $t$ steps of generalized DQW



# *Discrete-time quantum walks in 2D*

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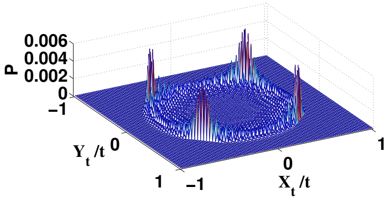
Gorver walk on 2D and its limitations :

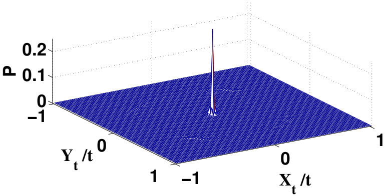
$|\Psi_{in}\rangle = \frac{1}{2}\left(|0\rangle - |1\rangle - |2\rangle + |3\rangle\right) \otimes |x = 0, y = 0\rangle$

$$C = \sum_{x,y \in \mathbb{Z}} \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix} \otimes |x = 0, y = 0\rangle$$

$$S_{gv} = |0\rangle\langle 0| \otimes |x-1, y-1\rangle\langle x, y| + |1\rangle\langle 1| \otimes |x+1, y+1\rangle\langle x, y| \\ + |2\rangle\langle 2| \otimes |x-1, y+1\rangle\langle x, y||3\rangle\langle 3| \otimes |x+1, y-1\rangle\langle x, y|$$

Probability of Grover walk



# *Self avoiding quantum walks in subspace*

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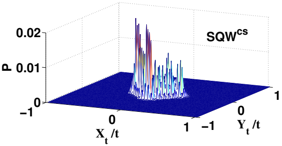
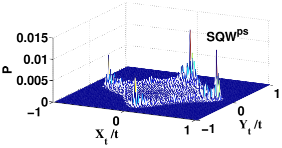
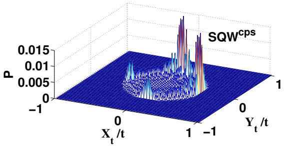
$$C^{sc} = \frac{1}{\sqrt{3}} \begin{bmatrix} 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & -1 & 0 & -1 \\ -1 & -1 & 1 & 0 \end{bmatrix} \; ; \; C^{sp} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 & -1 & 1 & 0 \\ 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$$C^{scp} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Probability of self avoiding quantum walk in coin, position and coin-position space



From discrete-time quantum walk to relativistic equations
:Klein-Gordon, Dirac

(free quantum field dynamics)

# *Symmetric evolution of DQW and hyperbolic PDE*

$$|\Psi_{in}\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow\rangle \pm i|\downarrow\rangle\Big] \otimes |x=0\rangle \qquad B(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

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In the form of left moving and right moving component

$$\psi^{\uparrow}_{x,t+1} = \cos(\theta)\psi^{\uparrow}_{x+1,t} - i\sin(\theta)\psi^{\downarrow}_{x-1,t}$$
$$\psi^{\downarrow}_{x,t+1} = \cos(\theta)\psi^{\downarrow}_{x-1,t} - i\sin(\theta)\psi^{\uparrow}_{x+1,t}$$

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Differential equation form in continuum limit :Klein-Gordon equation

$$\left[\frac{\partial^2}{\partial t^2} - \cos(\theta)\frac{\partial^2}{\partial x^2} + 2[1-\cos(\theta)]\right]\psi^{\uparrow(\downarrow)}_{x,t} = 0$$

CMC, SB and RS, PRA, **81** 062340 (2010)

# *Continuum limit and simulation of Dirac equation*

## Dirac equation

$$\left(i\hbar\frac{\partial}{\partial t} - \hat{\mathbf{H}}_{\mathbf{D}}\right)\Psi = \left(i\hbar\frac{\partial}{\partial t} + i\hbar c\hat{\alpha}\cdot\frac{\partial}{\partial x} - \hat{\beta}mc^2\right)\Psi = 0$$

# Continuum limit and simulation of Dirac equation

Dirac equation

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From DTQW

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when $\theta = 0$, the expression in continuum limit takes the form

$$\left[i\hbar\frac{\partial}{\partial t} - i\hbar\sigma_3\frac{\partial}{\partial x}\right]\Psi(x,t) = 0$$

Massless Dirac equation

David Mayer (1996) ; Fredrick Strauch (2006) ; CMC (2010)

# *Massive Dirac equation*

# Massive Dirac equation

Dirac Equation in 1D : For $\theta \neq 0$

Giuseppe Molfetta - Fabrice Debbasch (2013) and CMC (2013)

$$W_\theta = S(B_\theta \otimes \mathbb{1}) = \exp(-i\hat{H}(\theta)\tau)$$

$$\hat{H}_{\sigma_z}(\theta) = \hat{R}^\dagger(\tfrac{\theta}{2})\hat{H}(\theta)\hat{R}(\tfrac{\theta}{2}) = -i\hat{\sigma}_z \cdot \frac{\partial}{\partial z} + \hat{\sigma}_y \sin(\theta)$$

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Using different Pauli basis it can be extended to higher dimensions

Scientific Reports 3, 2829 (2013) Evolution in 2D:

$$|\Psi^{sq}(t)\rangle = [\hat{W}_{\sigma_x}(\theta)\hat{W}_{\sigma_z}(\theta)]^t|\Psi^{sq}_{in}\rangle$$

Corresponding Hamiltonian:

$$\hat{H}^{sq}(\theta) = \hat{H}_{\sigma_x}(\theta) + \hat{H}_{\sigma_z}(\theta) = -i\left(\hat{\alpha}_x \cdot \frac{\partial}{\partial x} + \hat{\alpha}_z \cdot \frac{\partial}{\partial z}\right) + (\hat{\beta}_x + \hat{\beta}_z)\sin(\theta)$$

# *Discretization of space and time*

- Early Proposal to simplify the computation of field theories
  - Divisibility of Space and Time., Yukawa, H. Atomistics and the Prog. Theor. Phys. Suppl. 37 and 38, 512 (1966)
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- Quantum cellular automaton and quantum lattice gas
  - From quantum cellular automata to quantum lattice gases, Meyer, D. A. J., Stat. Phys. 85, 551 (1996)
  - The Feynman path integral for the Dirac equation, Riazanov, G. V., Sov. Phys. JETP 6 1107-1113 (1958)

Discretization of Field (Dirac) equation $\neq$ DQW

# *Dirac Cellular Automaton*

DH from the QCA by constructing the evolution operator for a system which is (1) unitary, (2) invariant under space translation, (3) covariant under parity transformation, (4) covariant under time reversal and (5) has a minimum of two internal degrees of freedom (spinor).

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$$U_{DA} = \begin{pmatrix} \alpha T_- & -i\beta \\ -i\beta & \alpha T_+ \end{pmatrix} = \alpha\{T_- \otimes |\uparrow\rangle\langle\uparrow| + T_+ \otimes |\downarrow\rangle\langle\downarrow|\} - i\beta(I \otimes \sigma_x)$$

where $\alpha$ corresponds to the hopping strength, $\beta$ corresponds to the mass term.

# Dirac Cellular Automaton

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$$U_{DA} = \begin{pmatrix} \alpha T_{-} & -i\beta \\ -i\beta & \alpha T_{+} \end{pmatrix} = \alpha\{T_{-} \otimes \left|\uparrow\right\rangle \left\langle\uparrow\right| + T_{+} \otimes \left|\downarrow\right\rangle \left\langle\downarrow\right|\} - i\beta(I \otimes \sigma_x)$$

where $\alpha$ corresponds to the hopping strength, $\beta$ corresponds to the mass term.Associated Hamiltonian in momentum basis, produces DH,

$$H(k) = \frac{a}{c\tau} \begin{pmatrix} -kc & mc^2 \\ mc^2 & kc \end{pmatrix}$$

with the identification $\beta = \frac{mac}{\hbar}$, $k$ is a eigenvalue of momentum operator.


•Derivation of the Dirac equation from principles of information processing, D Ariano, G. M. and Perinotti, P. Phys. Rev. A 90, 062106 (2014)

•Quantum field as a quantum cellular automaton: The Dirac free evolution in one dimension, Bisio, A., DAriano,G. M., Tosini, A. Annals of Physics 354, 244264 (2015)

# $DTQW$

The general form of $C \equiv B(\theta)$ is,

$$C = C(\xi,\theta,\phi,\delta) = e^{i\xi} e^{-i\theta\sigma_x} e^{-i\phi\sigma_y} e^{-i\delta\sigma_z} = e^{i\xi} \times$$

$$\begin{pmatrix} e^{-i\delta}(\cos(\theta)\cos(\phi) - i\sin(\theta)\sin(\phi)) & -e^{i\delta}(\cos(\theta)\sin(\phi) + i\sin(\theta)\cos(\phi)) \\ e^{-i\delta}(\cos(\theta)\sin(\phi) - i\sin(\theta)\cos(\phi)) & e^{i\delta}(\cos(\theta)\cos(\phi) + i\sin(\theta)\sin(\phi)) \end{pmatrix}$$

$$= e^{i\xi} \begin{pmatrix} F_{\theta,\phi,\delta} & G_{\theta,\phi,\delta} \\ -G^*_{\theta,\phi,\delta} & F^*_{\theta,\phi,\delta} \end{pmatrix}$$

# *DTQW*

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$$= e^{i\xi}\begin{pmatrix} F_{\theta,\phi,\delta} & G_{\theta,\phi,\delta} \\ -G^*_{\theta,\phi,\delta} & F^*_{\theta,\phi,\delta} \end{pmatrix}$$

The general form of the evolution operator

$$U_{QW} = e^{i\xi}\begin{pmatrix} F_{\theta,\phi,\delta}\ T_- & G_{\theta,\phi,\delta}\ T_- \\ -G^*_{\theta,\phi,\delta}\ T_+ & F^*_{\theta,\phi,\delta}\ T_+ \end{pmatrix} \neq \begin{pmatrix} \alpha T_- & -i\beta \\ -i\beta & \alpha T_+ \end{pmatrix}$$

$$U_{QW} = F_\theta\{\, T_- \otimes |\uparrow\rangle\langle\uparrow| + T_+ \otimes |\downarrow\rangle\langle\downarrow|\,\} + G_\theta\{\, T_- \otimes |\uparrow\rangle\langle\downarrow|) + T_+ \otimes |\downarrow\rangle\langle\uparrow|\,\}$$

By taking the value of $\theta \to 0$ the off-diagonal terms can be ignored and a massless DH can be recovered.

# *Split-step QW - 2-period QW*

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$$C(\theta_1,\phi_1,\delta_1) = \begin{pmatrix} F_{\theta_1,\phi_1,\delta_1} & G_{\theta_1,\phi_1,\delta_1} \\ -G^*_{\theta_1,\phi_1,\delta_1} & F^*_{\theta_1,\phi_1,\delta_1} \end{pmatrix},$$

$$C(\theta_2,\phi_2,\delta_2) = \begin{pmatrix} F_{\theta_2,\phi_2,\delta_2} & G_{\theta_2,\phi_2,\delta_2} \\ -G^*_{\theta_2,\phi_2,\delta_2} & F^*_{\theta_2,\phi_2,\delta_2} \end{pmatrix}$$

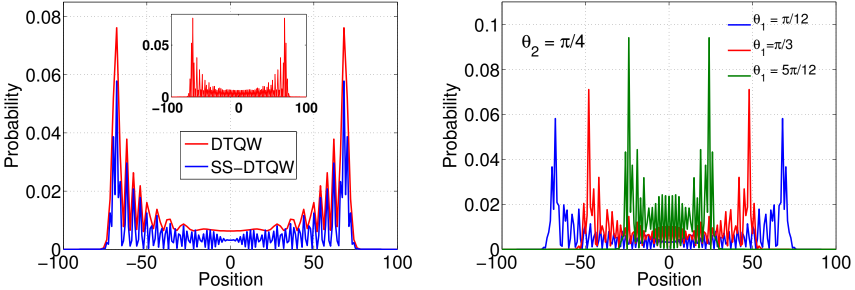
and a two half-shift operators,

$$S_- = \begin{pmatrix} T_- & 0 \\ 0 & I \end{pmatrix}, \qquad S_+ = \begin{pmatrix} I & 0 \\ 0 & T_+ \end{pmatrix}$$

$$U_{SQW} = S_+\Big(I \otimes C(\theta_2,\phi_2,\delta_2)\Big)S_-\Big(I \otimes C(\theta_1,\phi_1,\delta_1)\Big)$$

$$= \begin{pmatrix} F_{\theta_2,\phi_2,\delta_2}F_{\theta_1,\phi_1,\delta_1}\ T_- - G_{\theta_2,\phi_2,\delta_2}G^*_{\theta_1,\phi_1,\delta_1}\ I & F_{\theta_2,\phi_2,\delta_2}G_{\theta_1,\phi_1,\delta_1}\ T_- + G_{\theta_2,\phi_2,\delta_2}F^*_{\theta_1,\phi_1,\delta_1}\ I \\ -G^*_{\theta_2,\phi_2,\delta_2}F_{\theta_1,\phi_1,\delta_1}\ I - F^*_{\theta_2,\phi_2,\delta_2}G^*_{\theta_1,\phi_1,\delta_1}\ T_+ & -G^*_{\theta_2,\phi_2,\delta_2}G_{\theta_1,\phi_1,\delta_1}\ I + F^*_{\theta_2,\phi_2,\delta_2}F^*_{\theta_1,\phi_1,\delta_1}\ T_+ \end{pmatrix}$$

# *DCA and SS-QW*




SSQW $(\theta_1 = 0, \theta_2 = \pi/4)$ = DCA $\alpha = \beta = \frac{1}{\sqrt{2}}$ Substituting $\theta_1 = \phi_1 = \delta_1 = \delta_2 = 0$ we get,

$$U_{SQW} = \begin{pmatrix} \cos(\theta_2)T_- & -i\sin(\theta_2)I \\ -i\sin(\theta_2)I & \cos(\theta_2)T_+ \end{pmatrix}$$

which is in the same form as $U_{DA}$ where $\beta = \sin(\theta_2) \equiv \frac{mca}{\hbar}$ and $\alpha = \cos(\theta_2)$.

# *DCA and SS-QW cont.*

From the unitary operator we will recover the DH in the form,

$$H_{SQW} = -\frac{\hbar\cos^{-1}\left(\cos(\theta_2)\cos\left(\frac{ka}{\hbar}\right)\right)}{\tau\sqrt{1-(\cos(\theta_2)\cos\left(\frac{ka}{\hbar}\right))^2}}\left[\cos(\theta_2)\sin\left(\frac{ka}{\hbar}\right)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} - \sin(\theta_2)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\right]$$

For smaller mass, $\theta_2 \approx 0$ and for smaller momentum, $k \approx 0$, $\sin\theta_2 \approx \theta_2, \cos\theta_2 \approx 1, \sin\left(\frac{ka}{\hbar}\right) \approx \frac{ka}{\hbar}, \cos\left(\frac{ka}{\hbar}\right) \approx 1$.

$$H_{SQW} \approx -\frac{a}{\tau}k\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} + \frac{\hbar}{\tau}\theta_2\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

which is in a form of one-dimensional Dirac equation for a $\frac{1}{2}$ spinor, with the identifications, $\frac{a}{\tau} = c$ and $\frac{\hbar\theta_2}{\tau} = mc^2$, so, $m = \frac{\hbar\theta_2\tau}{a^2}$.

Scientific Reports 6, 25779 (2016)

# SS-QW and DE

$$\left[\frac{\partial}{\partial t} - \cos(\theta_2)\begin{bmatrix}\cos(\theta_1) & -i\sin(\theta_1)\\ i\sin(\theta_1) & -\cos(\theta_1)\end{bmatrix}\frac{\partial}{\partial x} - \begin{bmatrix}\cos(\theta_1+\theta_2)-1 & -i\sin(\theta_1+\theta_2)\\ -i\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2)-1\end{bmatrix}\right]\begin{bmatrix}\psi^{\downarrow}_{x,t}\\ \psi^{\uparrow}_{x,t}\end{bmatrix} = 0$$

1. Setting $\theta_1 = 0$ and $\theta_2$ to a small value (mass of sub-atomic particles) :

$$i\hbar\left[\frac{\partial}{\partial t} - \left(1-\frac{\theta_2^2}{2}\right)\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\frac{\partial}{\partial x} + i\theta_2\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\right]\begin{bmatrix}\psi^{\downarrow}_{x,t}\\ \psi^{\uparrow}_{x,t}\end{bmatrix} \approx 0$$

2. Setting $\cos(\theta_1+\theta_2) = 1$ :

$$i\hbar\left[\frac{\partial}{\partial t} - \cos(\theta_2)\begin{bmatrix}\cos(\theta_1) & -i\sin(\theta_1)\\ i\sin(\theta_1) & -\cos(\theta_1)\end{bmatrix}\frac{\partial}{\partial x}\right]\begin{bmatrix}\psi^{\downarrow}_{x,t}\\ \psi^{\uparrow}_{x,t}\end{bmatrix} = 0$$

3. Setting $\theta_1$ to be extremely small and $\cos(\theta_1+\theta_2) = 1$ :

$$i\hbar\left[\frac{\partial}{\partial t} - \cos(\theta_2)\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\frac{\partial}{\partial x}\right]\begin{bmatrix}\psi^{\downarrow}_{x,t}\\ \psi^{\uparrow}_{x,t}\end{bmatrix} \approx 0$$

Bounds on the dynamics of periodic quantum walks and emergence of gapless and gapped Dirac equation

N. Pradeep Kumar, Radhakrishna Balu, Raymond Laflamme and CMC, Phys. Rev. A 97, 012116 (2018)

# *Accelerating quantum walks*

Replace $\theta$ in coin operation by $\theta(t) = \frac{\pi}{2} f(t)$ where $e^{-at} \geq f(t) \geq 0$.

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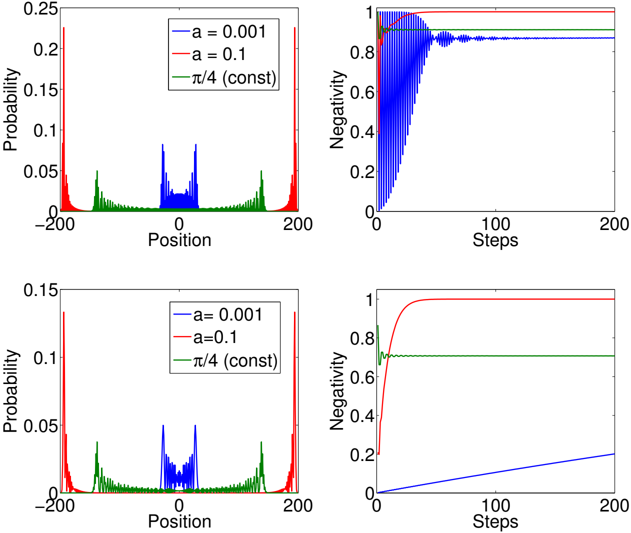

# *Entanglement generation from accelerated QW*

- Entanglement


$$|\Psi\rangle_{AB} = |a\rangle_A \otimes |b\rangle_B = |a\rangle_A |b\rangle_B$$

$$|a\rangle_A |b\rangle_B \to \sum c_{ab} |a\rangle_A |b\rangle_B$$

$$|\Psi\rangle_{AB} = \alpha|0\rangle_A|0\rangle_B \pm \beta|1\rangle_A|1\rangle_B$$

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$|\Psi_{in}\rangle = |00\rangle \rightarrow$ accelerated QW on two particle $\rightarrow$ entangled state
coin operation

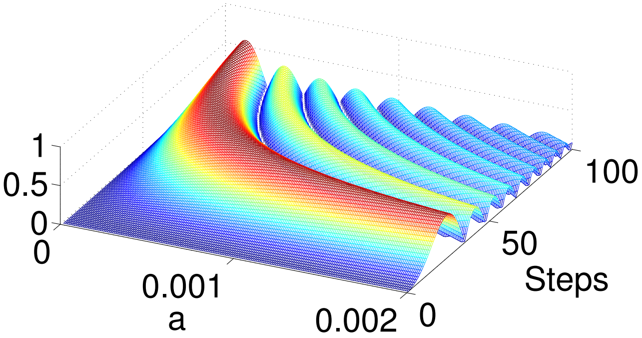
$$C[\theta(t)] = \begin{pmatrix} \cos[\theta(t)] & 0 & 0 & -i\sin[\theta(t)] \\ 0 & \cos[\theta(t)] & -i\sin[\theta(t)] & 0 \\ 0 & -i\sin[\theta(t)] & \cos[\theta(t)] & 0 \\ -i\sin[\theta(t)] & 0 & 0 & \cos[\theta(t)] \end{pmatrix}$$

shift-operator

$$S^1 \otimes S^2$$

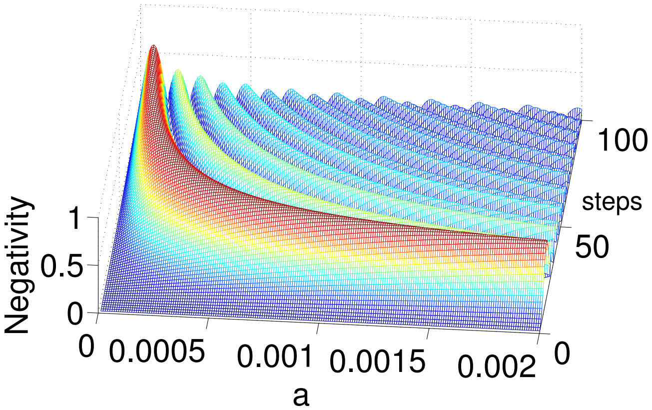
# *Entanglement Generation*

Standard QW



# *Entanglement Generation*

Split-step QW



# *Conclusion*

- QW can be used to generate entanglement between the massive particles

THANK YOU