Entanglement and coherence in distributed quantum networks

Alexander Streltsov

Gdańsk University of Technology, Poland
National Quantum Information Centre of Gdańsk, Poland

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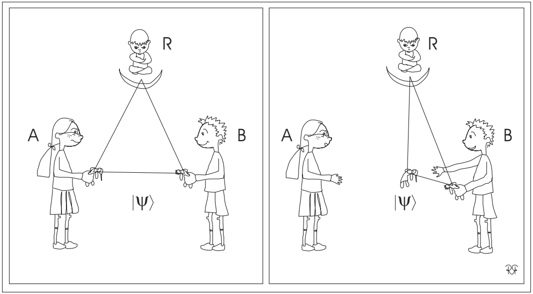
Outline

1. Quantum state merging and assisted entanglement distillation
2. Multipartite quantum state conversion
3. Assisted coherence distillation and incoherent state merging

Outline

Quantum state merging



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- Setting: Alice, Bob, and a referee share many copies of a pure state $|\psi\rangle^{RAB}$

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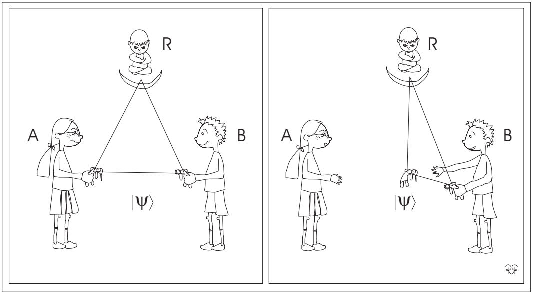

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- <u>Setting</u>: Alice, Bob, and a referee share many copies of a pure state $|\psi\rangle^{RAB}$
- <u>Aim of quantum state merging</u>: send Alice's system to Bob while preserving the total state, i.e., the final state $|\psi\rangle^{RBB'}$ is the same as $|\psi\rangle^{RAB}$ up to relabeling A and B'

Quantum state merging[1]

- For achieving quantum state merging, Alice and Bob have access to <u>shared singlets and a classical channel</u>

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- The minimal number of singlets, asymptotically needed per copy of the state $|\psi\rangle^{RAB}$, is given by the conditional entropy:

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- $S(A|B) \geq 0$: merging is possible with singlets at rate $S(A|B)$

- $S(A|B) < 0$: merging is possible without singlets, and Alice and Bob can obtain additional singlets at rate $-S(A|B)$

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Assisted entanglement distillation[1]

- Setting: Alice, Bob, and Charlie share many copies of a pure state $|\psi\rangle^{ABC}$

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- Solution: given a pure state $|\psi\rangle^{ABC}$, the optimal entanglement distillation rate between Alice and Bob with assistance of Charlie is equal to the regularized entanglement of assistance

$$E_a^{\infty}\left(\rho^{AB}\right) = \min\left\{S(\rho^A), S(\rho^B)\right\} \tag{2}$$

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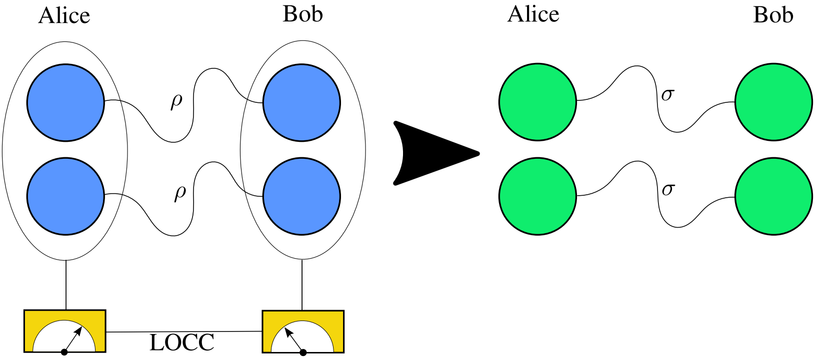
Multipartite quantum state conversion



- <u>Setting</u>: *N* parties share many copies of a multipartite state ρ

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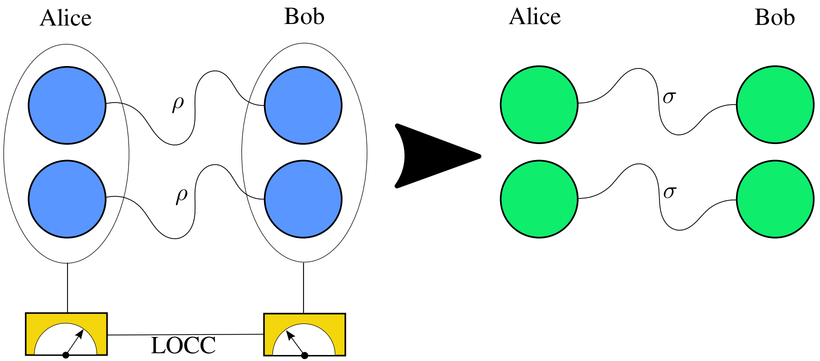

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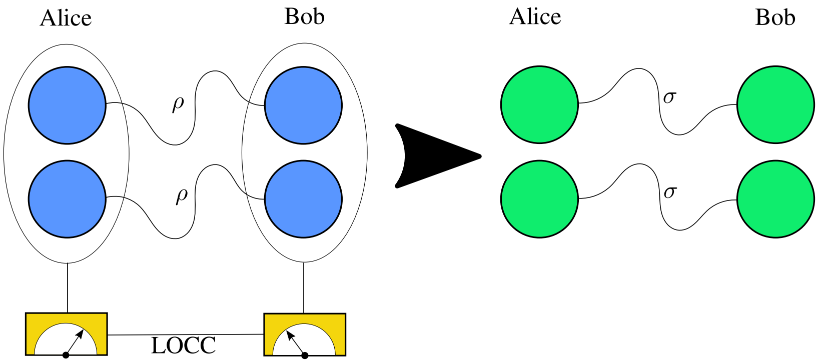

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- <u>Aim of multipartite state conversion</u>:
  creation of another state σ via N-partite LOCC operations
- For $N = 2$: <u>entanglement distillation</u> if $\sigma = |\Psi^+\rangle\langle\Psi^+|$,
  <u>entanglement dilution</u> if $\rho = |\Psi^+\rangle\langle\Psi^+|$

Multipartite quantum state conversion

- Conversion rate:

$$R(\rho \to \sigma) = \sup\left\{r : \lim_{n\to\infty}\left(\inf_{\Lambda}\left\|\Lambda\left(\rho^{\otimes n}\right) - \sigma^{\otimes \lfloor rn \rfloor}\right\|_1\right) = 0\right\} \quad (3)$$

[1] M. B. Plenio and S. Virmani, Quant. Inf. Comp. 2006

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- $N = 2$: $R(\rho \to |\Psi^+\rangle\langle\Psi^+|)$ is distillable entanglement[1] of ρ, and $R(|\Psi^+\rangle\langle\Psi^+| \to \sigma)^{-1}$ is entanglement cost of σ

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- General solution[2] for $N = 2$:

$$R(\psi^{AB} \to \phi^{AB}) = \frac{S(\psi^A)}{S(\phi^A)} \quad (4)$$

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- Entanglement theory is reversible for bipartite pure states:

$$R(\psi^{AB} \to \phi^{AB}) = R(\phi^{AB} \to \psi^{AB})^{-1} \quad (5)$$

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Multipartite quantum state conversion[1]

- Surprisingly little was known for $N > 2$

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- Bounds for $N = 3$:

$$R(\psi^{ABC} \to \phi^{ABC}) \leq \min\left\{\frac{S(\psi^A)}{S(\phi^A)}, \frac{S(\psi^B)}{S(\phi^B)}, \frac{S(\psi^C)}{S(\phi^C)}\right\} \tag{6}$$

$$R(\psi^{ABC} \to \phi^{ABC}) \geq \min\left\{\frac{S(\psi^A)}{S(\phi^B) + S(\phi^C)}, \frac{S(\psi^B)}{S(\phi^B)}, \frac{S(\psi^C)}{S(\phi^C)}\right\} \tag{7}$$

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- The bound can be further improved by interchanging the parties A, B, C

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- The bounds coincide whenever $\min\left\{\frac{S(\psi^A)}{S(\phi^B)+S(\phi^C)}, \frac{S(\psi^B)}{S(\phi^B)}, \frac{S(\psi^C)}{S(\phi^C)}\right\}$ is equal to $\frac{S(\psi^B)}{S(\phi^B)}$ or $\frac{S(\psi^C)}{S(\phi^C)}$

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- In these cases we get the exact rate for tripartite quantum state conversion:

$$R(\psi^{ABC} \to \phi^{ABC}) = \min\left\{\frac{S(\psi^B)}{S(\phi^B)}, \frac{S(\psi^C)}{S(\phi^C)}\right\} \tag{8}$$

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- Results can be extended to $N > 3$

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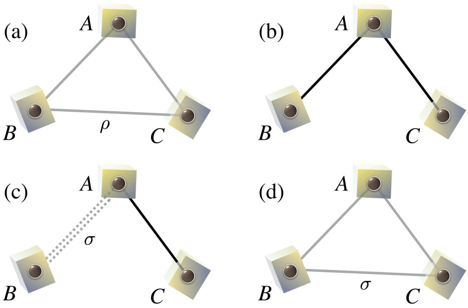

Sketch of the proof:

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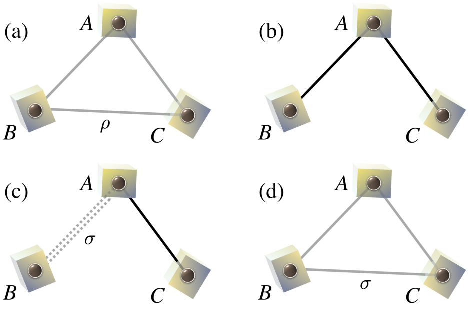

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- The parties apply quantum state merging and assisted entanglement distillation to distill singlets between *AB* and *AC*

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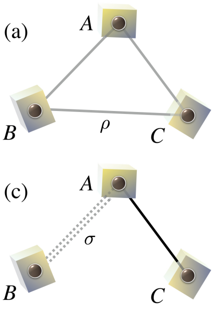

Sketch of the proof:

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- Alice and Bob use their singlets to create the final state σ, remaining singlets between Alice and Charlie are used to teleport parts of σ to Charlie

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Outline

Incoherent states and measurements[1]

- Incoherent states: states which are diagonal in a preferred basis:

$$\sigma = \sum_i p_i |i\rangle \langle i| \tag{9}$$

[1] T. Baumgratz, M. Cramer, and M. B. Plenio, PRL (2014)

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- Incoherent measurements: quantum measurements which do not create coherence

$$\Lambda[\rho] = \sum_i K_i \rho K_i \tag{10}$$

with incoherent Kraus operators K_i, i.e., $K_i |m\rangle \sim |n\rangle$

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Alternative frameworks of coherence[6]

- <u>Maximally incoherent operations (MIO)</u>[1]: most general set, contains all operations which cannot create coherence: $\Lambda[\rho_{\rm i}] \in \mathcal{I}$, where $\mathcal{I}$ is the set of all incoherent states.

---

[1] J. Åberg, arXiv 2006

[2] A. Winter and D. Yang, PRL 2016; B. Yadin, J. Ma, D. Girolami, M. Gu, V. Vedral, PRX 2016

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[4] B. Regula, M. Piani, M. Cianciaruso, T. R. Bromley, A. S., G. Adesso, arXiv 2017

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- Translationally invariant operations (TIO)[3]: Quantum operations which commute with time translations, i.e., $e^{-iHt}\Lambda[\rho]e^{iHt} = \Lambda[e^{-iHt}\rho e^{iHt}]$ for a given Hamiltonian H.

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- Theory of multilevel coherence: coherence between $N > 2$ levels of a quantum system[4][5]

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Coherence distillation and dilution

- Distillable coherence[1]: maximal rate for extracting the state $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ via incoherent operations

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- Single-shot coherence distillation[2] and dilution[3] has also been considered

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Coherence theory of a single qubit[1,2,3]

- Incoherent operations on a single qubit admit a decomposition into 5 Kraus operators:

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- Open question: it is not known if 5 Kraus operators are indeed required, or if the number can be reduced to 4

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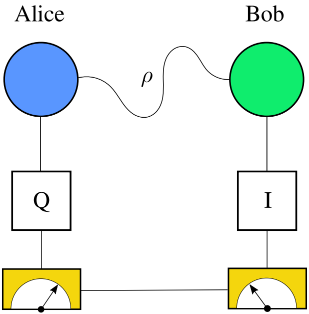
Entanglement and coherence in distributed scenarios[1][2][3][4]

LQICC: Local quantum-incoherent operations and classical communication



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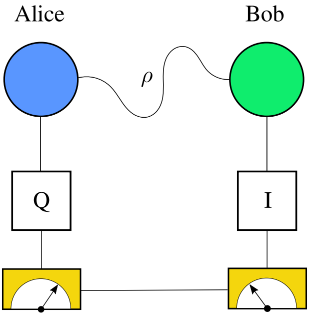

LQICC operations preserve the set of quantum-incoherent states:
$\rho_{\mathrm{qi}}^{AB} = \sum_i p_i \sigma_i^A \otimes |i\rangle\langle i|^B$

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Assisted coherence distillation

- Setting: Alice and Bob share many copies a bipartite state ρ^{AB} and can perform bipartite LQICC operations[1][2]

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- Optimal entanglement-coherence pairs (E, C): pairs of entanglement and coherence rates for which merging is possible, but neither E nor C can be reduced
- Main problem: determine all optimal pairs (E, C) for a given quantum state

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Incoherent quantum state merging[1]

Theorem

Given a tripartite quantum state ρ^{RAB}, any achievable pair (E, C) fulfills the following inequality:

$$E + C \geq S\left(\Delta^{AB}[\rho^{RAB}]\right) - S\left(\Delta^{B}[\rho^{RAB}]\right). \tag{11}$$

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S is the von Neumann entropy and Δ^X denotes full decoherence of a (possibly multipartite) subsystem X:

$$\Delta^X[\rho] = \sum_i |i\rangle\langle i|^X \rho\, |i\rangle\langle i|^X . \tag{12}$$

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Since the right-hand side of Eq. (11) is nonnegative, the sum $E + C$ is also nonnegative: **no merging procedure can gain coherence and entanglement at the same time**

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For pure states $|\psi\rangle^{RAB}$ we have

$$E \geq E_{\min} = S(\rho^{AB}) - S(\rho^{B}) \tag{13}$$

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The bound in Eq. (14) is achievable for all pure states

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Theorem

Any pure state $|\psi\rangle^{RAB}$ can be merged without local coherence by using singlets at rate

$$E_0 = S(\Delta^{AB}[\rho^{AB}]) - S(\Delta^{B}[\rho^{B}]). \tag{15}$$

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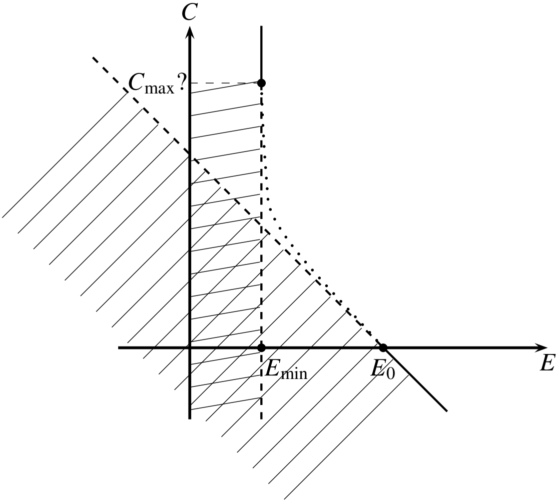

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Incoherent quantum state merging[1]

- Conjecture: it is possible to save a large amount of local coherence by using little extra entanglement, i.e., for some states the pairs $(E, C \gg 0)$ and $(E' = E + \varepsilon, C' \ll C)$ are both optimal for small ε

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- Possible candidate:

$$\rho = \frac{1}{d_B} \sum_{i=0}^{d_B - 1} |i\rangle\langle i|^R \otimes |\phi_i\rangle\langle\phi_i|^A \otimes |\psi_i\rangle\langle\psi_i|^B , \qquad (18)$$

where $|\psi_i\rangle$ are mutually orthogonal maximally coherent states of arbitrary dimension d_B, and $|\phi_i\rangle$ are single-qubit states

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- Our results imply an incoherent version of Schumacher compression: $S(\Delta[\rho])$ is the optimal compression rate if the decompression is performed via incoherent operations only

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