Quantum Correlations

Aditi Sen De

Harish-Chandra Research Institute, Allahabad

Phys. Rev. A 91, 062119 (2015); Phys. Rev. A 94, 042310 (2016); arXiv:1610.00730; Phys. Rev. A 97, 012316 (2018)

Frozen
Quantum Correlations

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Saptarshi

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Titas

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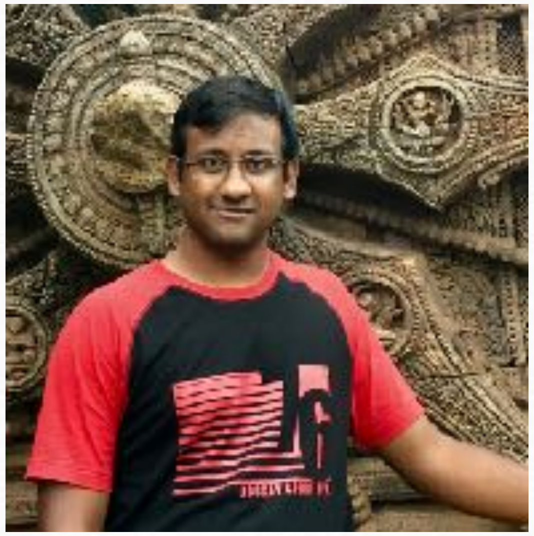

Debasis

Tamoghna

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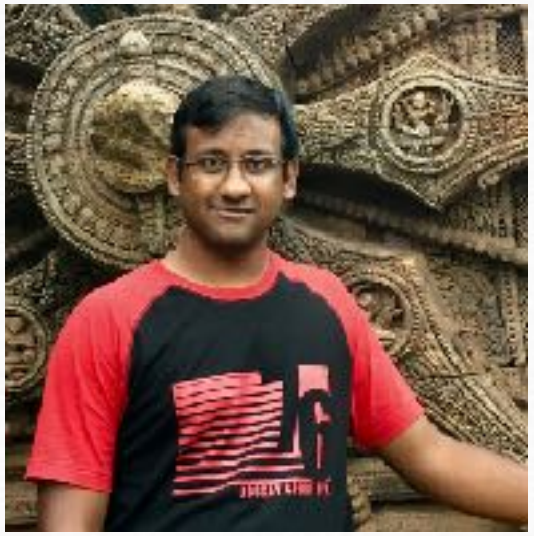
Saptarshi

Titas

Amit



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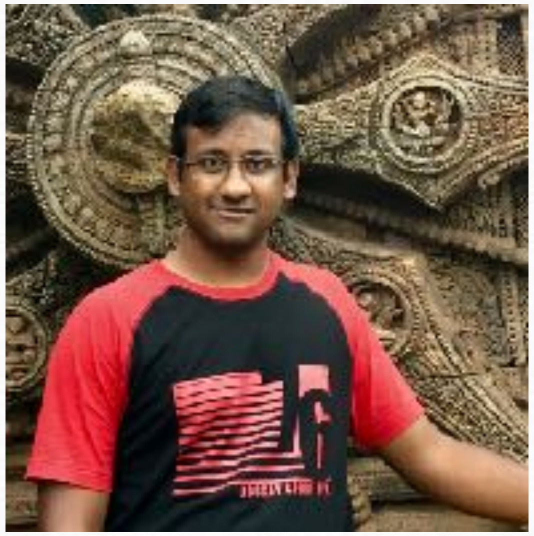

Debasis

Tamoghna

Ujjwal

Outline

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Open Quantum System

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What is freezing?

Freezing of QC measures

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Freezing of QC measures

Freezing of ent

Environment

System: Quantum phases & Critical Lines

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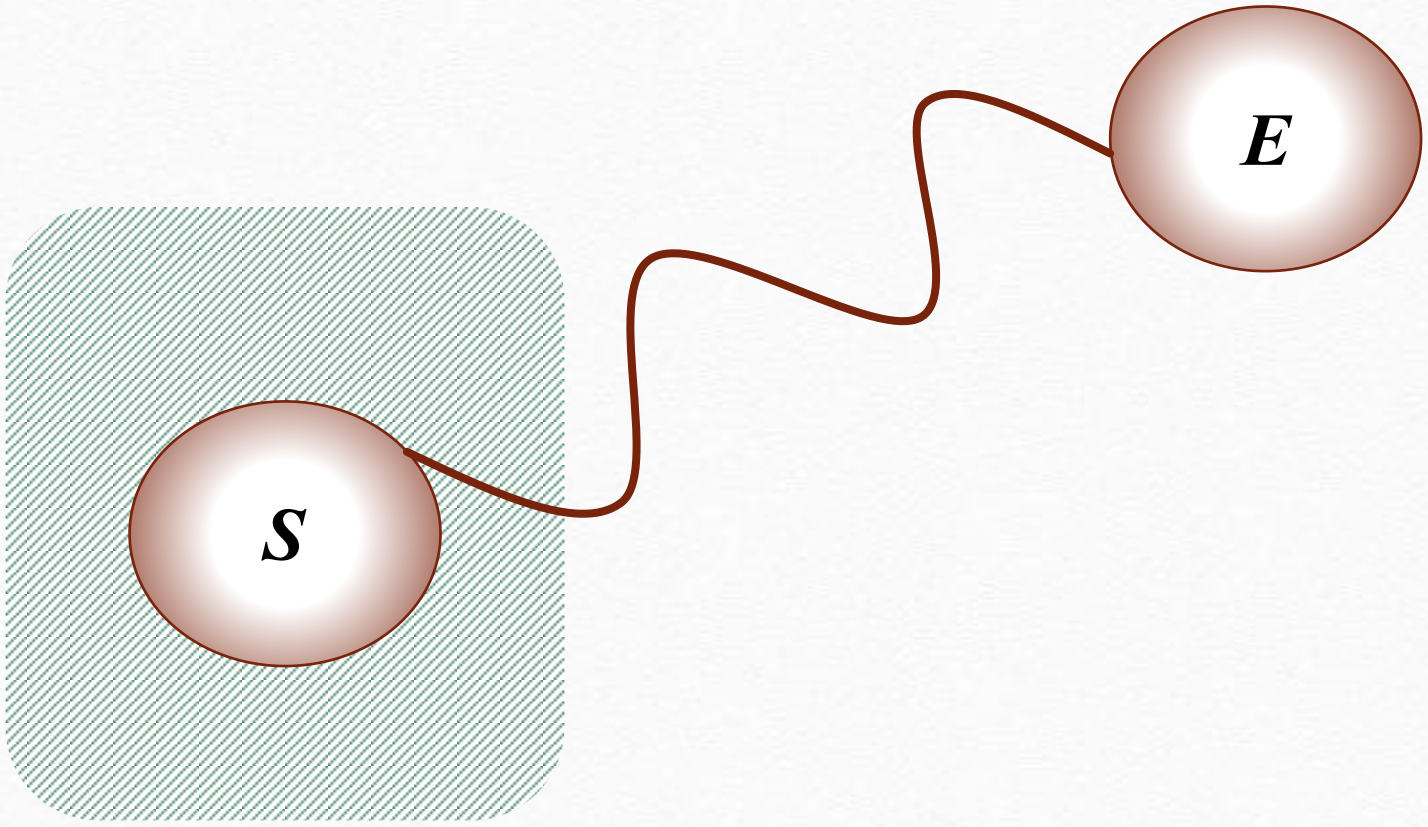
Open Quantum System: Effects of environment

Open System Dynamics

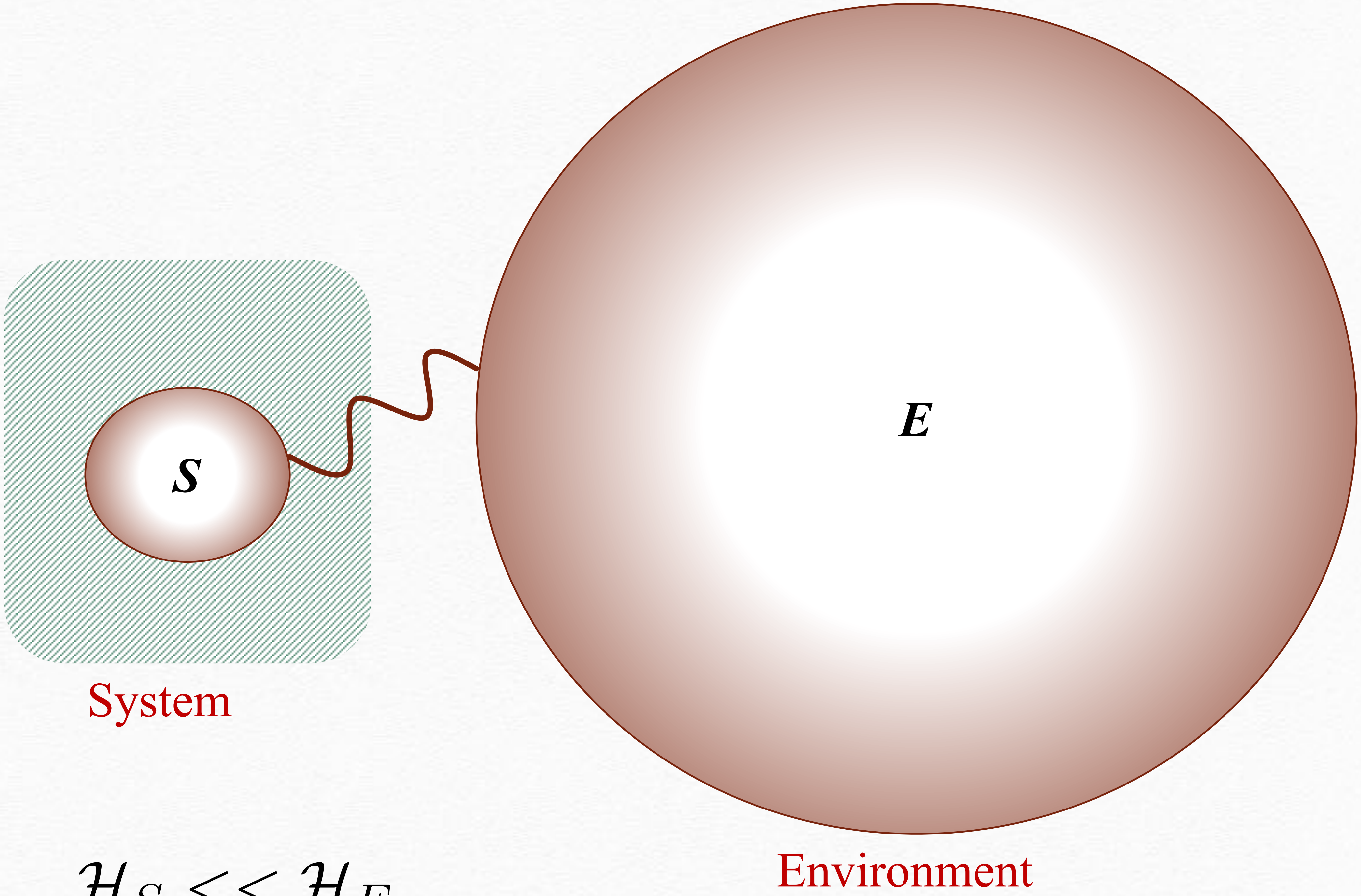
$$H_{total} = H_S + H_E + H^I_{SE}$$



Open quantum system

Open System Dynamics

$$H_{total} = H_S + H_E + H^I_{SE}$$



$$\mathcal{H}_S << \mathcal{H}_E$$

Open System Dynamics

Dynamical evolution…

Kraus operator representation:

$$\rho_S(t) = \sum_i K_i(t)\rho_S(0)K_i(t)^\dagger$$

with

$$\sum_i K_i(t)^\dagger K_i(t) = \mathbb{I}$$

Open System Dynamics

Dynamical evolution…

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with

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Master equation:

$$\frac{d\rho_S(t)}{dt} = -\frac{i}{\hbar}[H_s, \rho_S(t)] + \mathcal{D}_t[\rho_S(t)]$$

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other

Freezing of ent

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Quantum Correlations

Entanglement-Separability Paradigm

Ent of formation,
Logarithmic negativity

Information theoretic measures

Discord
Work deficit

Quantum Correlations

Entanglement-Separability Paradigm

Ent of formation,
Logarithmic negativity

$$\mathcal{L}(\rho_{AB}) = \log_2[2\mathcal{N}(\rho_{AB}) + 1]$$

$$\mathcal{N}(\rho_{AB}) = \frac{||\rho_{AB}^{T_A}||_1 - 1}{2}$$

Information theoretic measures

Discord
Work deficit

Quantum Correlations

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Information theoretic measures

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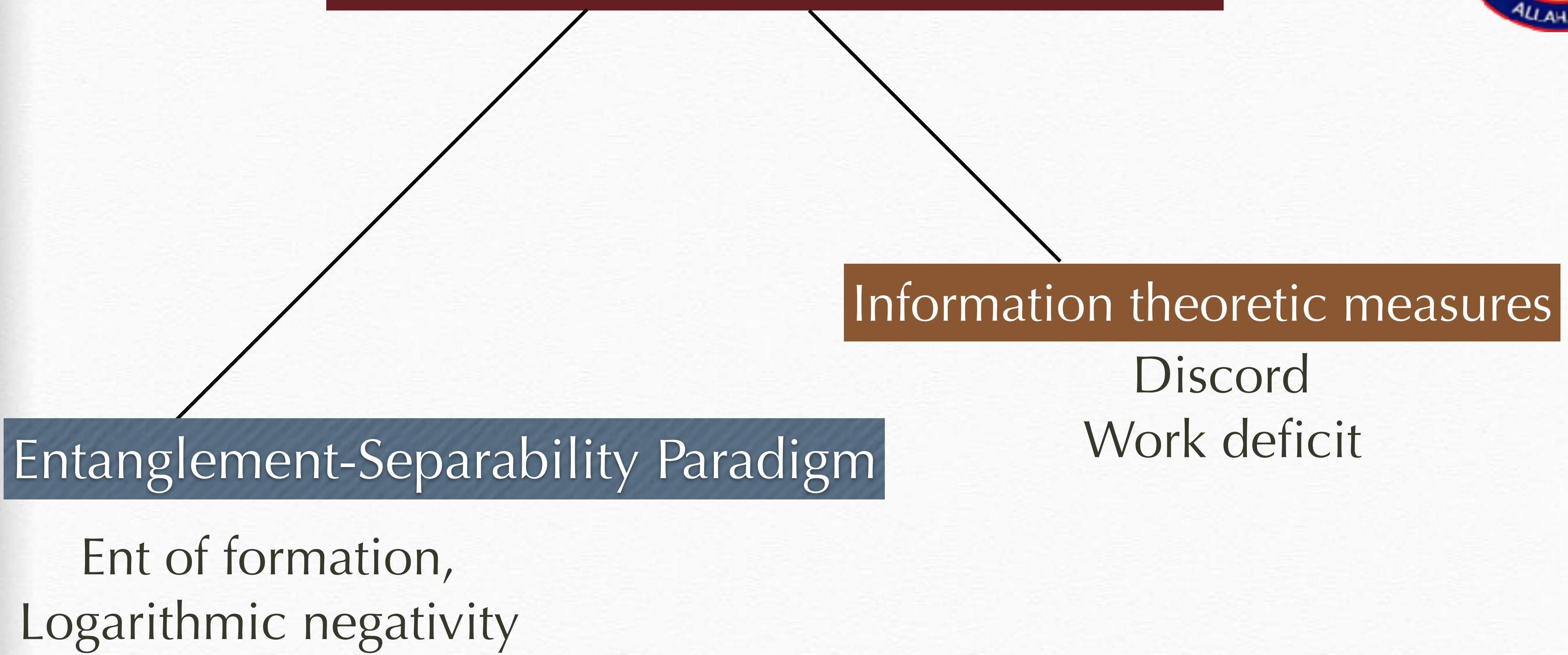
A Bera, T Das, D Sadhukhan, S Singha Roy, ASD, U Sen, ROPP **81,** 024001 (2018)

Quantum Correlations



Information theoretic measures

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Entanglement-Separability Paradigm

Ent of formation,

Logarithmic negativity

Quantum mutual information: Two inequivalent definitions

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

$$J(\rho_{AB}) = S(\rho_B) - S(\rho_{B|A})$$

$$S(\rho_{B|A}) = \sum_k p_k S(\rho^k_{AB})$$

$$\boxed{D(\rho_{AB}) = I(\rho_{AB}) - J(\rho_{AB})}$$

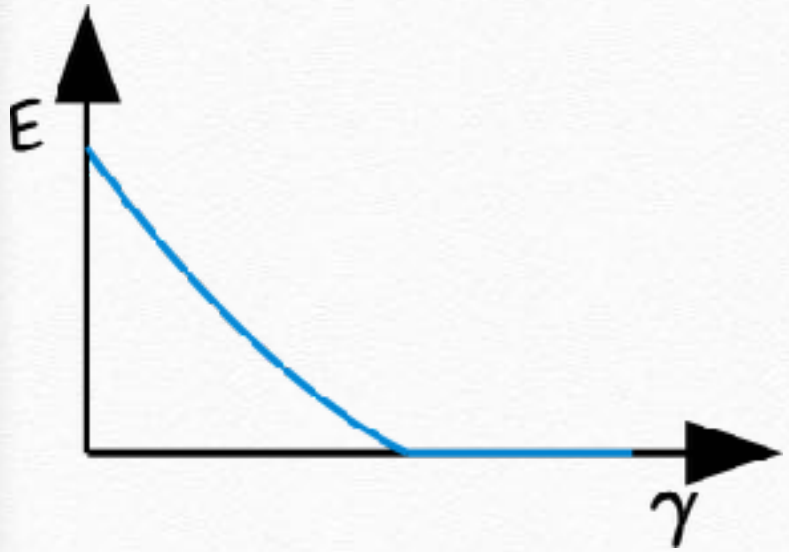
A Bera, T Das, D Sadhukhan, S Singha Roy, ASD, U Sen, ROPP **81,** 024001 (2018)

Dynamics $\overset{\text{QCs}}{\wedge}$ under noisy channels

Dynamics under noisy channels

Entanglement usually decays and dies..

Yu & Eberly, Science (2009)



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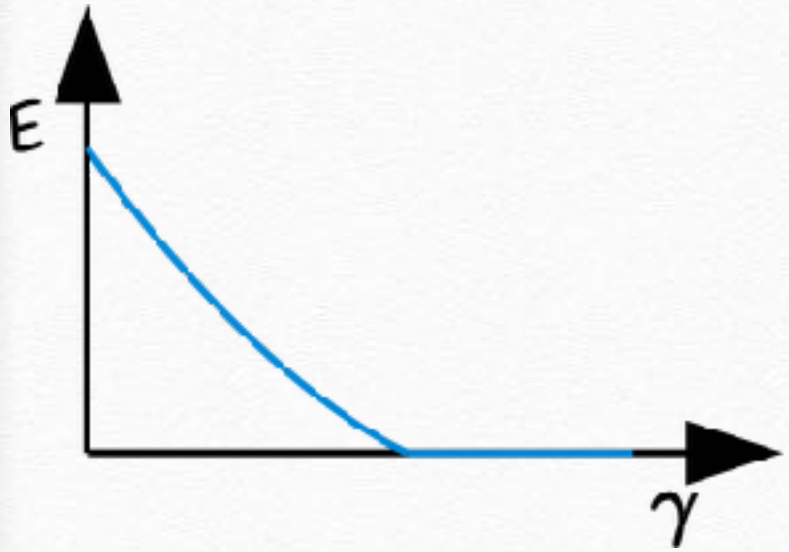
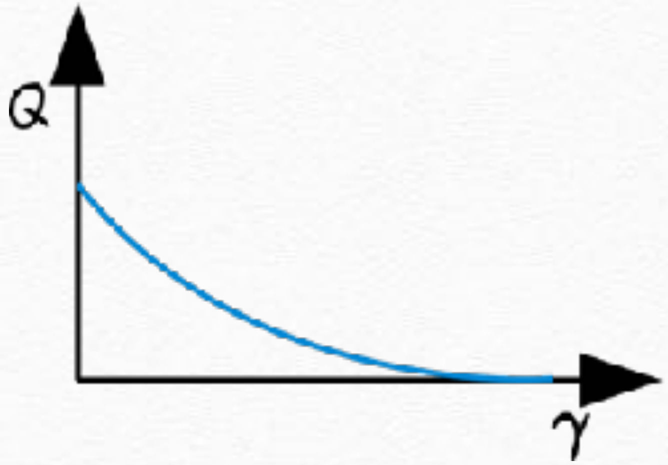


Quantum correlations are robust!

Werlang et. al., PRA (2009)



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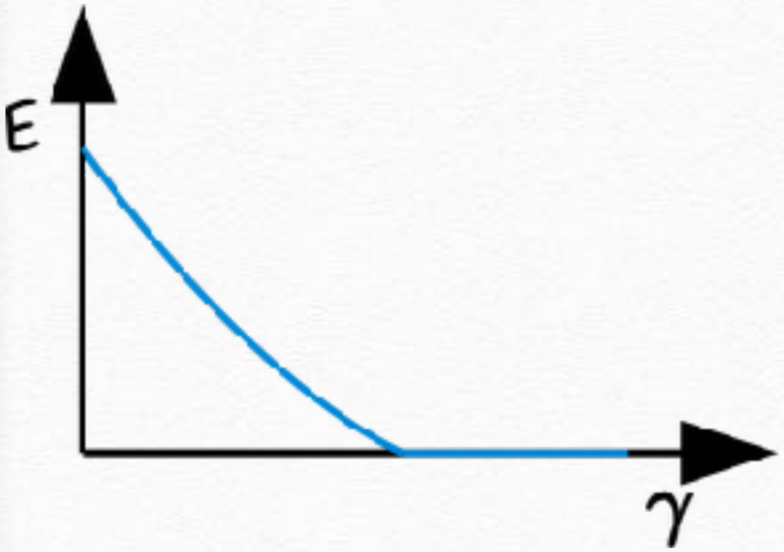

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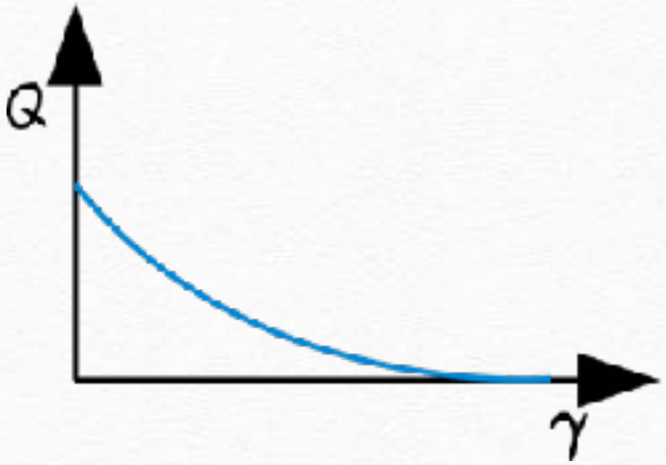
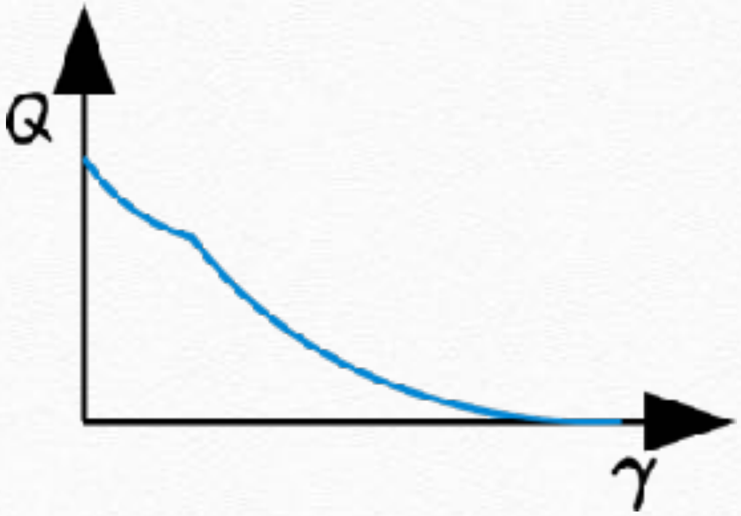


Play with the initial state

Maziero et. al., PRA (2009)



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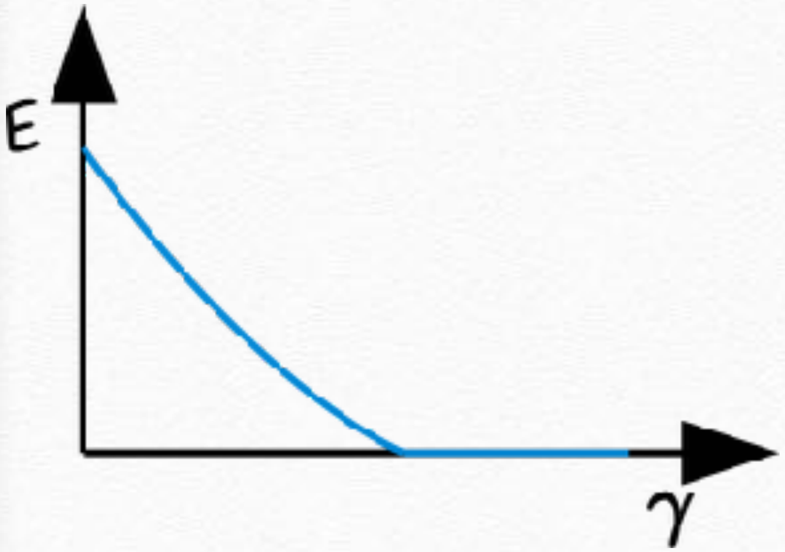

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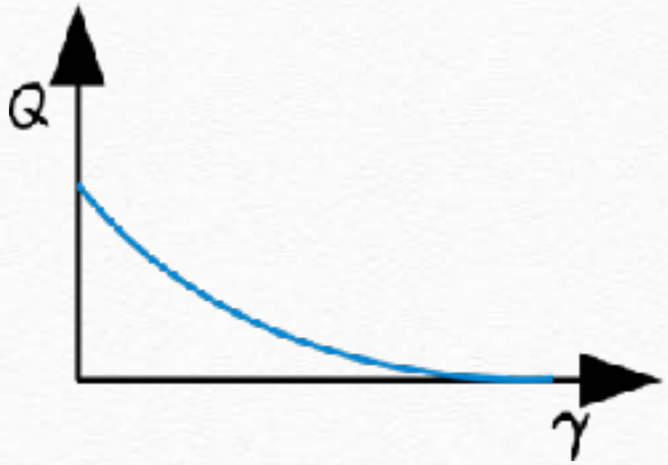

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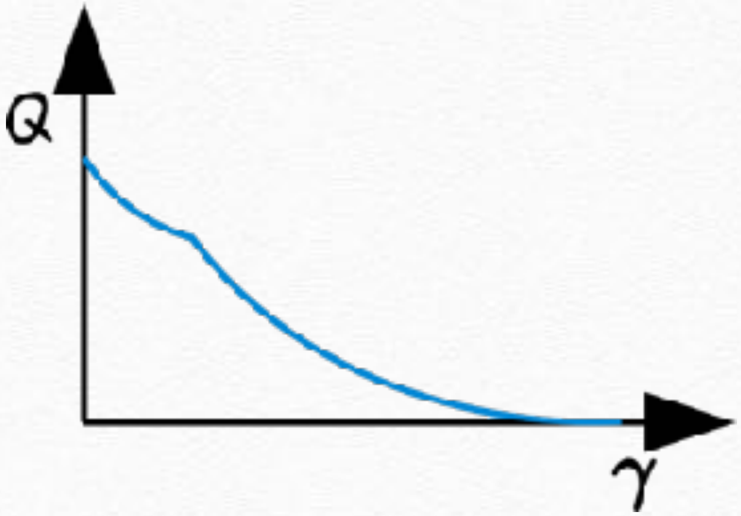
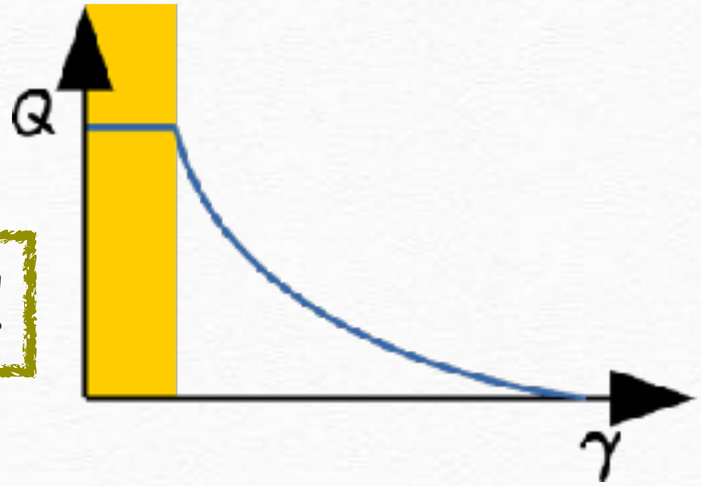


However other quantum correlation can **freeze!**

Mazzola et. al., PRL (2010),
Aaronson et. al., PRA (2013),
Cianciaruso et. al., Sci. Rep. (2015)
T. Chanda, AK. Pal, A. Biswas, ASD, U. Sen, PRA 2015

Surprising!!!!!



Freezing of Discord and Its Allies

- Initial two qubit state: ρ_{AB}
- Independent local environments act on each qubit

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- Evolution:

$$\rho_{AB}(\gamma) = \sum_{\mu\nu} K_\mu^A(\gamma) K_\nu^B(\gamma) \rho_{AB} K_\mu^{A^\dagger}(\gamma) K_\nu^{B^\dagger}(\gamma)$$

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- Bit-flip (BF), phase-flip (PF), bit-phase-flip channels

$$K_0(\gamma) = \sqrt{1-\gamma/2}\,\mathbb{I} \ \textbf{and} \ K_1 = \sqrt{\gamma/2}\,\sigma_\alpha$$

$$\alpha = 1 \ \text{(bit-flip)}, \ \alpha = 2 \ \text{(bit-phase-flip)}, \ \alpha = 3 \ \text{(phase-flip)}$$

Freezing of Discord and Its Allies

Initial state → Bell diagonal (BD) state

$$\rho_{AB} = \frac{1}{4}\left[\mathbb{I}_A \otimes \mathbb{I}_B + \sum_{\alpha=1}^{3} c_{\alpha\alpha}\sigma_A^{\alpha} \otimes \sigma_B^{\alpha}\right]$$

Universal for (almost) all the discord-like measures…

Aaronson et. al., PRA (2013), Cianciaruso et. al., Sci. Rep. (2015)

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Two sets of conditions:

1. $c_{22}/c_{33} = -c_{11}, \textbf{ with } |c_{33}| < |c_{11}|$
2. $c_{33}/c_{22} = -c_{11}, \textbf{ with } |c_{22}| < |c_{11}|$

Universal for (almost) all the discord-like measures…

Aaronson et. al., PRA (2013), Cianciaruso et. al., Sci. Rep. (2015)

Freezing of Discord and Its Allies

General two-qubit state (upto LU):

$$\rho_{AB} = \frac{1}{4}\Big[\mathbb{I}_A \otimes \mathbb{I}_B + \sum_{\alpha=1}^{3} c_{\alpha\alpha}\sigma_A^{\alpha} \otimes \sigma_B^{\alpha} + \sum_{\alpha=1}^{3} c_{\alpha 0}\sigma_A^{\alpha} \otimes \mathbb{I}_B + \sum_{\beta=1}^{3} c_{0\beta}\mathbb{I}_A \otimes \sigma_B^{\beta}\Big]$$

Discord is hard to compute!!!
No analytical closed form!!!

Freezing of Discord and Its Allies

BD state + magnetization in *x* direction:

$$\rho_{AB} = \frac{1}{4}\Big[\mathbb{I}_A \otimes \mathbb{I}_B + \sum_{\alpha=1}^{3} c_{\alpha\alpha}\sigma_A^\alpha \otimes \sigma_B^\alpha + c_{10}\sigma_A^1 \otimes \mathbb{I}_B + c_{01}\mathbb{I}_A \otimes \sigma_B^1\Big]$$

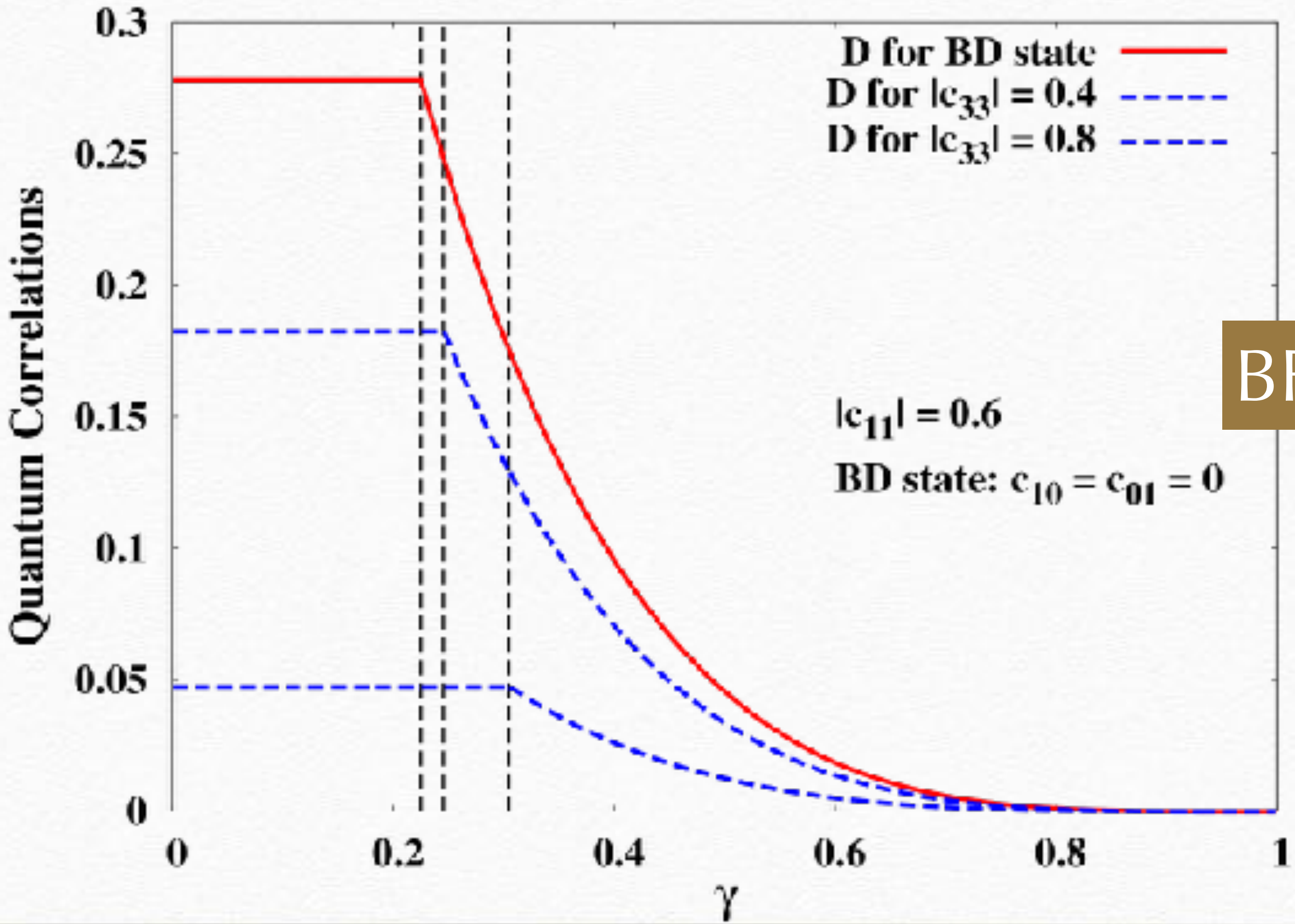
Closed form can be found for all (almost) parameter values

T Chanda, T Das, D Sadhukhan, A K Pal, ASD, U Sen, PRA **92**, 062301 (2015)

Freezing of Discord and Its Allies

BD state + magnetization in *x* direction:

$$\rho_{AB} = \frac{1}{4}\Big[\mathbb{I}_A \otimes \mathbb{I}_B + \sum_{\alpha=1}^{3} c_{\alpha\alpha}\sigma_A^{\alpha} \otimes \sigma_B^{\alpha} + c_{10}\sigma_A^{1} \otimes \mathbb{I}_B + c_{01}\mathbb{I}_A \otimes \sigma_B^{1}\Big]$$

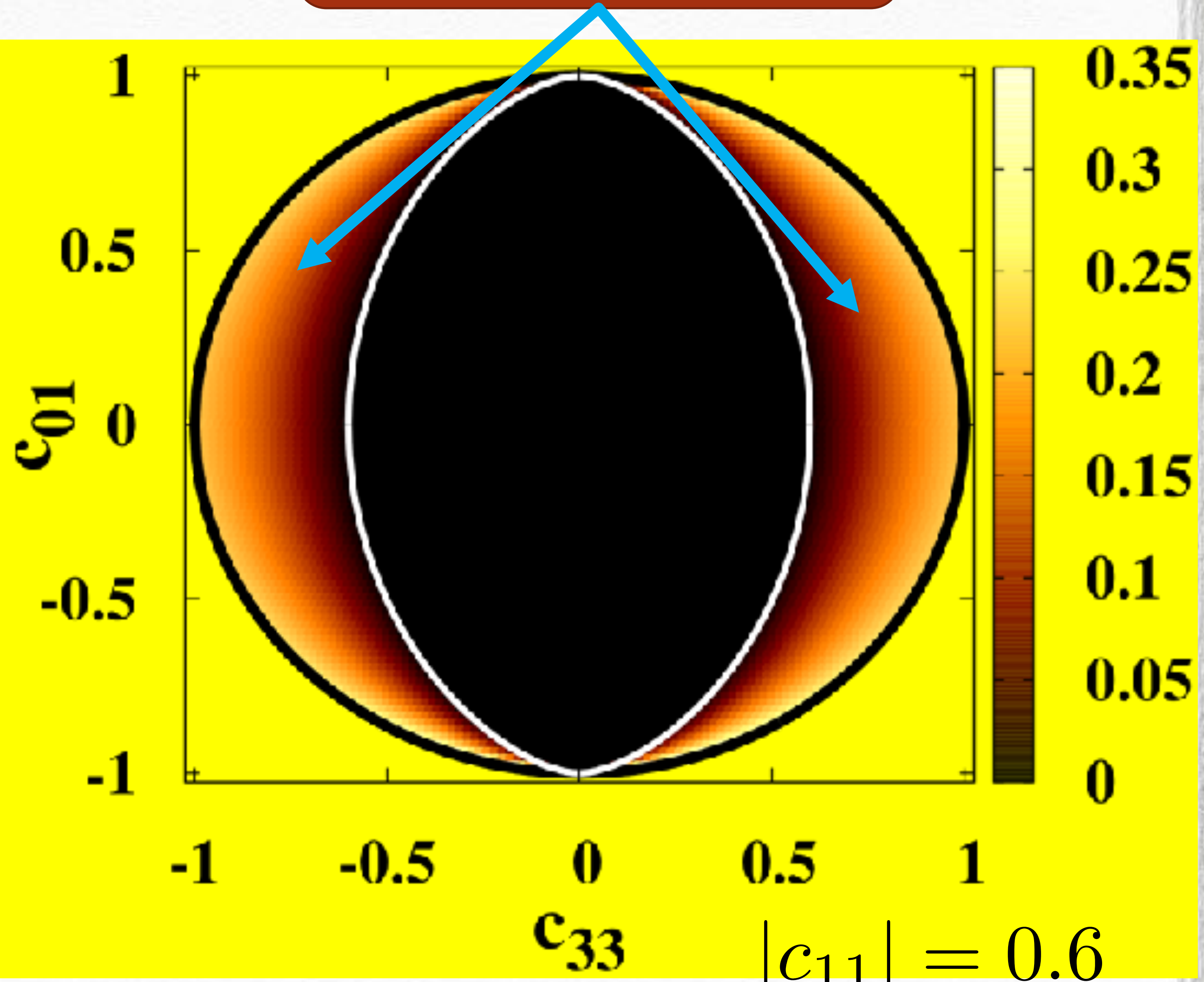


BF channel

Freezing of Discord and Its Allies

Two sets of conditions: Necessary and sufficient

Freezing



$|c_{11}| = 0.6$

Freezing of Discord and Its Allies

Two sets of conditions: Necessary and sufficient

Freezing

$$(1)\ (c_{22}/c_{33}) = -(c_{10}/c_{01}) = -c_{11}$$

$$(2)\ c_{33}^2 + c_{01}^2 \leq 1$$

$$(3)\ F(\sqrt{c_{33}^2 + c_{01}^2}) < F(c_{11}) + F(c_{01}) - F(c_{10})$$

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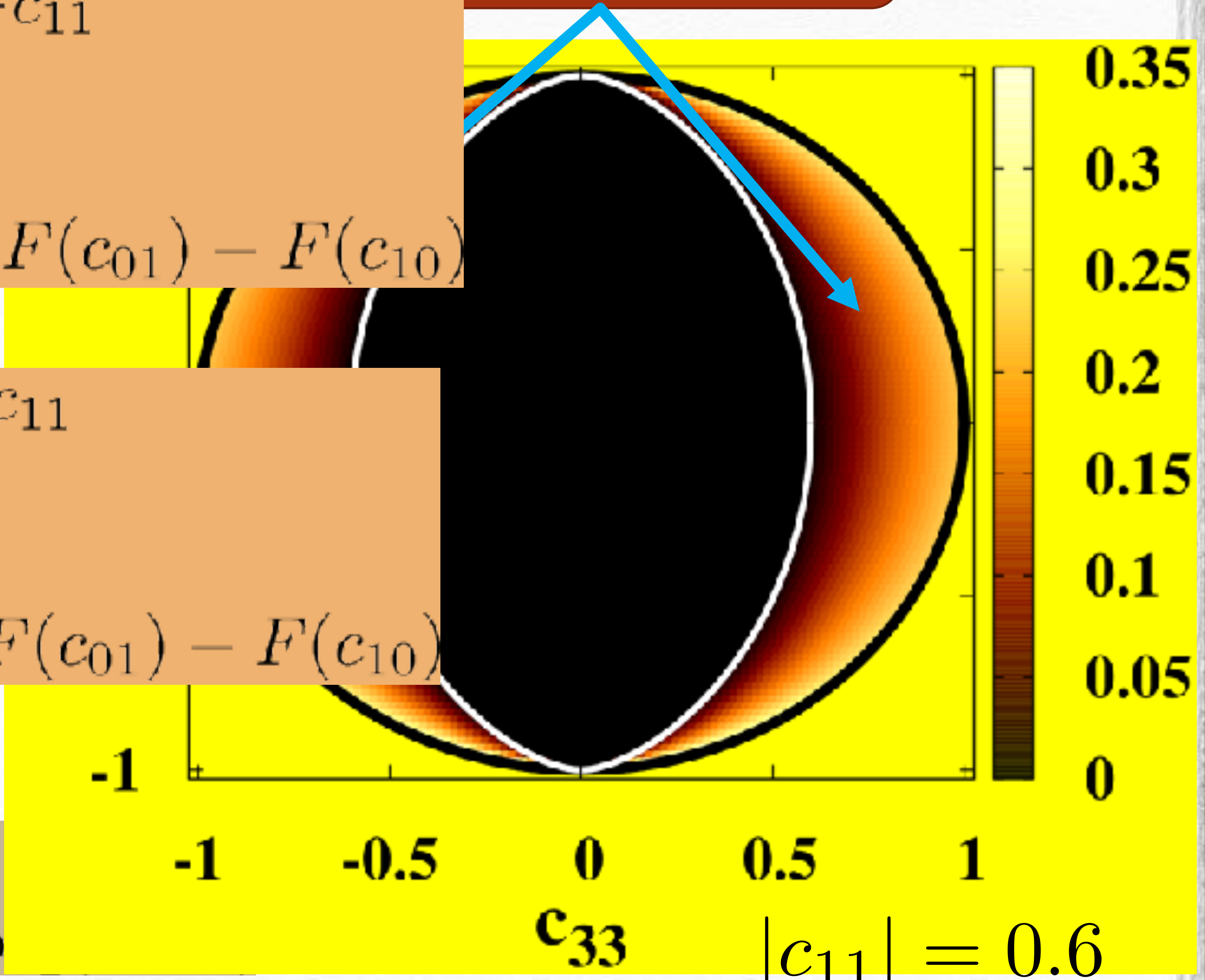


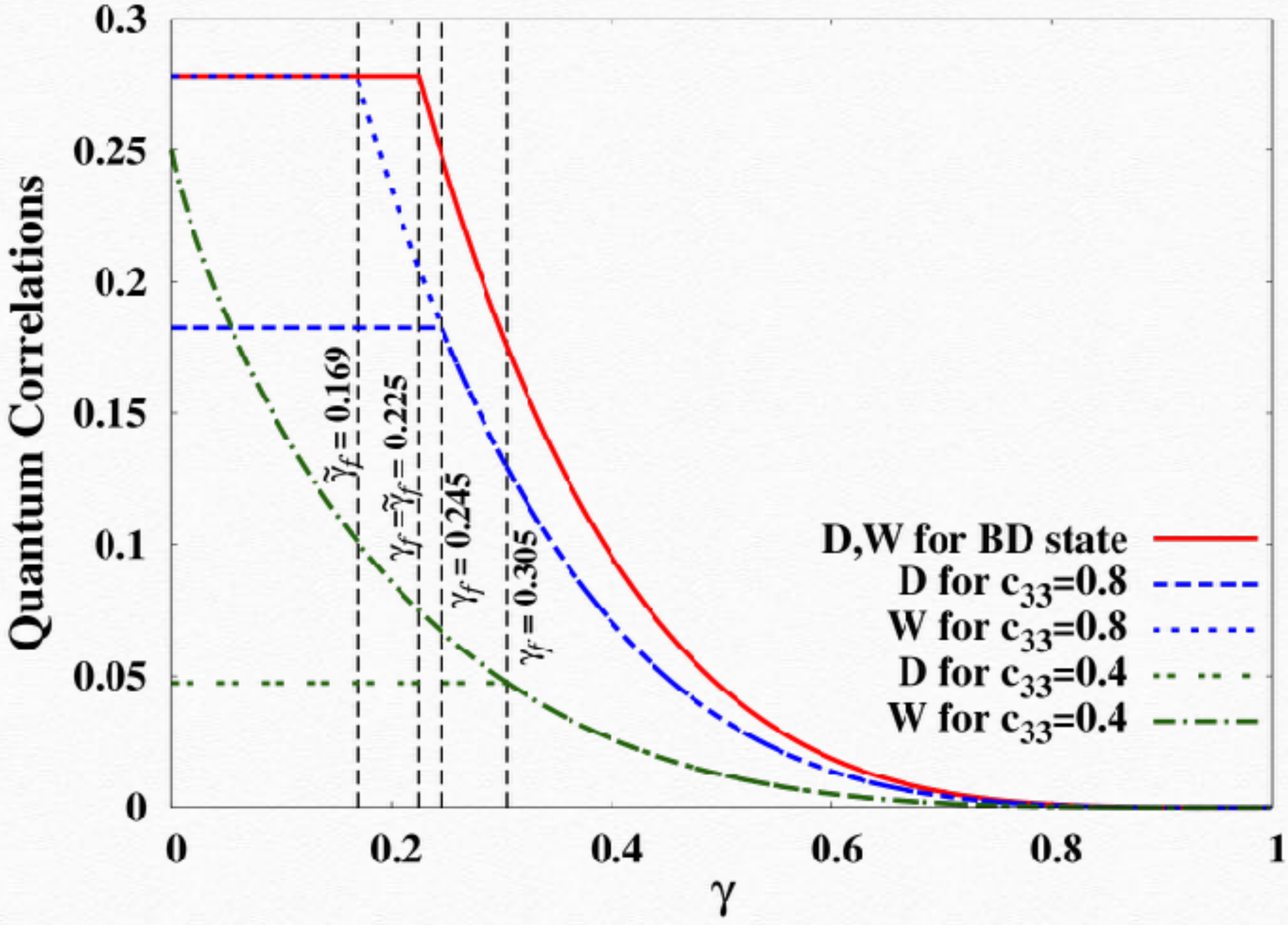
$|c_{11}| = 0.6$

$$F(y) = 2\,(H((1+y)/2) - 1)$$

$$H(\alpha) = -\alpha\log_2\alpha - (1-\alpha)\,\mathrm{lo}$$

Freezing of Discord and Its Allies

Universality no longer exists…
One-way quantum work deficit…



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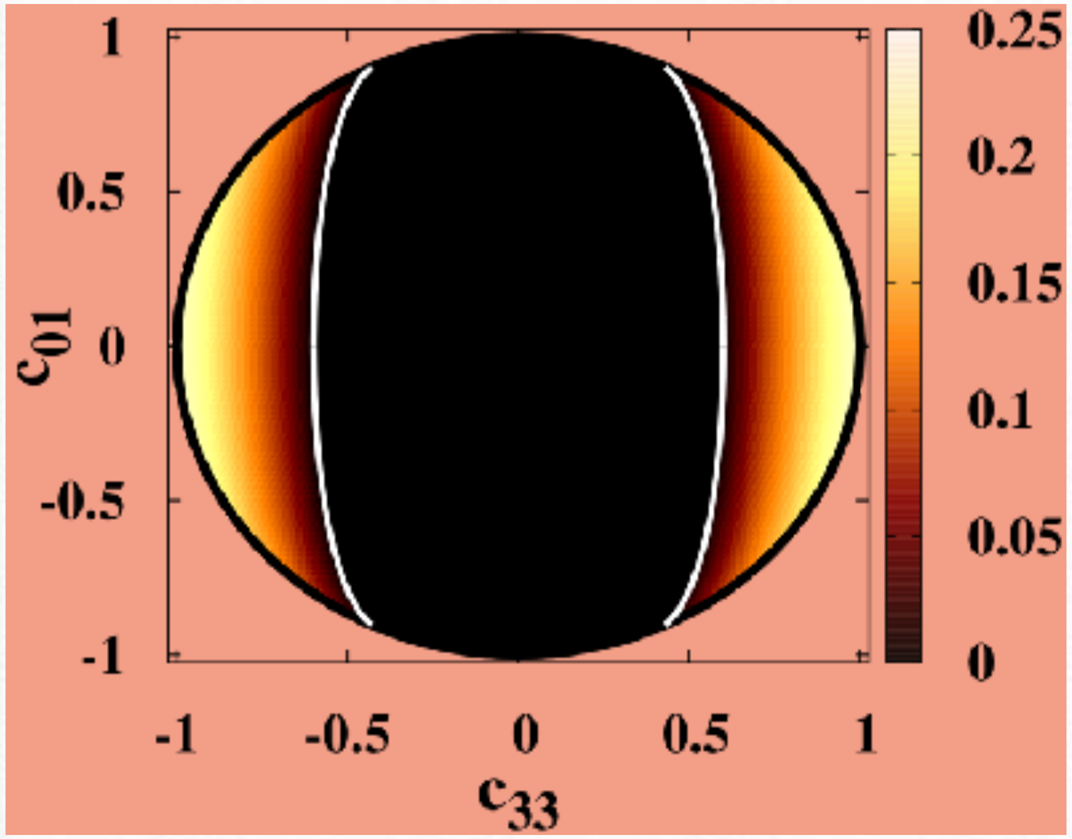

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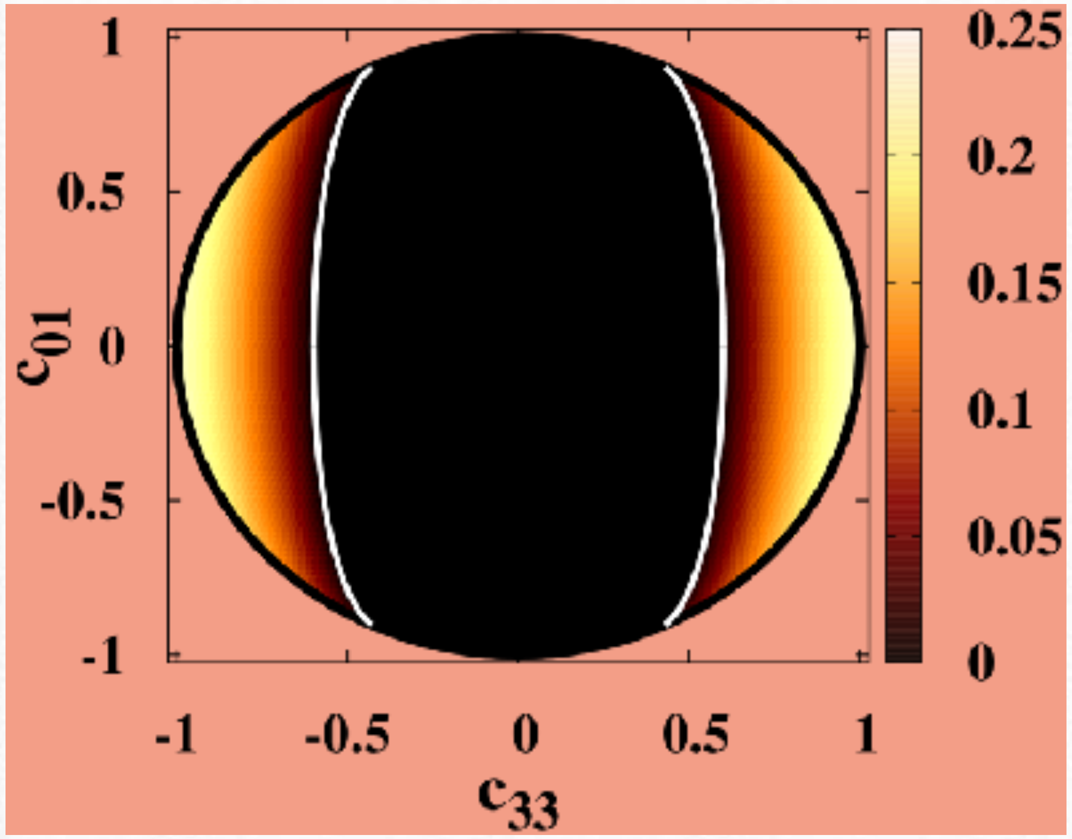
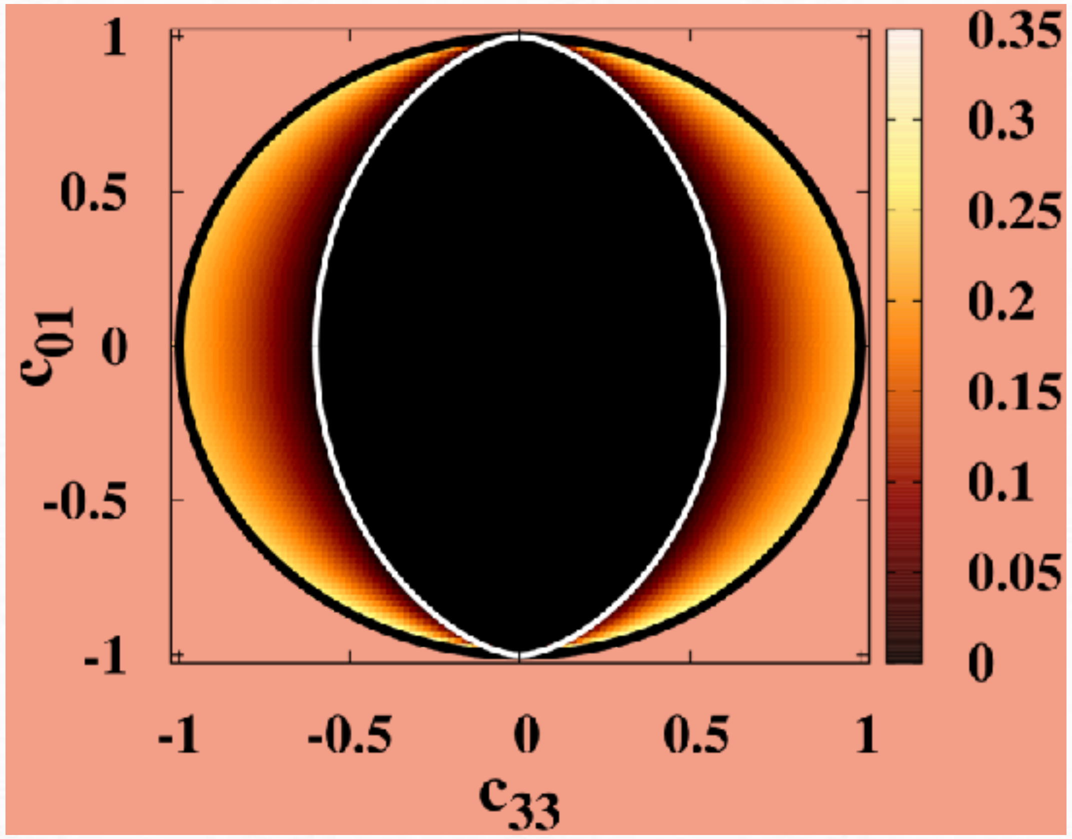


Discord



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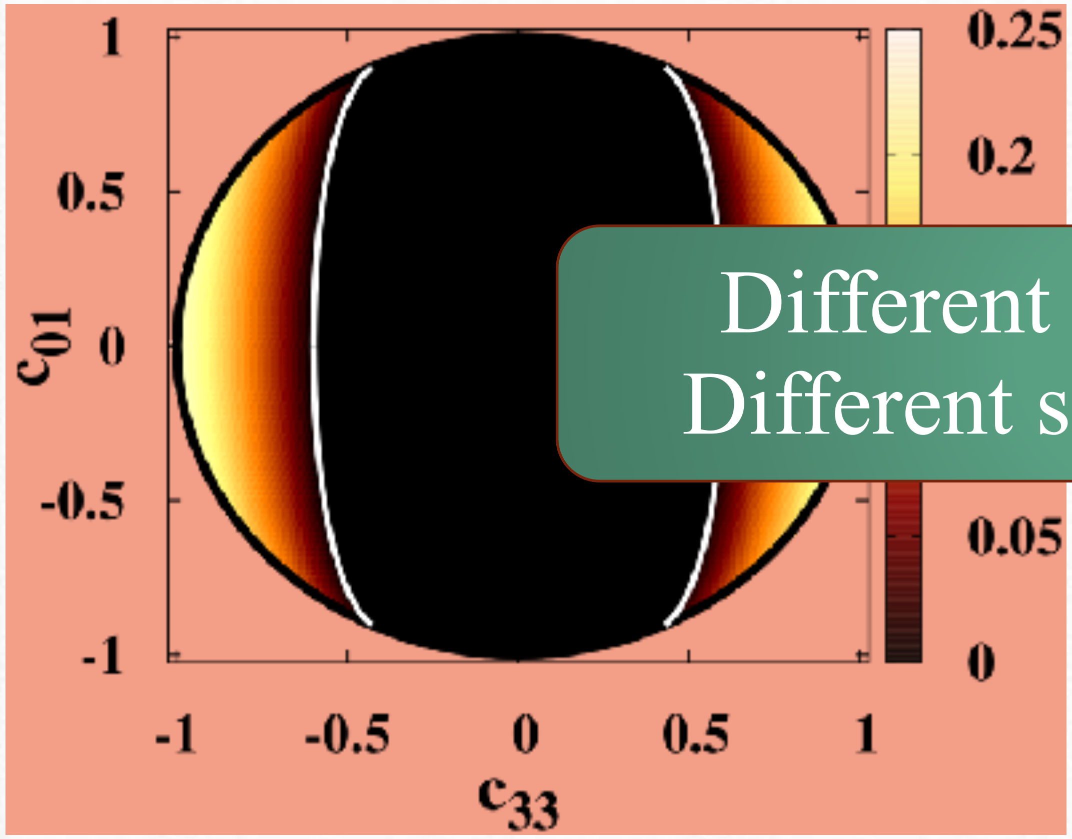

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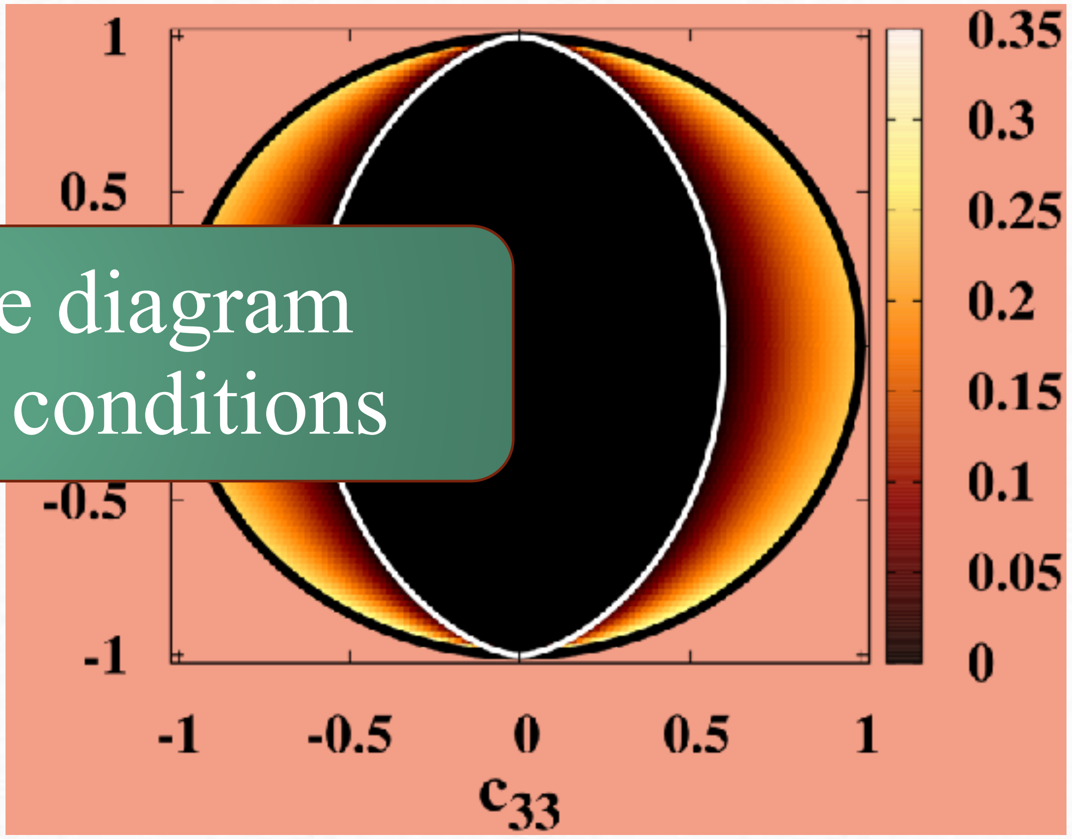

Different phase diagram
Different set of conditions

T Chanda, A K Pal, A Biswas, ASD, U Sen, PRA **91,** 062119 (2015)

Outline

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Freezing of other QC measures

Freezing of ent

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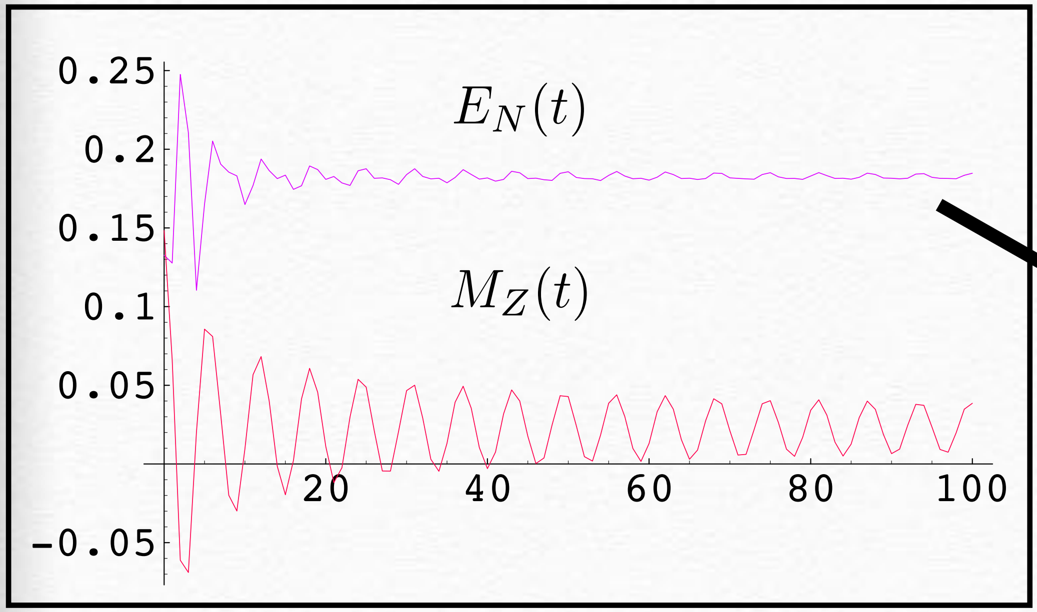
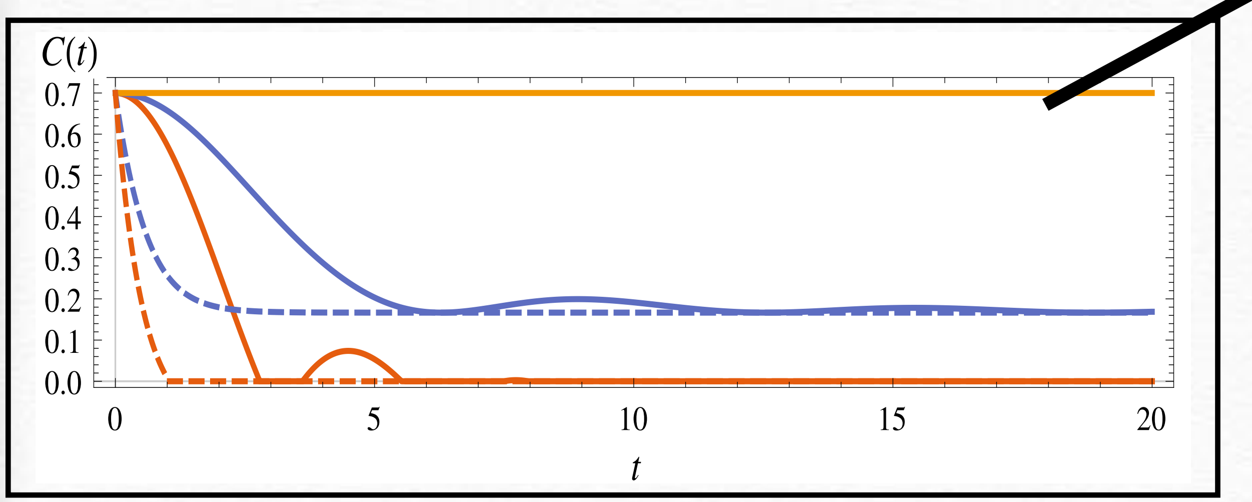
Summary

Freezing of entanglement?





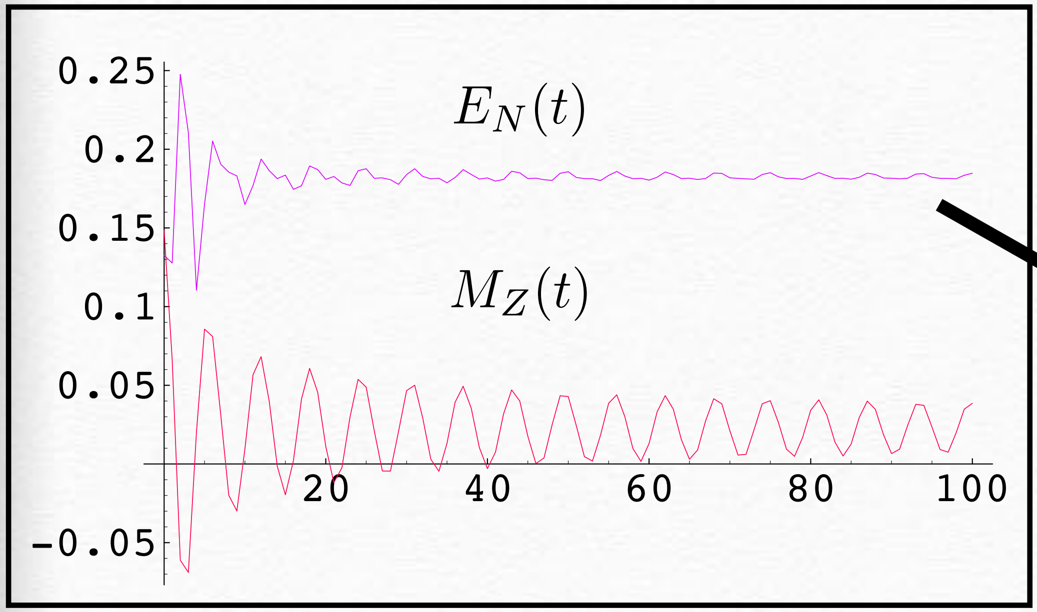
Saturation/Preservation

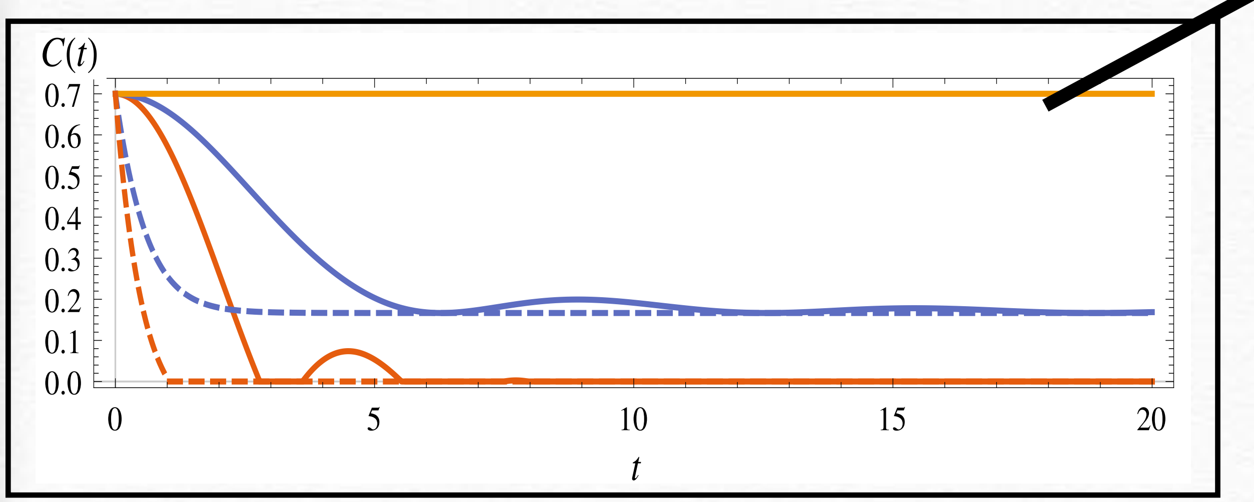
ASD, Sen, & Lewenstein **PRA (Rap. Comm.) 70, 060304 (2004)**

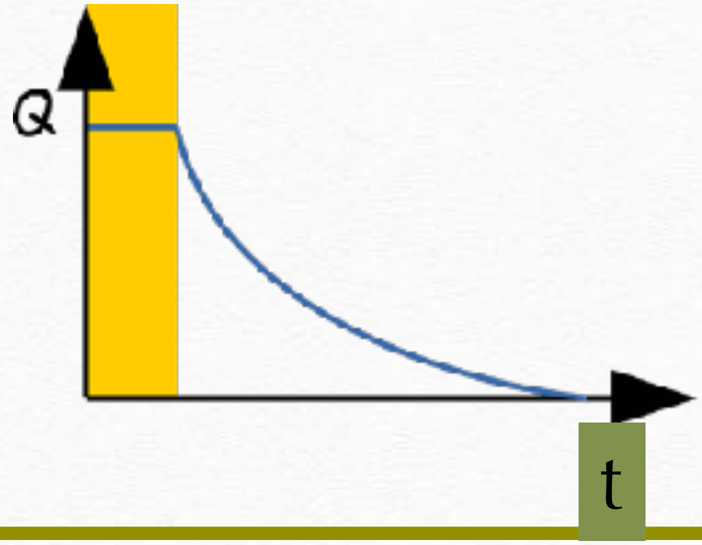
Apollaro, Cuccoli, Franco, Paternostro, Plastina, & Verrucchi, **NJP 12, 083046 (2010)**

Carnio, Buchlightner, & Gessner, **PRL 115, 010404 (2015)**

Freezing of entanglement?







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Freezing at the beginning of the dynamics?

Freezing at the beginning of the dynamics?

1. Proper choice of system as well as environment

Scale-invariant freezing of entanglement,
T. Chanda, T. Das, D. Sadhukhan, A. K. Pal, A. Sen(De), U. Sen, **arXiv:1610.00730 (2016)**

Freezing at the beginning of the dynamics?

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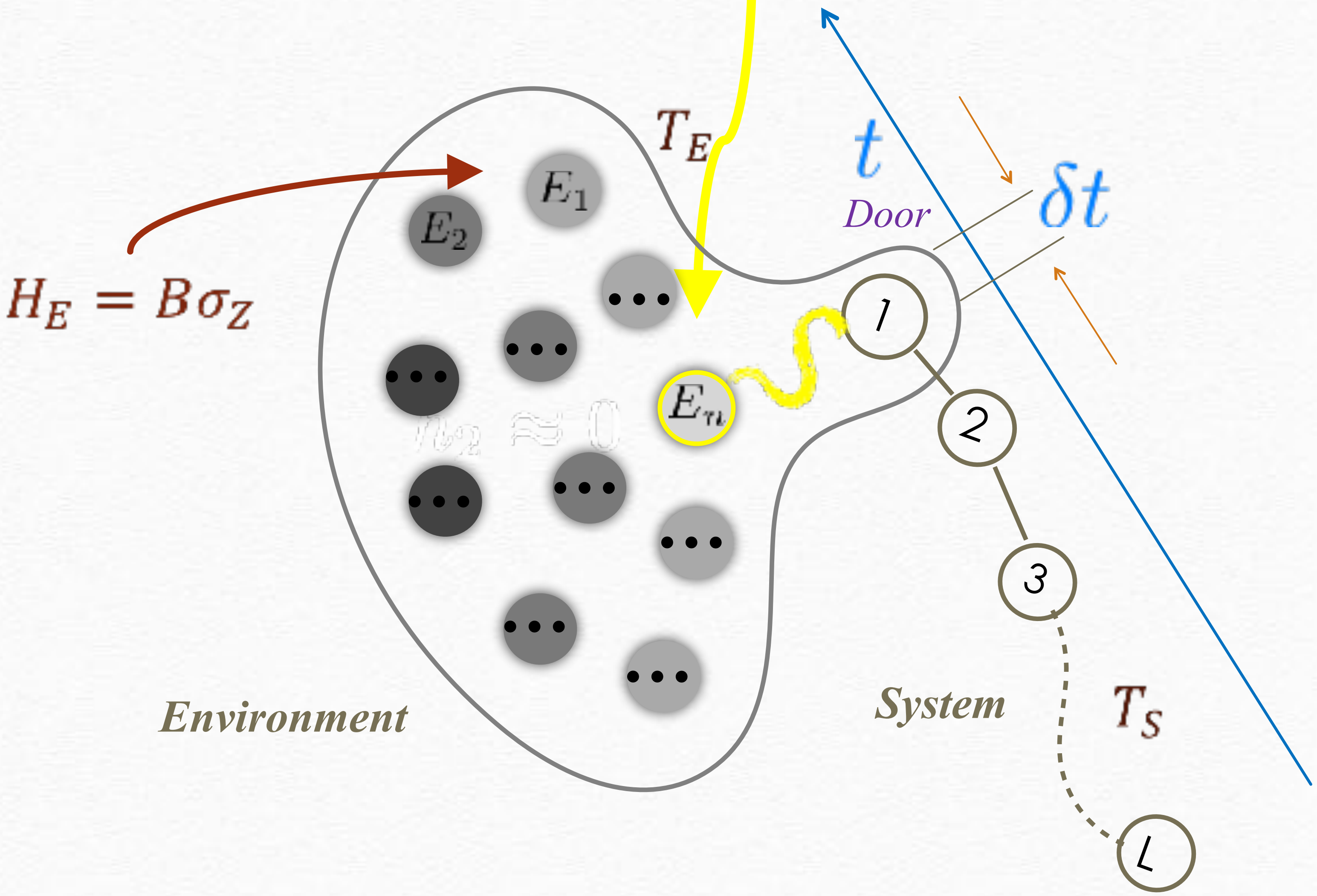
Freezing at the beginning of the dynamics?

1. Proper choice of system as well as environment

Repetitive quantum interaction

$$\hat{H}_{int} = k^{1/2}\delta t^{-1/2}(\hat{\sigma}_d^x\hat{\sigma}_E^x + \hat{\sigma}_d^y\hat{\sigma}_E^y)$$



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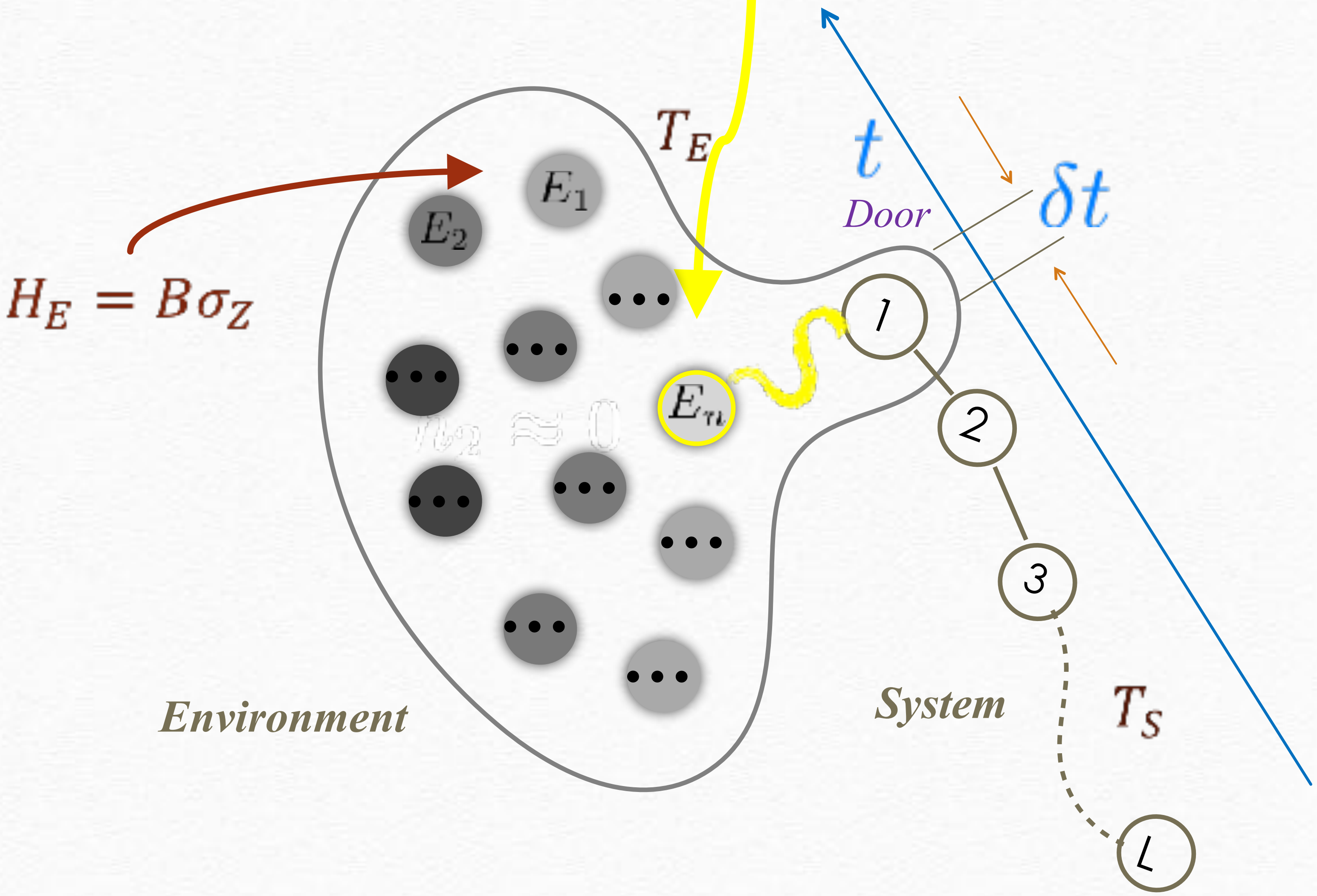

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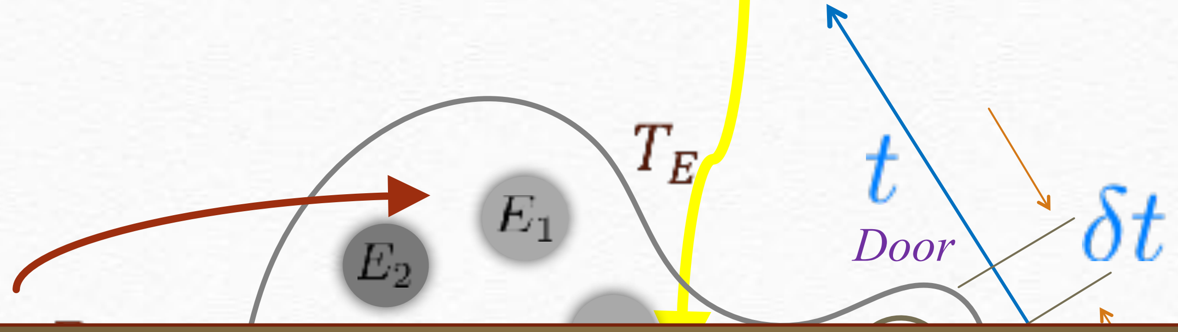

Quantum master equation:

$$\dot{\hat{\rho}}_S = -\frac{i}{\hbar}[\hat{H}_S, \hat{\rho}_S] + \mathcal{D}(\hat{\rho}_S)$$

$$\mathcal{D}(\hat{\rho}_S) = \frac{2k}{\hbar^2 Z_E}\sum_{l=1}^{N_d}\sum_{i=0}^{1} e^{(-1)^i \beta_E B}[2\hat{\eta}_{d_l}^{i+1}\hat{\rho}_S\hat{\eta}_{d_l}^{i} - \{\hat{\eta}_{d_l}^{i}\hat{\eta}_{d_l}^{i+1}, \hat{\rho}_S\}]$$

where…

$$Z_E = \mathbf{Tr}[\exp -\beta_E\hat{H}_E], \textbf{ and } \hat{\eta}_{d_l}^{\alpha} = \hat{\sigma}_{d_l}^x + (-1)^{\alpha}\hat{\sigma}_{d_l}^y$$

Freezing at the beginning of the dynamics?

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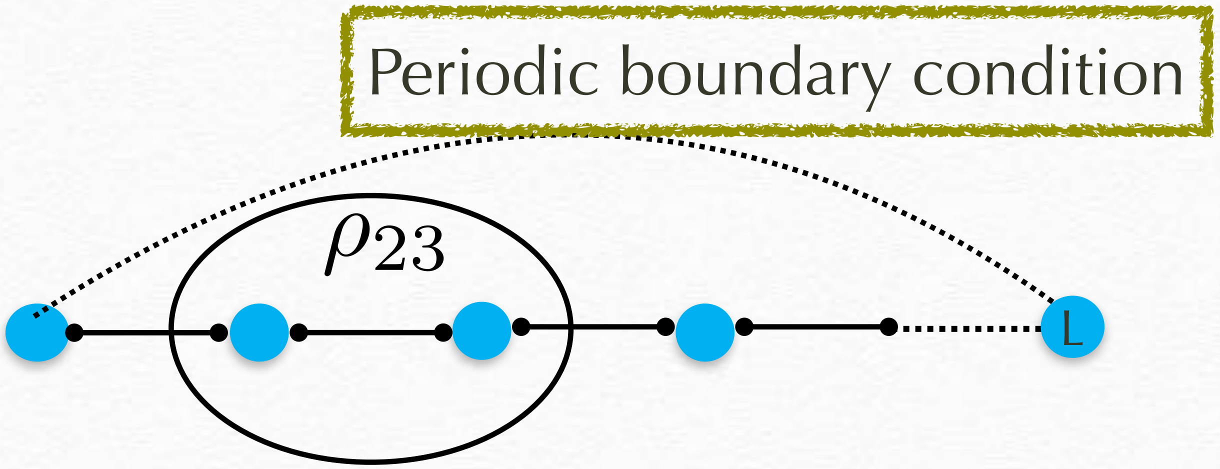

Periodic boundary condition

Quantum spin model

Chanda, Das, Sadhukhan, Pal, ASD, & Sen, **PRA 94, 042310 (2016)**

Motivation: Quantum information perspective

Study fundamental properties by using quantum info or vice-versa.

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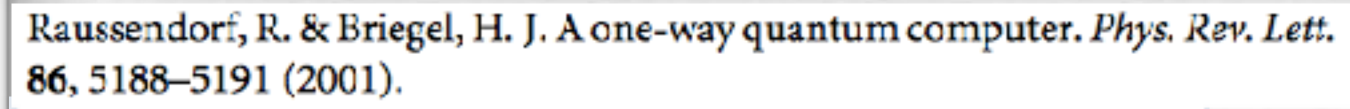
Potential candidate for realising quantum computer

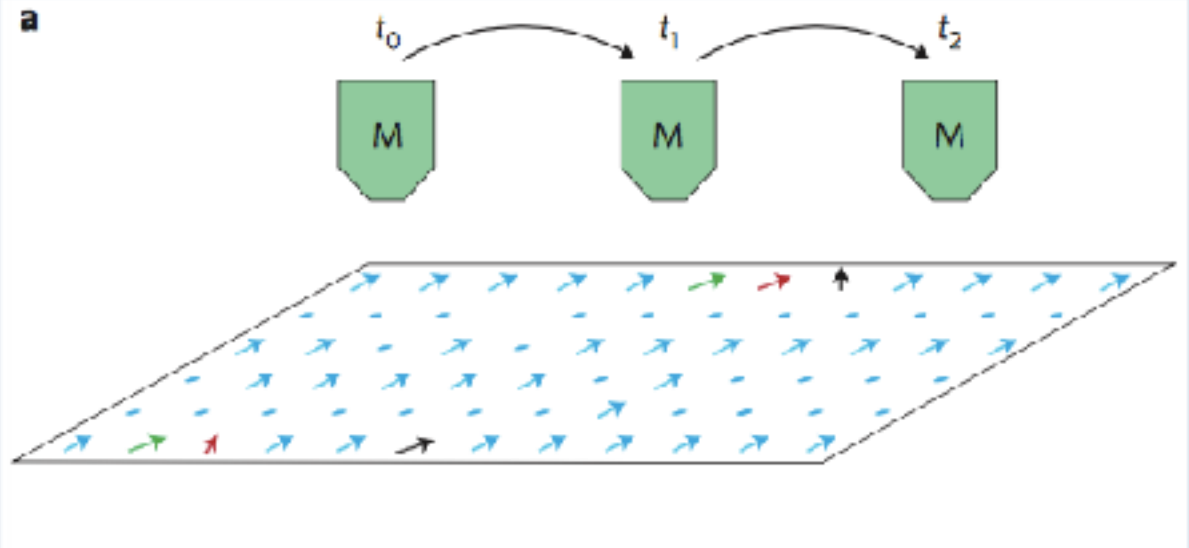
M. Lewenstwein, A. Sanpera, V. Ahufinger, B. Damski, ASD, and U. Sen, Adv. Phys. **56**, 243 ('06).

L. Amico, R. Fazio, A. Osterloh, and V.Vedral, Rev.Mod. Phys. **80**, 517 ('08).

Potential candidate for realising information processing tasks



Raussendorf, R. & Briegel, H. J. A one-way quantum computer. *Phys. Rev. Lett.* **86**, 5188–5191 (2001).

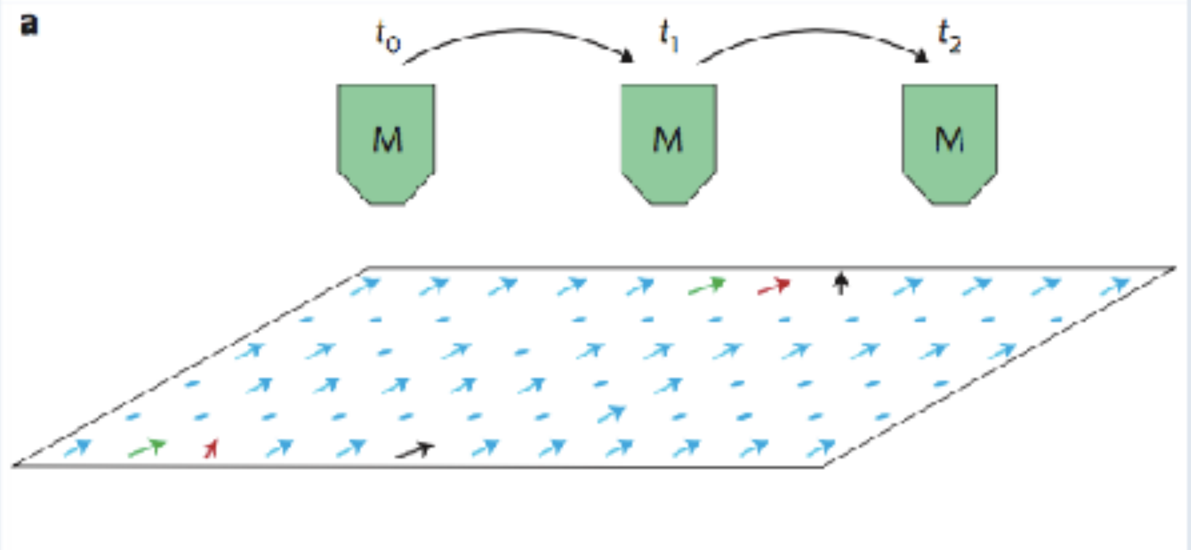


One-way quantum computer by using Ising chain: Cluster state preparation, followed by single qubit measurement leads to gate implementation; fidelity of gates =1

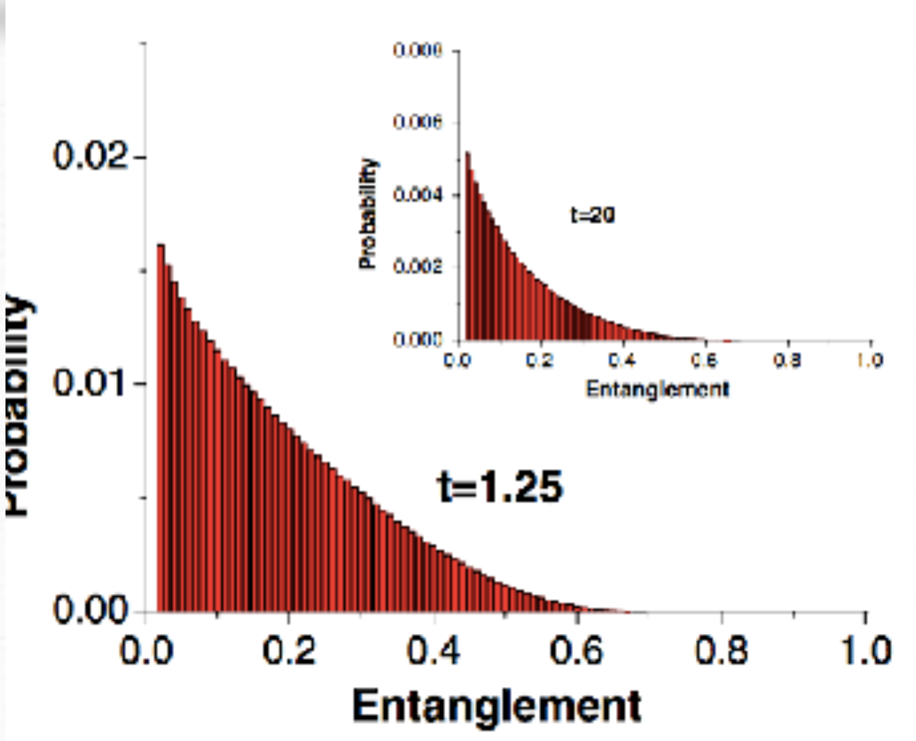
One-way quantum computer

Potential candidate for realising information processing tasks

Raussendorf, R. & Briegel, H. J. A one-way quantum computer. *Phys. Rev. Lett.* **86**, 5188–5191 (2001).



One-way quantum computer by using Ising chain: Cluster state preparation, followed by single qubit measurement leads to gate implementation; fidelity of gates =1



Gate implementations possible in disordered model; fidelity=0.85

ASD, U. Sen, V. Ahufinger, HJ. Briegel, A. Sanpera, & M. Lewenstein
Phys. Rev. A **74**, 062309 (2006)

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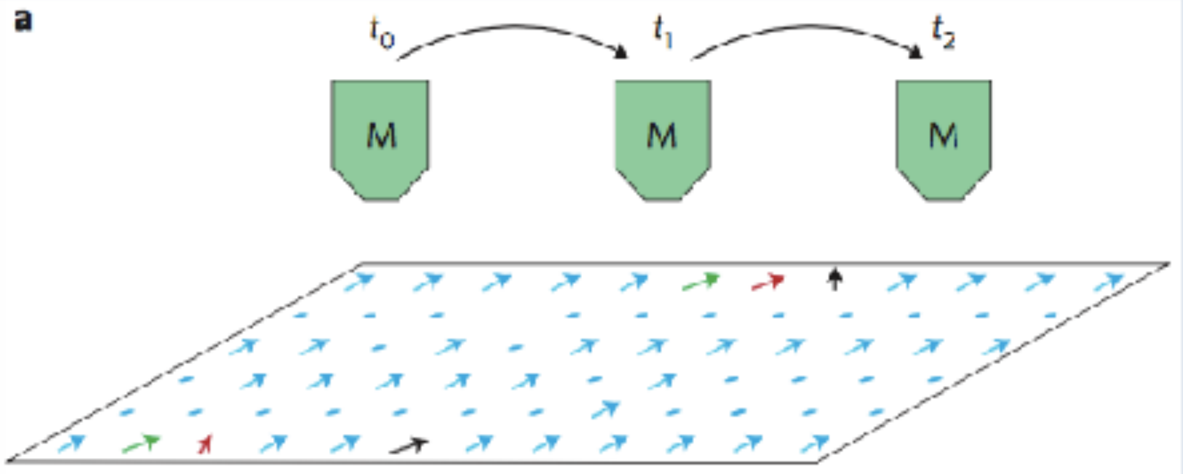

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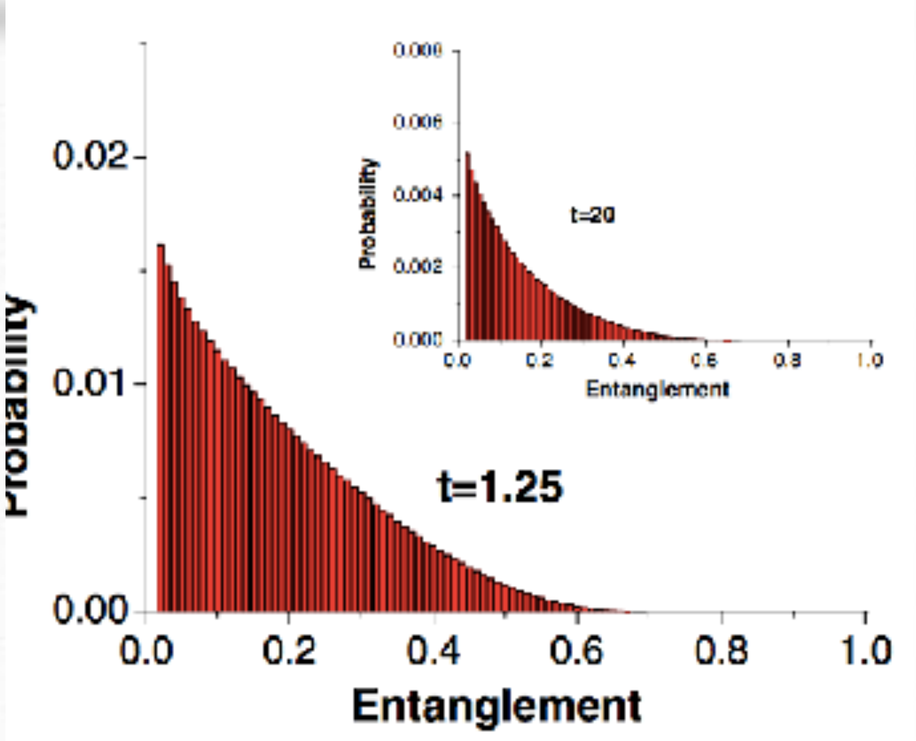

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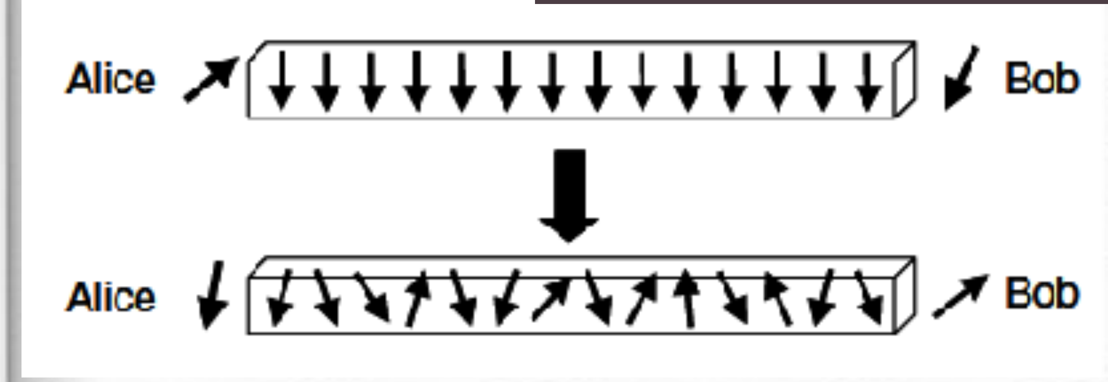
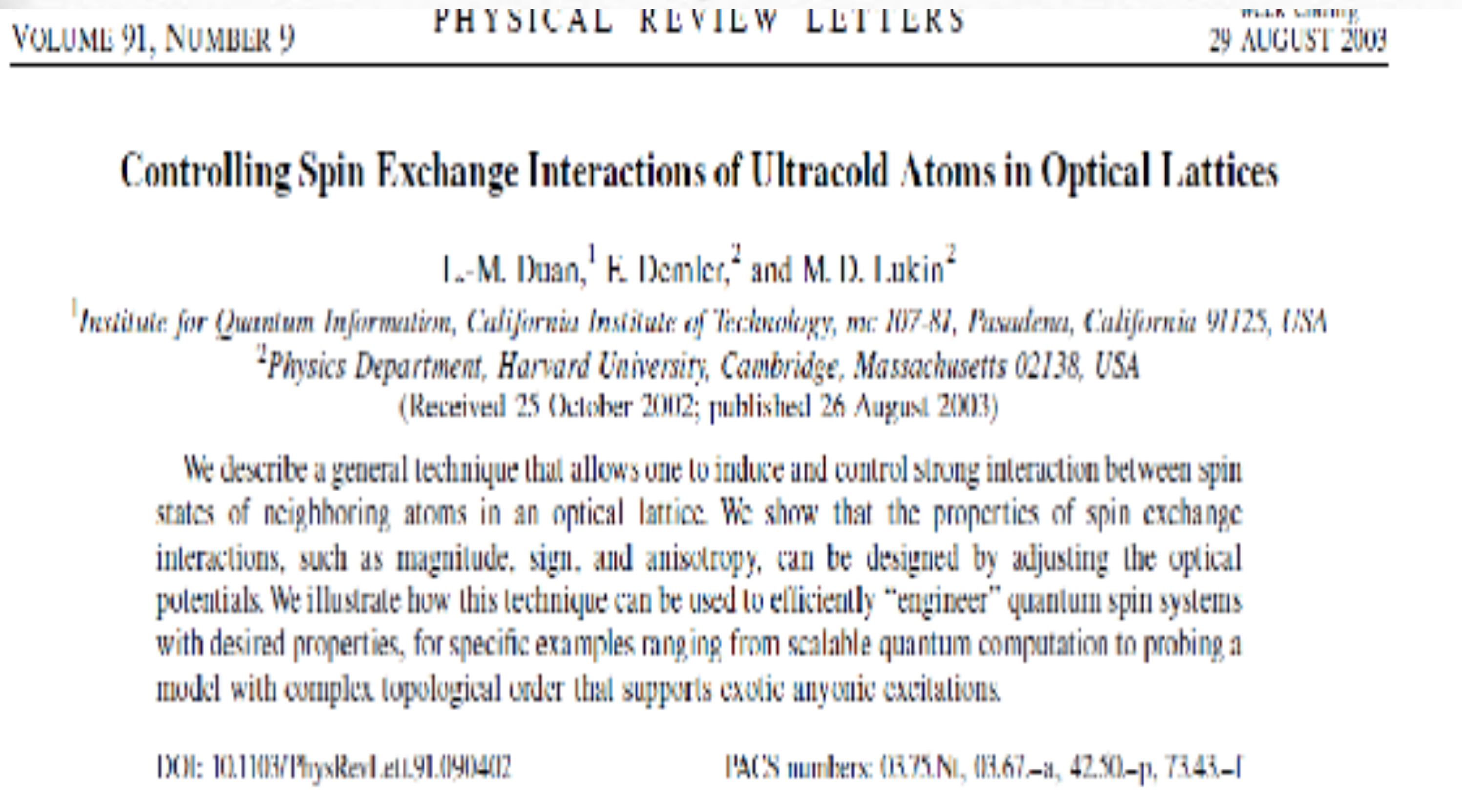
State Transmission



S. Bose, PRL 91, 207901 (2003)

Initially spin chain is in its ground state in an external magnetic field. Alice and Bob are at opposite ends of the chain. Alice places the quantum state she wants to communicate on the spin nearest to her. After a while, Bob receives this state with some fidelity on the spin nearest to him.

Implementation: Proposals



VOLUME 91, NUMBER 9 PHYSICAL REVIEW LETTERS 29 AUGUST 2003

Controlling Spin Exchange Interactions of Ultracold Atoms in Optical Lattices

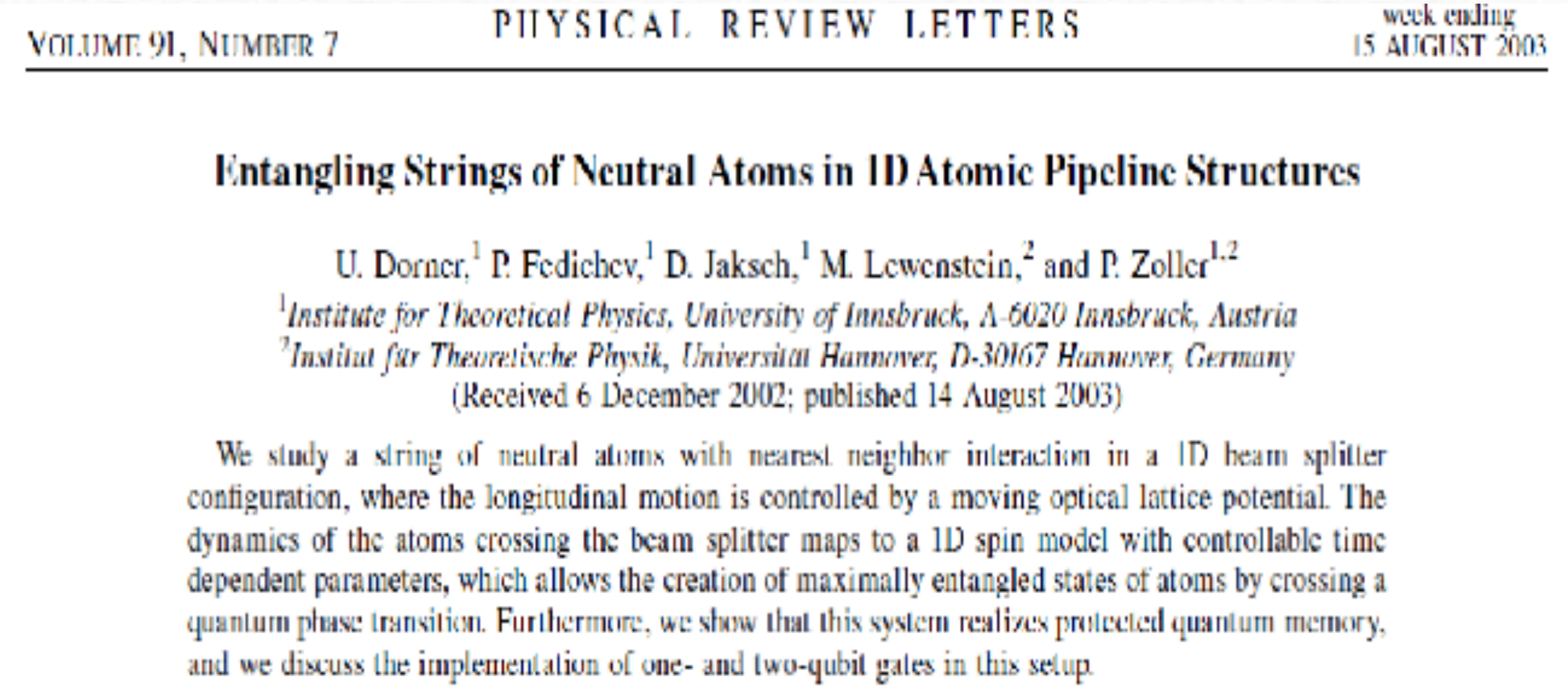
L.-M. Duan,[1] E. Demler,[2] and M. D. Lukin[2]
[1]*Institute for Quantum Information, California Institute of Technology, mc 107-81, Pasadena, California 91125, USA*
[2]*Physics Department, Harvard University, Cambridge, Massachusetts 02138, USA*

(Received 25 October 2002; published 26 August 2003)

We describe a general technique that allows one to induce and control strong interaction between spin states of neighboring atoms in an optical lattice. We show that the properties of spin exchange interactions, such as magnitude, sign, and anisotropy, can be designed by adjusting the optical potentials. We illustrate how this technique can be used to efficiently "engineer" quantum spin systems with desired properties, for specific examples ranging from scalable quantum computation to probing a model with complex topological order that supports exotic anyonic excitations.

DOI: 10.1103/PhysRevLett.91.090402 PACS numbers: 03.75.Nt, 03.67.–a, 42.50.–p, 73.43.–f

Optical Lattices



VOLUME 91, NUMBER 7 PHYSICAL REVIEW LETTERS week ending 15 AUGUST 2003

Entangling Strings of Neutral Atoms in 1D Atomic Pipeline Structures

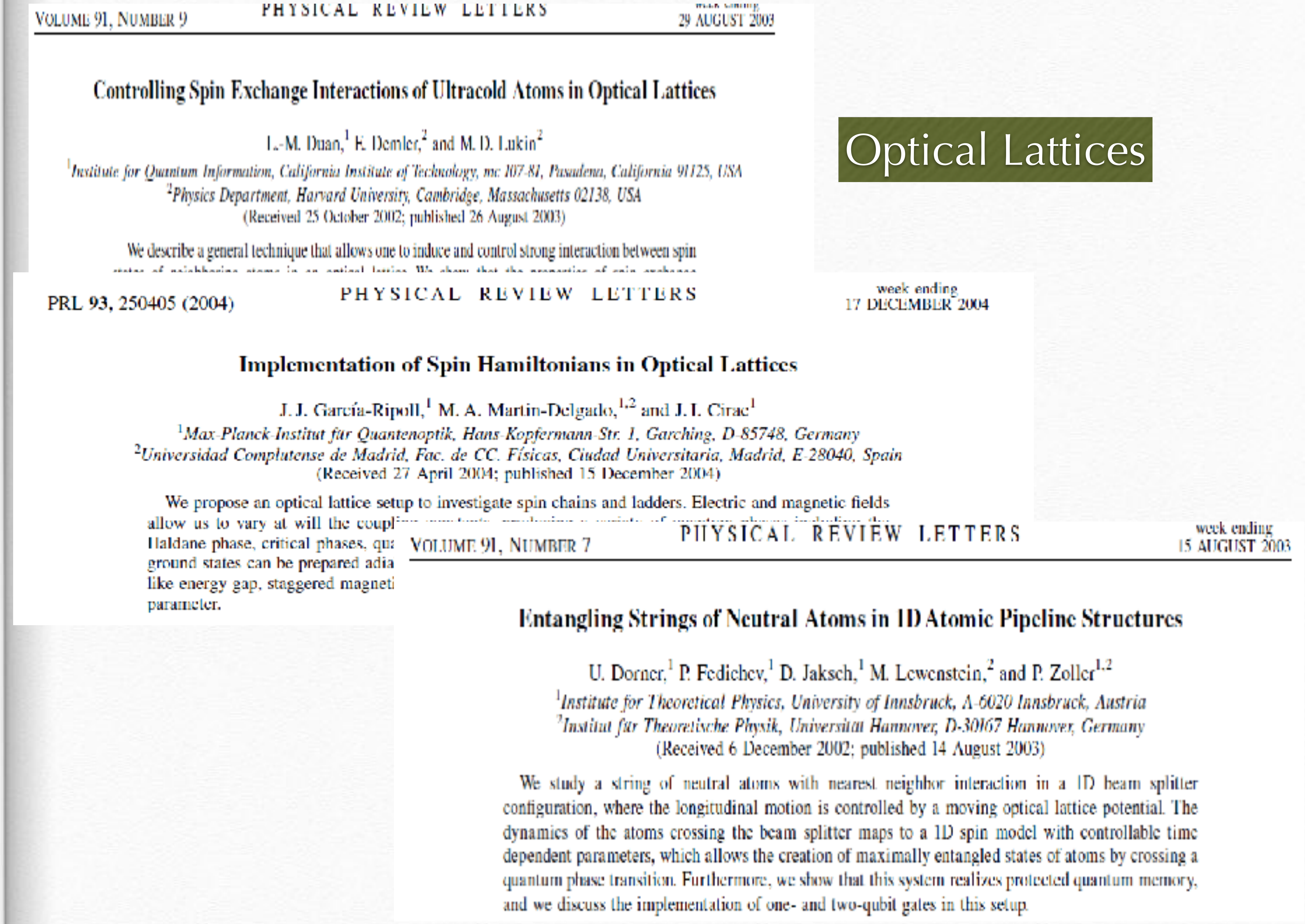
U. Dorner,[1] P. Fedichev,[1] D. Jaksch,[1] M. Lewenstein,[2] and P. Zoller[1,2]
[1]*Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria*
[2]*Institut für Theoretische Physik, Universität Hannover, D-30167 Hannover, Germany*

(Received 6 December 2002; published 14 August 2003)

We study a string of neutral atoms with nearest neighbor interaction in a 1D beam splitter configuration, where the longitudinal motion is controlled by a moving optical lattice potential. The dynamics of the atoms crossing the beam splitter maps to a 1D spin model with controllable time dependent parameters, which allows the creation of maximally entangled states of atoms by crossing a quantum phase transition. Furthermore, we show that this system realizes protected quantum memory, and we discuss the implementation of one- and two-qubit gates in this setup.

Implementation: Proposals

Optical Lattices



VOLUME 91, NUMBER 9 PHYSICAL REVIEW LETTERS week ending 29 AUGUST 2003

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[2]Physics Department, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 25 October 2002; published 26 August 2003)

We describe a general technique that allows one to induce and control strong interaction between spin

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PRL 93, 250405 (2004) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2004

Implementation of Spin Hamiltonians in Optical Lattices

J. J. García-Ripoll,[1] M. A. Martin-Delgado,[1,2] and J. I. Cirac[1]
[1]Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, Garching, D-85748, Germany
[2]Universidad Complutense de Madrid, Fac. de CC. Físicas, Ciudad Universitaria, Madrid, E-28040, Spain

(Received 27 April 2004; published 15 December 2004)

We propose an optical lattice setup to investigate spin chains and ladders. Electric and magnetic fields
allow us to vary at will the coupl
Haldane phase, critical phases, qu
ground states can be prepared adia
like energy gap, staggered magneti
parameter.

VOLUME 91, NUMBER 7 PHYSICAL REVIEW LETTERS week ending 15 AUGUST 2003

Entangling Strings of Neutral Atoms in 1D Atomic Pipeline Structures

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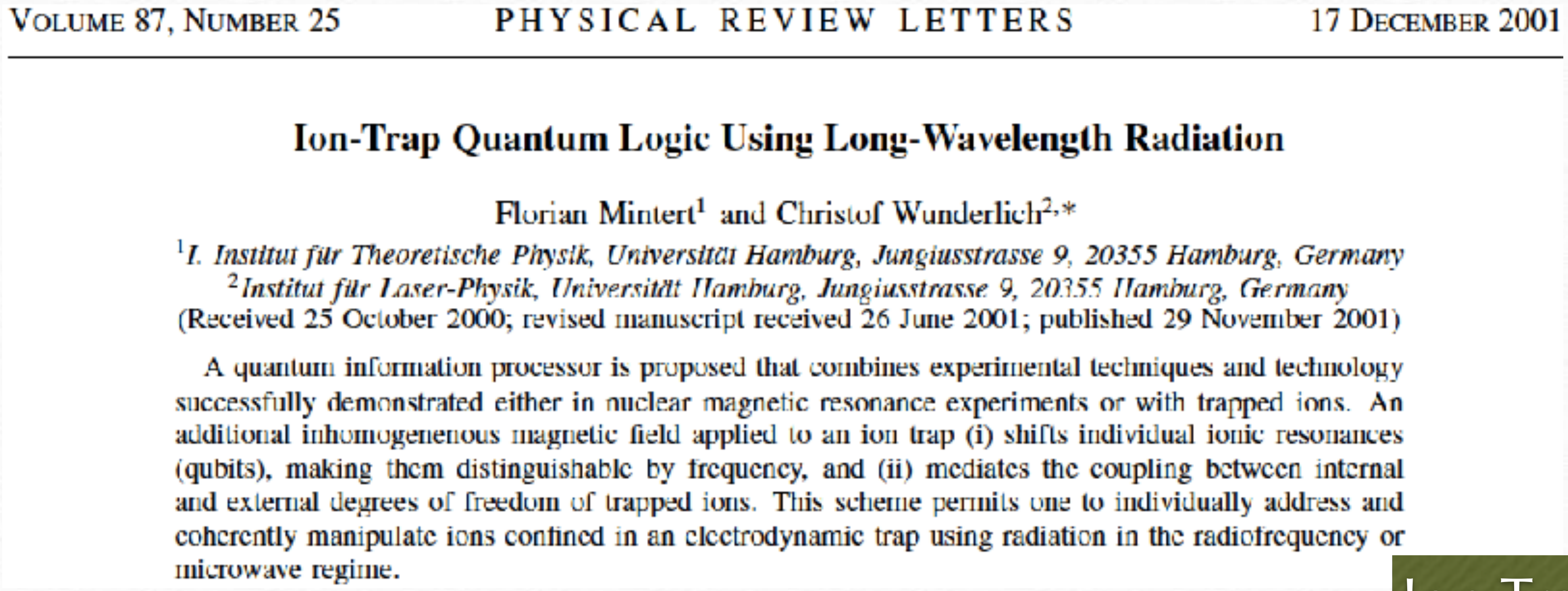
(Received 6 December 2002; published 14 August 2003)

We study a string of neutral atoms with nearest neighbor interaction in a 1D beam splitter configuration, where the longitudinal motion is controlled by a moving optical lattice potential. The dynamics of the atoms crossing the beam splitter maps to a 1D spin model with controllable time dependent parameters, which allows the creation of maximally entangled states of atoms by crossing a quantum phase transition. Furthermore, we show that this system realizes protected quantum memory, and we discuss the implementation of one- and two-qubit gates in this setup.

Implementation: Proposals



VOLUME 87, NUMBER 25 PHYSICAL REVIEW LETTERS 17 DECEMBER 2001

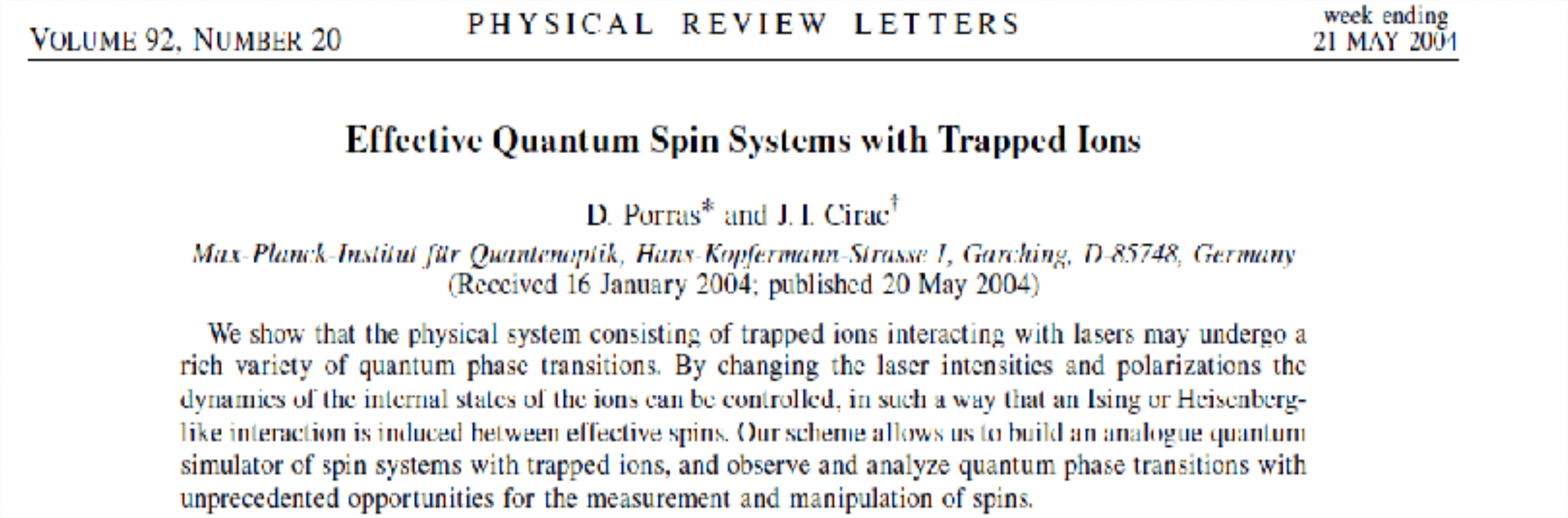
Ion-Trap Quantum Logic Using Long-Wavelength Radiation

Florian Mintert[1] and Christof Wunderlich[2,*]

[1]*I. Institut für Theoretische Physik, Universität Hamburg, Jungiusstrasse 9, 20355 Hamburg, Germany*
[2]*Institut für Laser-Physik, Universität Hamburg, Jungiusstrasse 9, 20355 Hamburg, Germany*
(Received 25 October 2000; revised manuscript received 26 June 2001; published 29 November 2001)

A quantum information processor is proposed that combines experimental techniques and technology successfully demonstrated either in nuclear magnetic resonance experiments or with trapped ions. An additional inhomogeneous magnetic field applied to an ion trap (i) shifts individual ionic resonances (qubits), making them distinguishable by frequency, and (ii) mediates the coupling between internal and external degrees of freedom of trapped ions. This scheme permits one to individually address and coherently manipulate ions confined in an electrodynamic trap using radiation in the radiofrequency or microwave regime.

Ion Traps



VOLUME 92, NUMBER 20 PHYSICAL REVIEW LETTERS week ending 21 MAY 2004

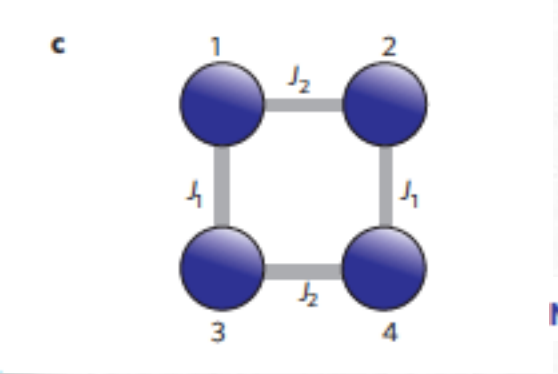
Effective Quantum Spin Systems with Trapped Ions

D. Porras* and J. I. Cirac†

Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, Garching, D-85748, Germany
(Received 16 January 2004; published 20 May 2004)

We show that the physical system consisting of trapped ions interacting with lasers may undergo a rich variety of quantum phase transitions. By changing the laser intensities and polarizations the dynamics of the internal states of the ions can be controlled, in such a way that an Ising or Heisenberg-like interaction is induced between effective spins. Our scheme allows us to build an analogue quantum simulator of spin systems with trapped ions, and observe and analyze quantum phase transitions with unprecedented opportunities for the measurement and manipulation of spins.

Implementation: Experiments



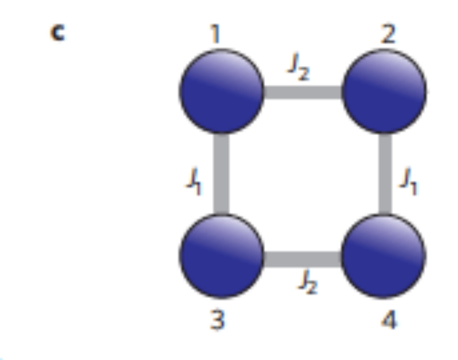
Quantum simulation of the wavefunction to probe frustrated Heisenberg spin systems

Photons

Xiao-song Ma[1,2†], Borivoje Dakic[2†], William Naylor[1,2], Anton Zeilinger[1,2,3] and Philip Walther[1,2*]

NATURE PHYSICS | VOL 7 | MAY 2011 |

Implementation: Experiments



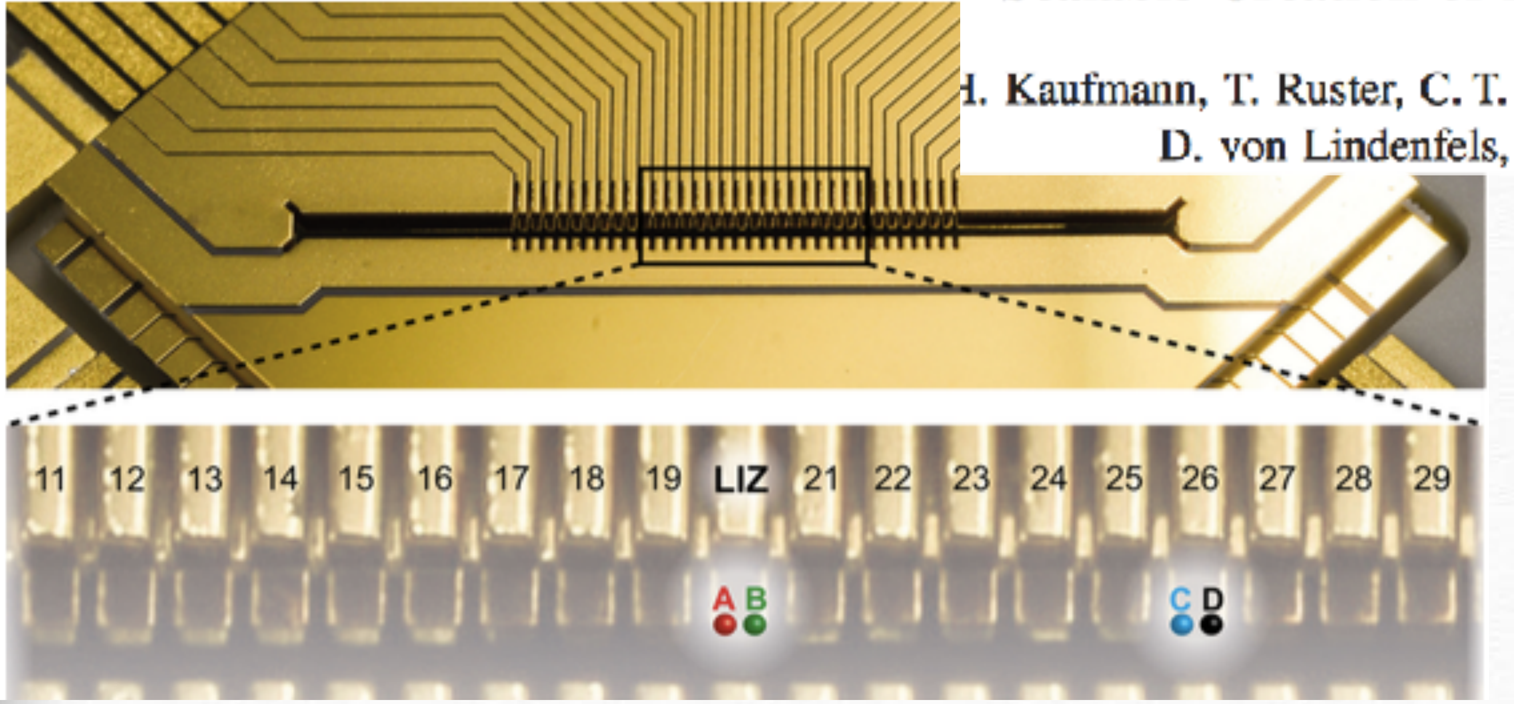
Quantum simulation of the wavefunction to probe frustrated Heisenberg spin systems

Xiao-song Ma[1,2†], Borivoje Dakic[2†], William Naylor[1,2], Anton Zeilinger[1,2,3] and Philip Walther[1,2*]

Photons

Scalable Creation of Long-Lived Multipartite Entanglement

H. Kaufmann, T. Ruster, C. T. Schmiegelow,[*] M. A. Luda,[†] V. Kaushal, J. Schulz, D. von Lindenfels, F. Schmidt-Kaler, and U. G. Poschinger[‡]

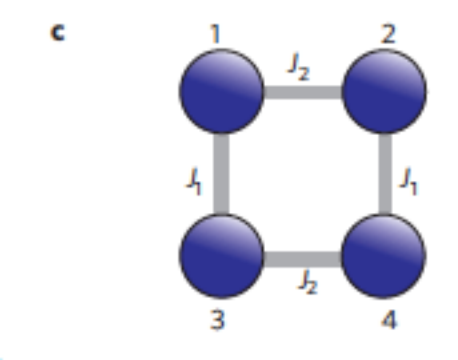


PRL 119, 150503 (2017)

1.1sec

Ions

Implementation: Experiments



Quantum simulation of the wavefunction to probe frustrated Heisenberg spin systems

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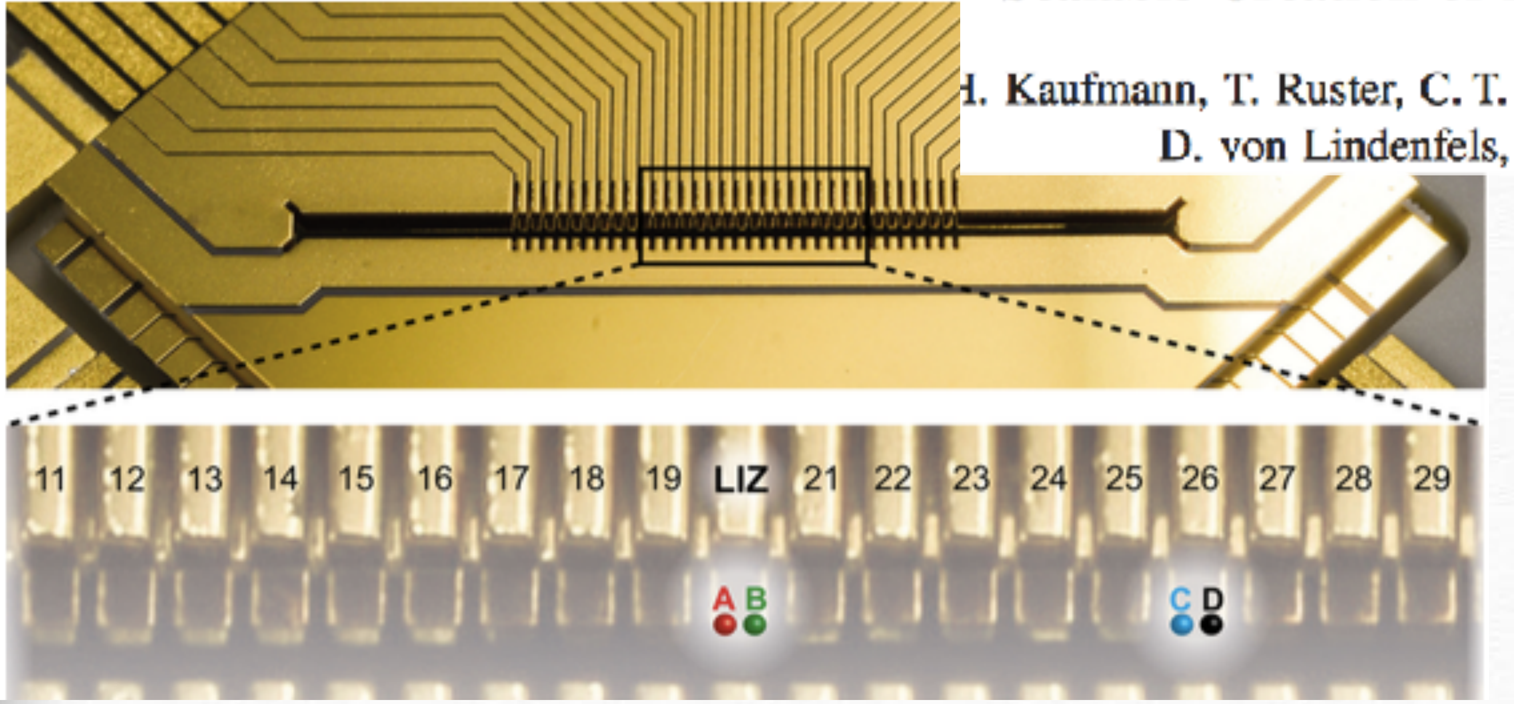
Photons



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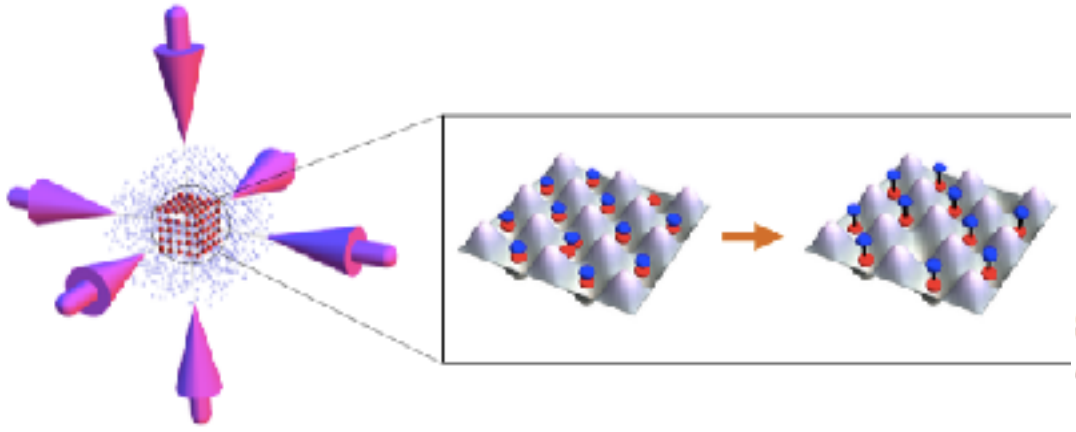
D. von Lindenfels, F. Schmidt-Kaler, and U. G. Poschinger[‡]

PRL 119, 150503 (2017)

Ions

Polar molecules

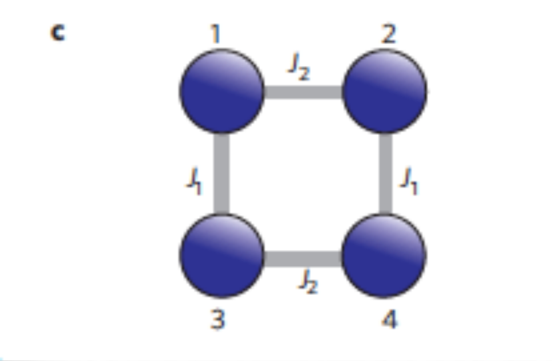


Creation of a low-entropy quantum gas of polar molecules in an optical lattice

Science **350**, 659 (2015)

Steven A. Moses, Jacob P. Covey, Matthew T. Miecnikowski, Bo Yan,[*] Bryce Gadway, Jun Ye,[‡] Deborah S. Jin[‡]

Implementation: Experiments



Quantum simulation of the wavefunction to probe frustrated Heisenberg spin systems

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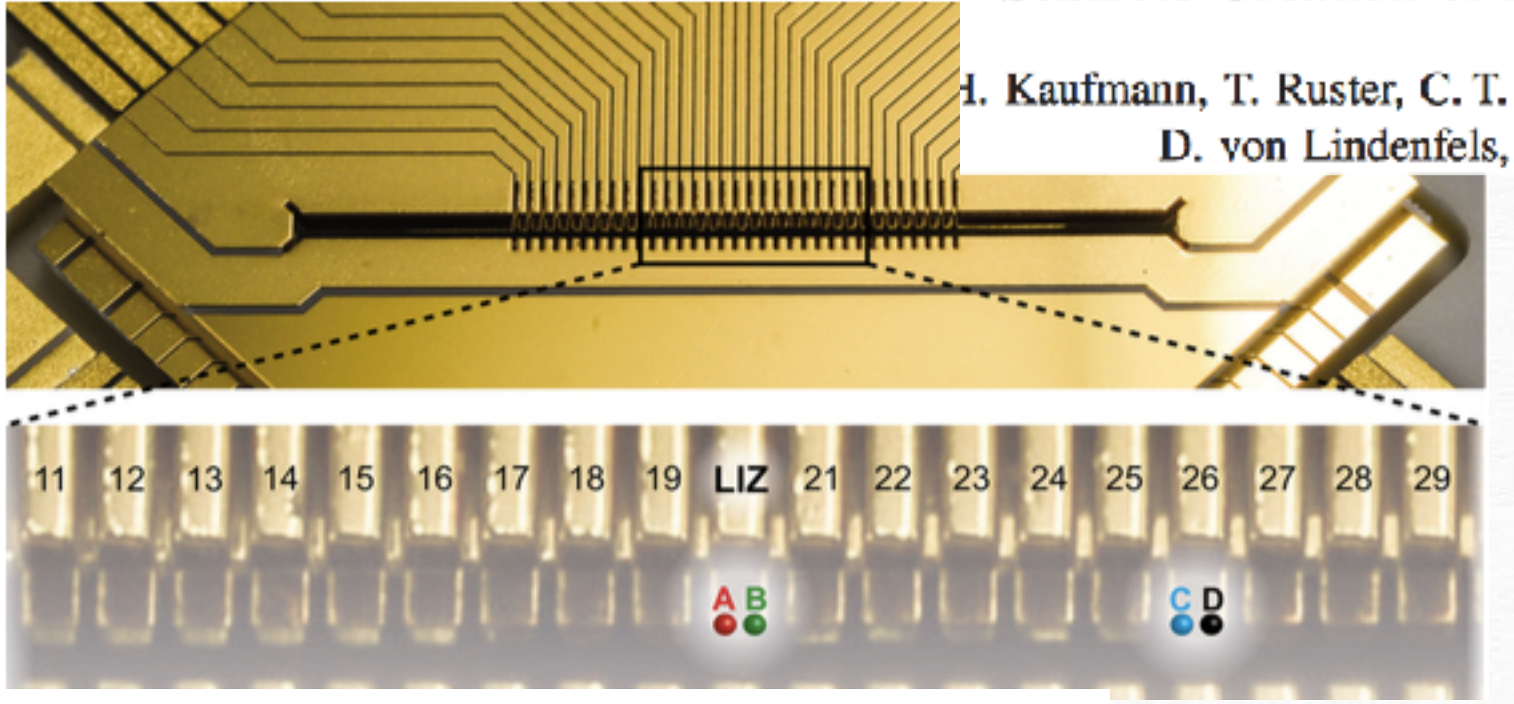
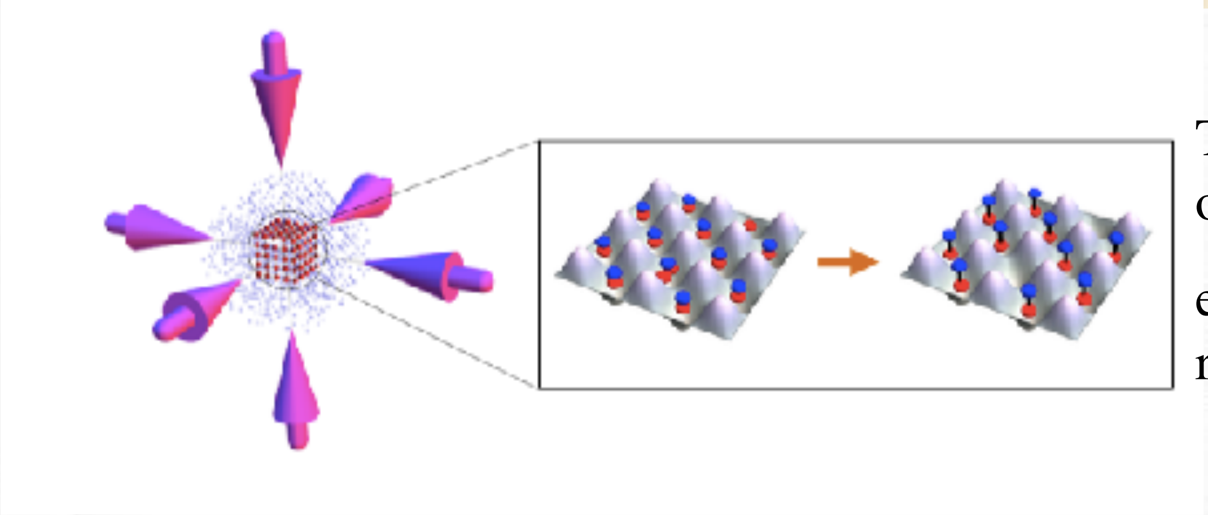
PRL 119, 150503 (2017)





The achieved filling fraction of 25% should enable future studies of transport &

entanglement propagation in a many-body system with long-range dipolar interactions.

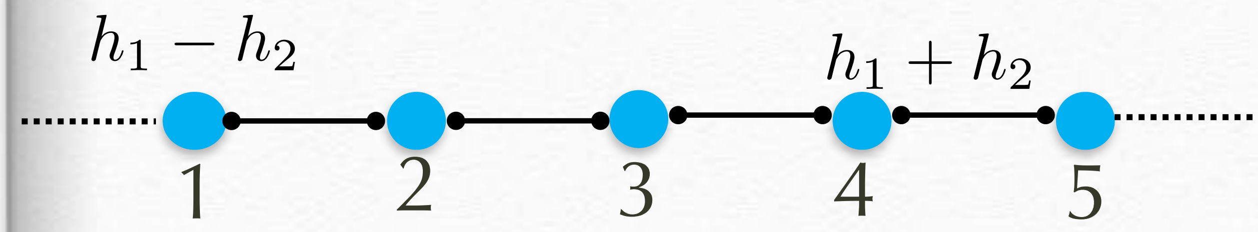
Freezing at the beginning of the dynamics?

1. Proper choice of system as well as environment

Quantum Spin Model

XY spin model with uniform & alternating field

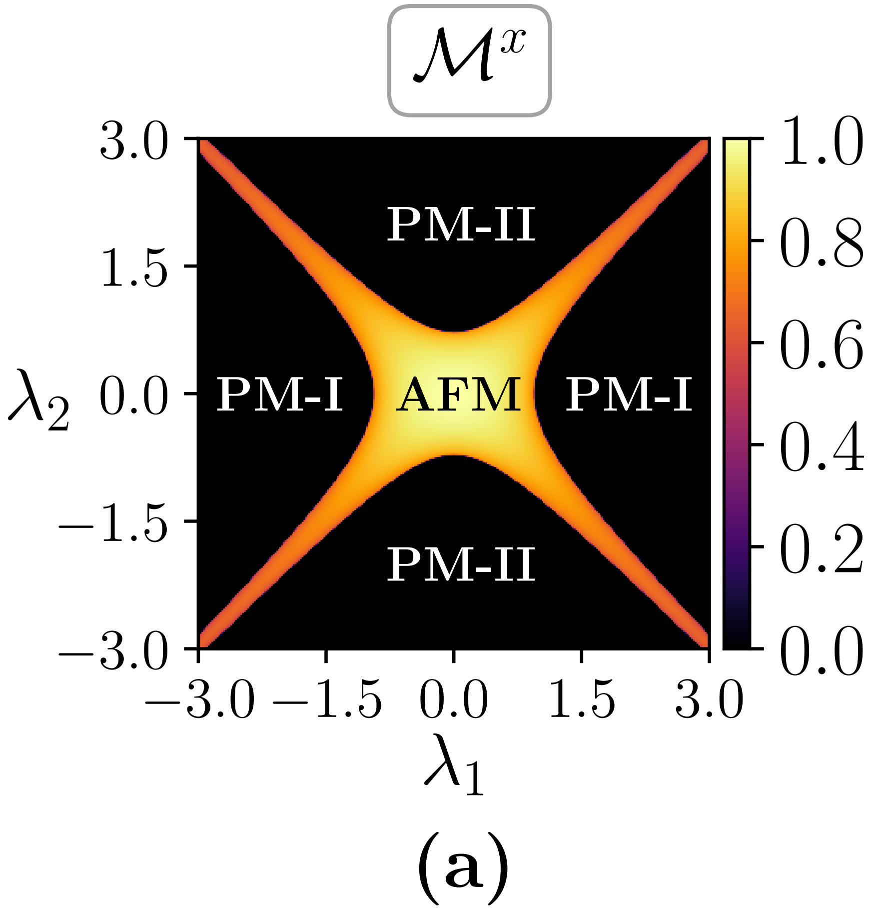
$$\hat{H}=\frac{1}{2}\sum_{j=1}^{N}\left[J\left\{\frac{1+\gamma}{2}\hat{\sigma}_j^x\hat{\sigma}_{j+1}^x+\frac{1-\gamma}{2}\hat{\sigma}_j^y\hat{\sigma}_{j+1}^y\right\}+\left(h_1+(-1)^j h_2\right)\hat{\sigma}_j^z\right]$$



ATXY spin model

Diagonalized by Jordan-Wigner, Fourier, Bogoliubov: Free fermionic model

ATXY model (Quantum phases)



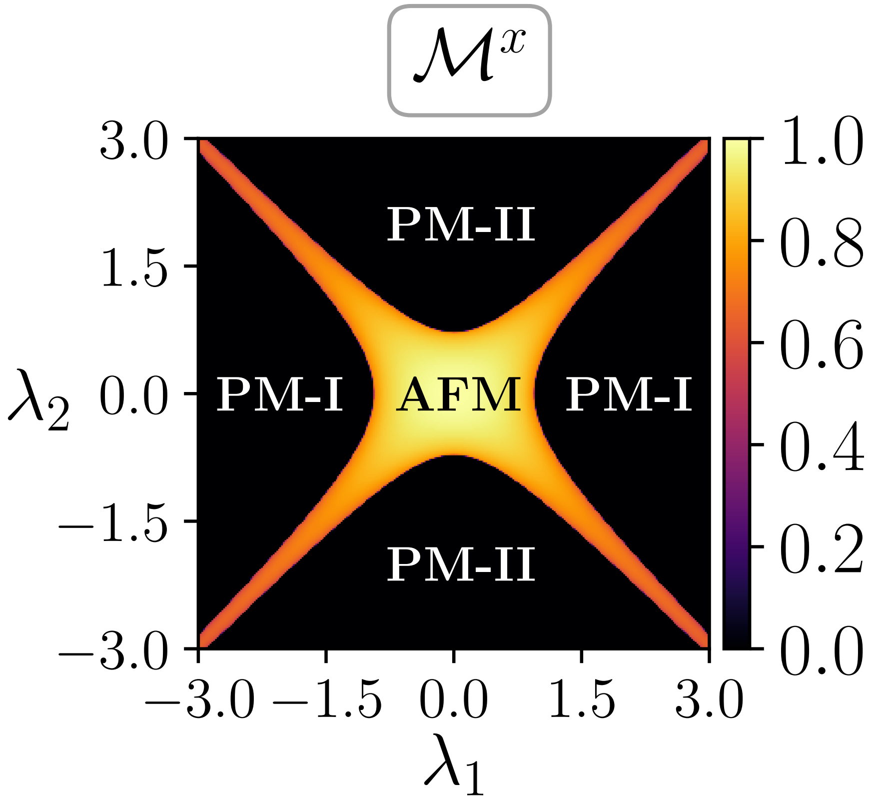
(a)

$$\mathcal{M}^x = \Big|\frac{1}{N}\sum_{j=1}^{N}(-1)^j\langle\hat{\sigma}_j^x\rangle\Big| = \Big|\frac{1}{N}\sum_{j=1}^{N}(-1)^j m_j^x\Big|.$$

Antiferromagnet

S. Roy, T. Chanda, T. Das, D. Sadhukhan, ASD, U.Sen, arXiv: 1710.11037

ATXY model (Quantum phases)



(a)

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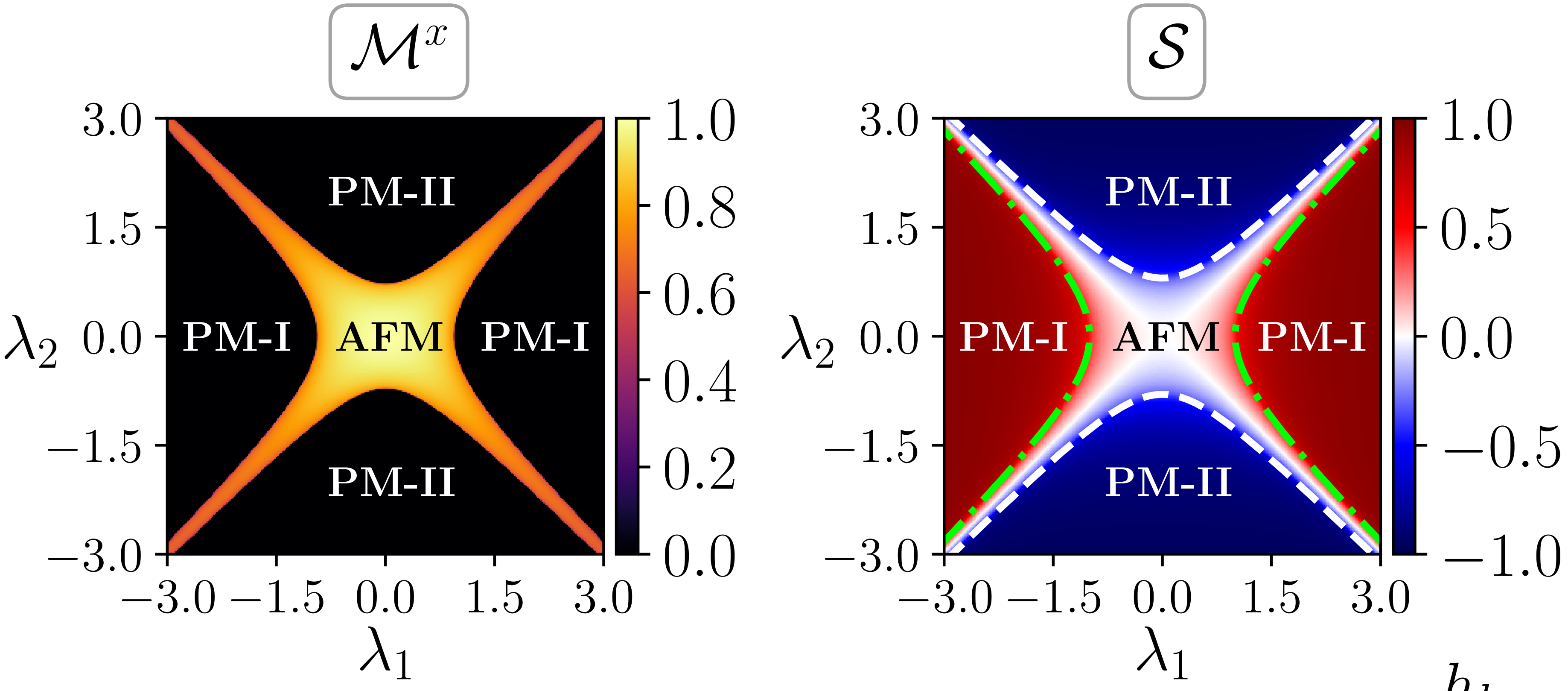
Antiferromagnet

$$\lambda_1^2 = 1 + \lambda_2^2 \quad (\text{AFM} \leftrightarrow \text{PM-I}),$$
$$\lambda_2^2 = \lambda_1^2 + \gamma^2 \quad (\text{AFM} \leftrightarrow \text{PM-II}).$$

S. Roy, T. Chanda, T. Das, D. Sadhukhan, ASD, U.Sen, arXiv: 1710.11037

ATXY model (Quantum phases)



(a) **(b)**

$$\lambda_k = \frac{h_k}{J};\ k = 1, 2$$

$$\mathcal{M}^x = \left|\frac{1}{N}\sum_{j=1}^{N}(-1)^j\langle\hat{\sigma}_j^x\rangle\right| = \left|\frac{1}{N}\sum_{j=1}^{N}(-1)^j m_j^x\right|.$$

Antiferromagnet (AFM)

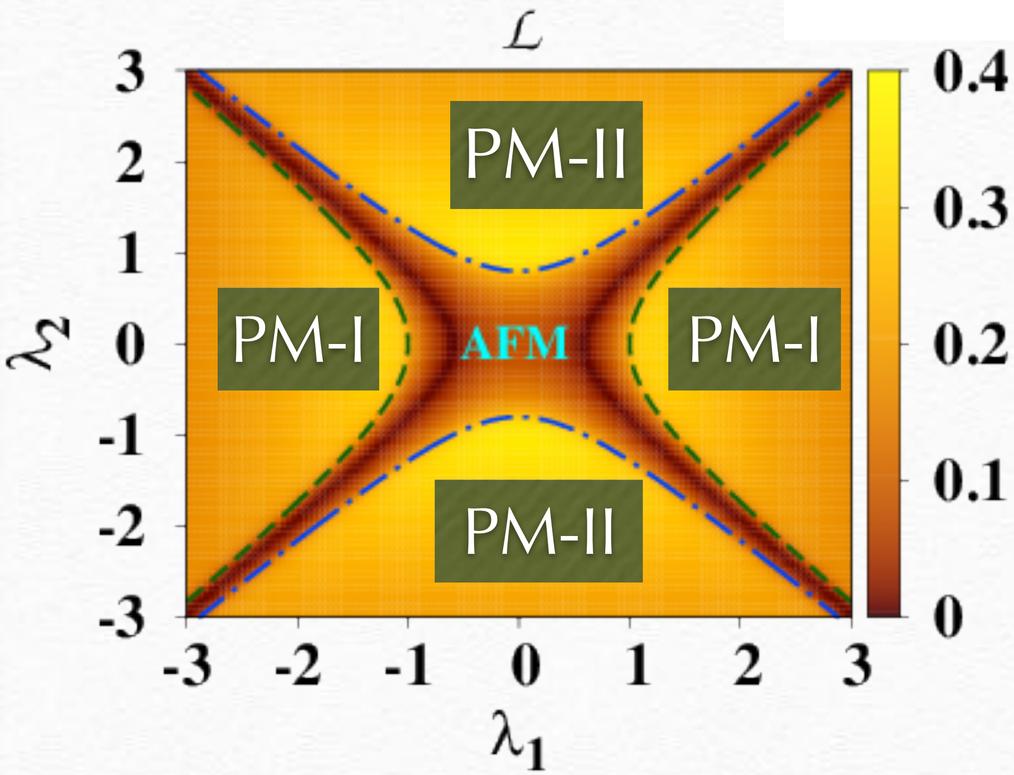
$$\mathcal{S} \equiv m_e^z m_o^z$$

$$\mathcal{S} > 0 \implies$$ Paramagnetic (PM-I)

$$\mathcal{S} < 0 \implies$$ PM-II

$$\lambda_1^2 = 1 + \lambda_2^2 \quad \textbf{(AFM} \leftrightarrow \textbf{PM-I)},$$
$$\lambda_2^2 = \lambda_1^2 + \gamma^2 \quad \textbf{(AFM} \leftrightarrow \textbf{PM-II)}.$$

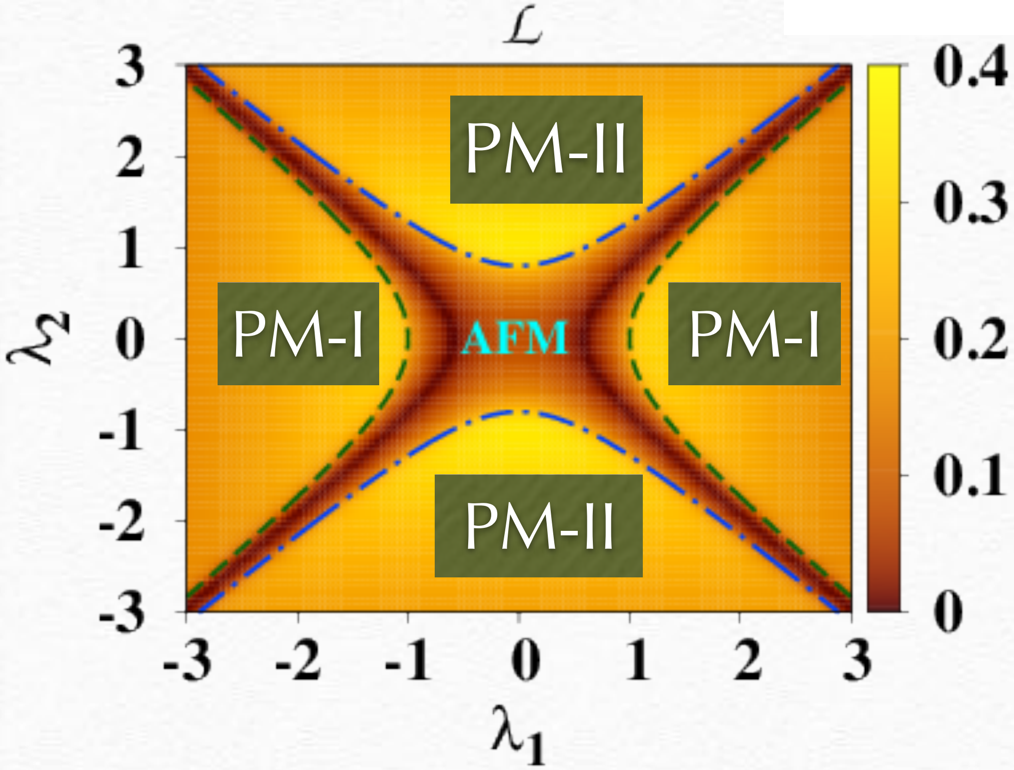
S. Roy, T. Chanda, T. Das, D. Sadhukhan, ASD, U.Sen, arXiv: 1710.11037

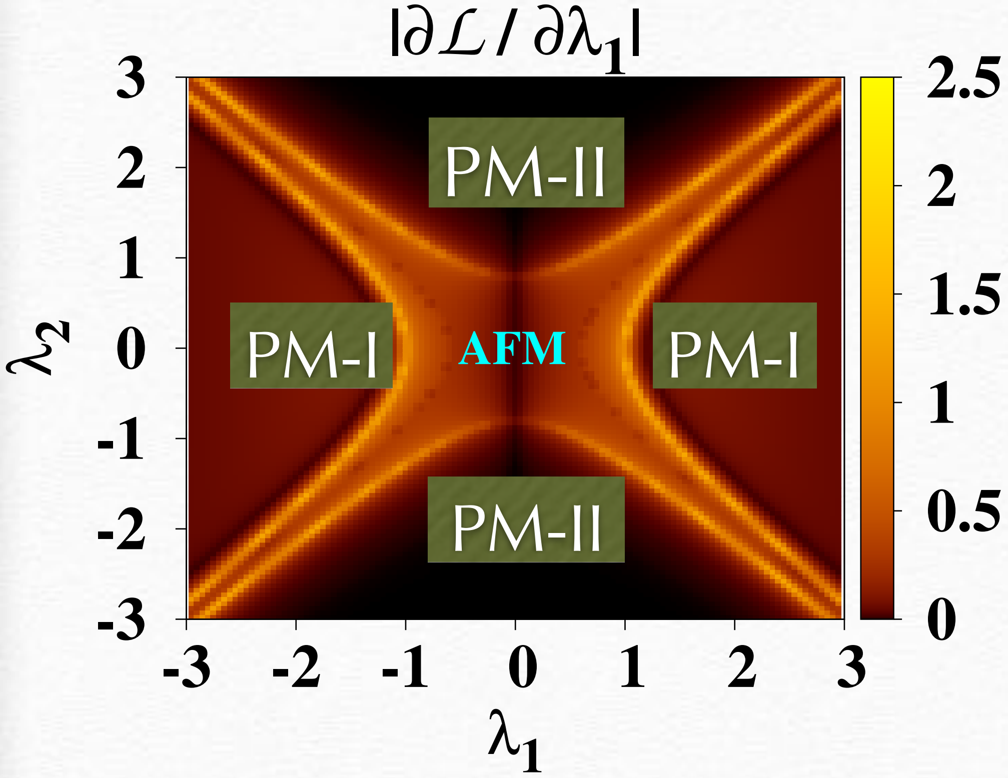


High ent in PM-II

Very low ent in AFM

T. Chanda, T. Das, D.Sadhukhan, A.K. Pal, ASD, & U. Sen, **PRA 94, 042310 (2016)**





High ent in PM-II

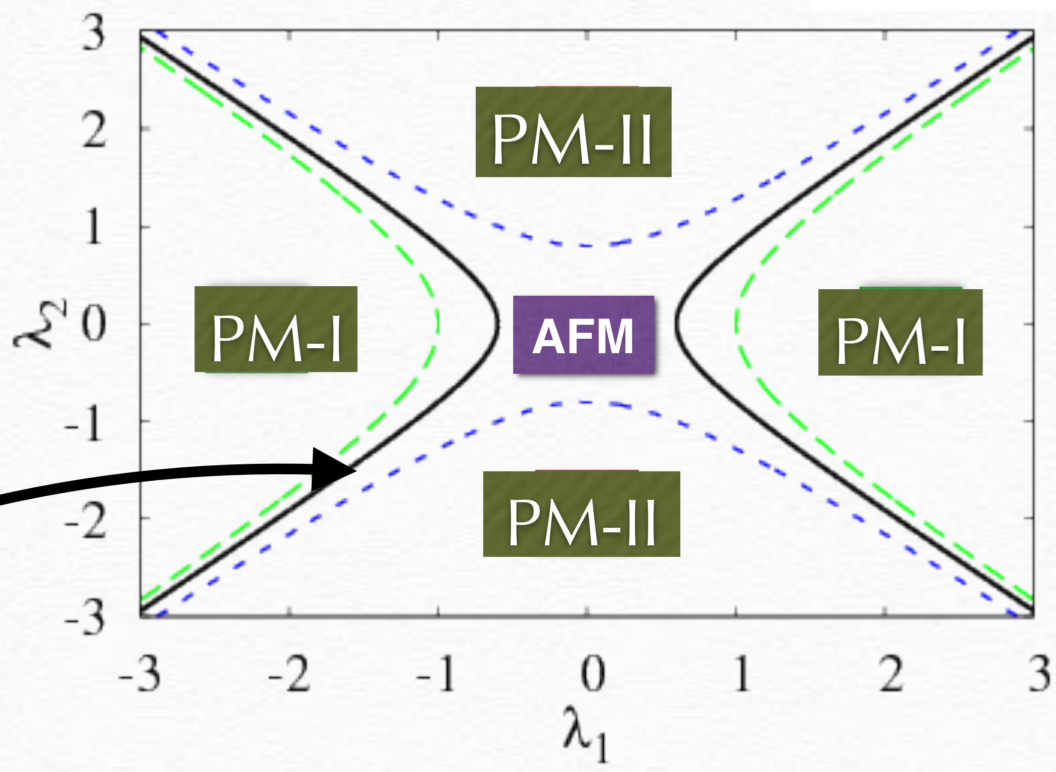
Very low ent in AFM

First derivative of ent diverges in critical lines

Finite size scaling analysis Performed

T. Chanda, T. Das, D.Sadhukhan, A.K. Pal, ASD, & U. Sen, **PRA 94, 042310 (2016)**

Factorization Surfaces



$$\lambda_1^2 = \lambda_2^2 + (1-\gamma)^2$$

We analytically found that
in the factorization surface,
the zero-temperature state is doubly degenerate

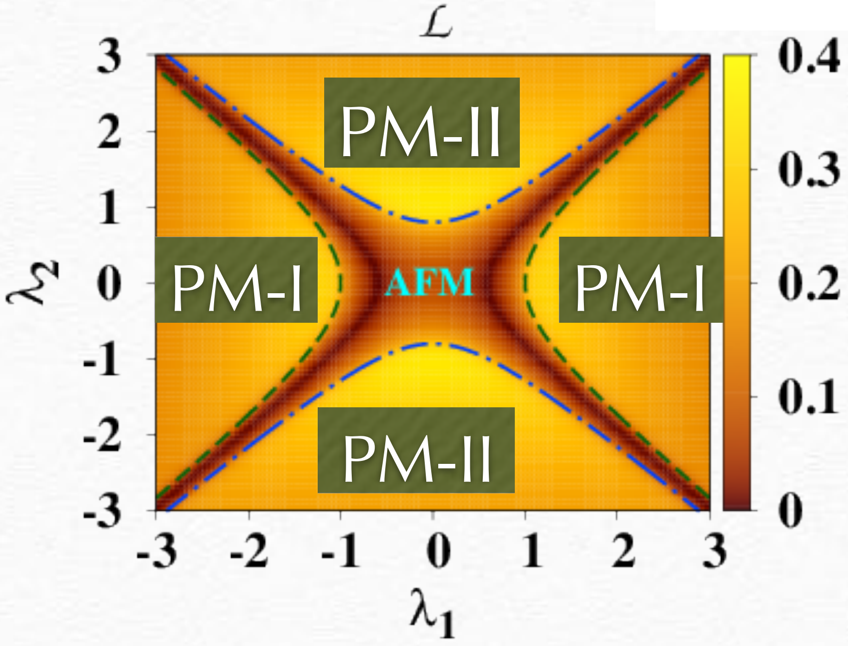
$$|\psi\rangle = \prod_{i=0}^{\frac{N}{2}-1} |\psi^o_{2i+1}\rangle \otimes |\psi^e_{2i+2}\rangle,$$

T. Chanda, T. Das, D.Sadhukhan, A.K. Pal, ASD, & U. Sen, **PRA 97, 012316 (2018)**
T. Chanda, T. Das, D.Sadhukhan, A.K. Pal, ASD, & U. Sen, **PRA 94, 042310 (2016)**

Ent is a fragile quantity

Common intuition: decrease with noise

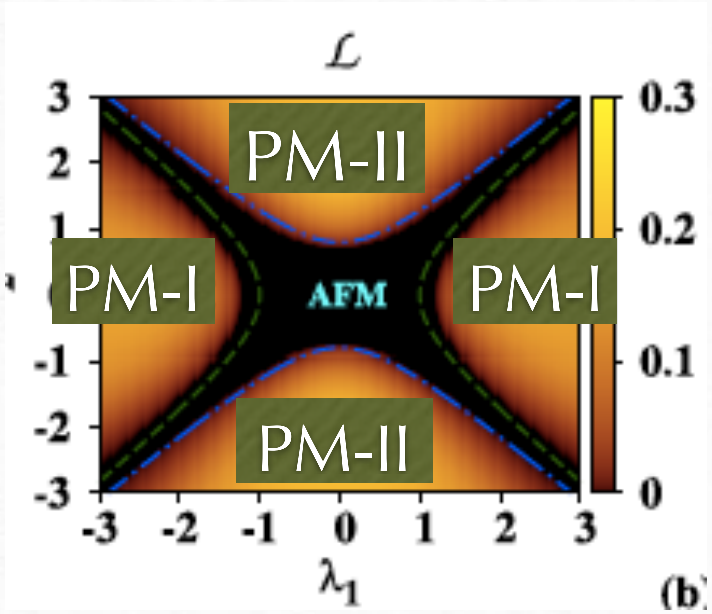
Ent increases with Temperature



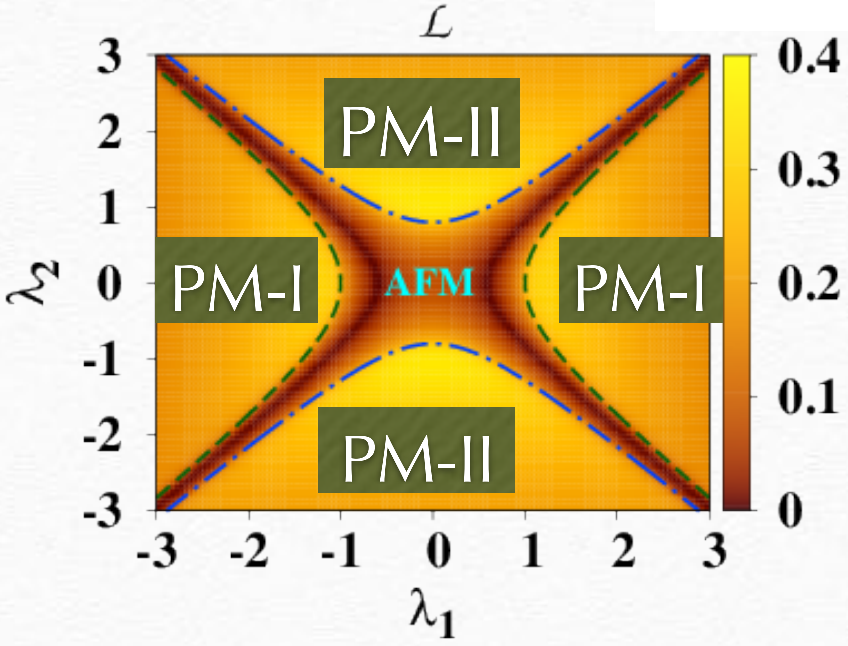
T increases

Ent decreases with temperature

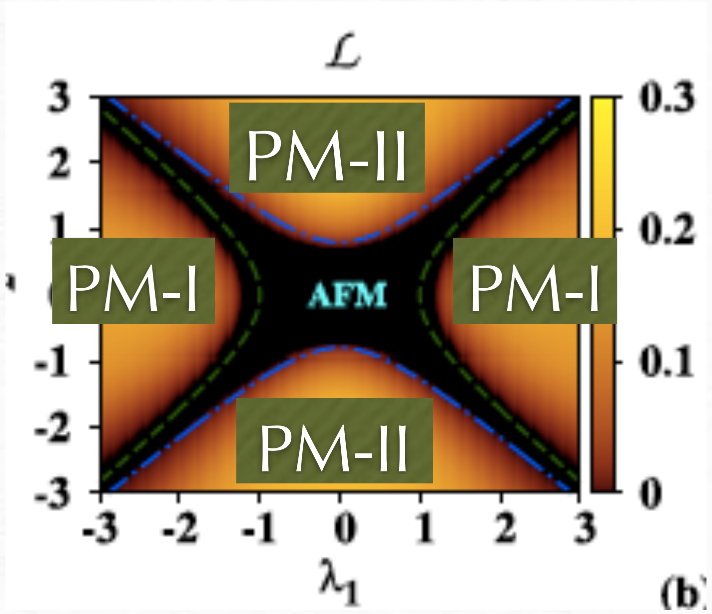
Intuition: ✓



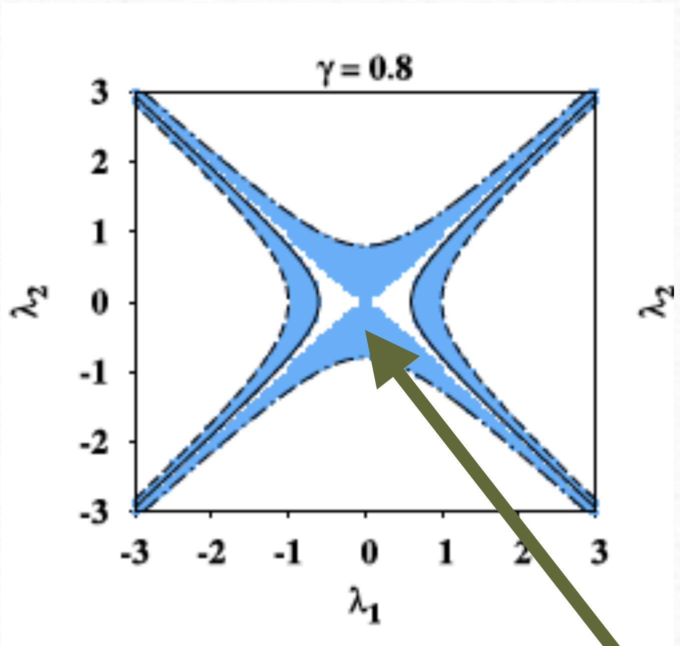
Ent increases with Temperature



T increases



Ent decreases
with temperature

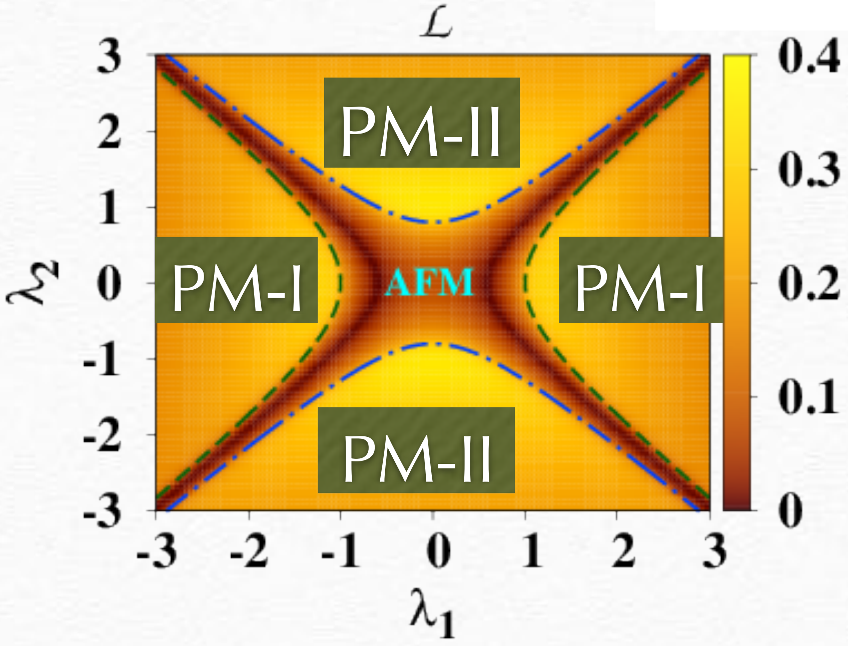


Counter-intuitive

Ent increases
with the increase of temperature

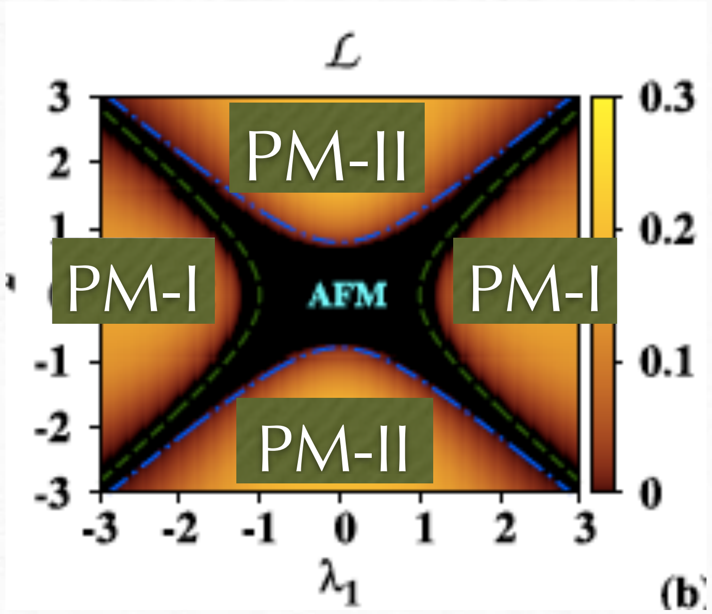
Nonmonotonic map of ent with temperature

Ent increases with Temperature

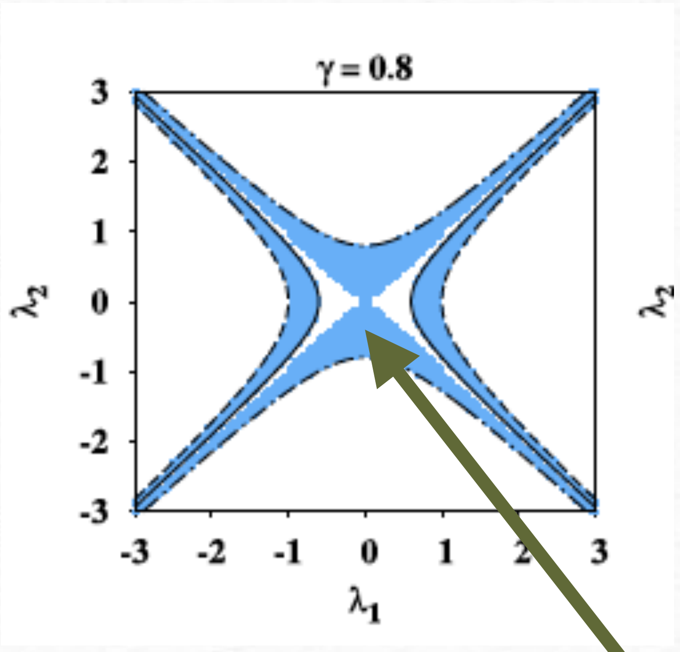


T increases

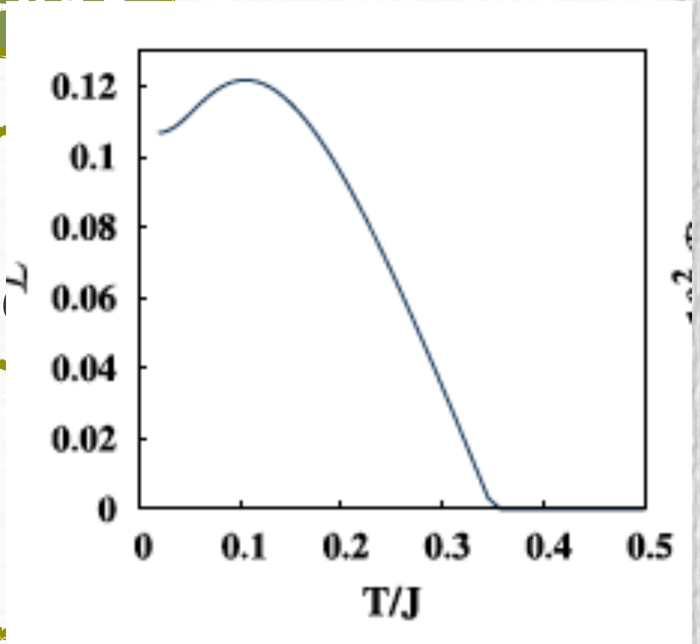
Ent decreases with temperature



Counter-intuitive



Ent increases
with the increase of temp

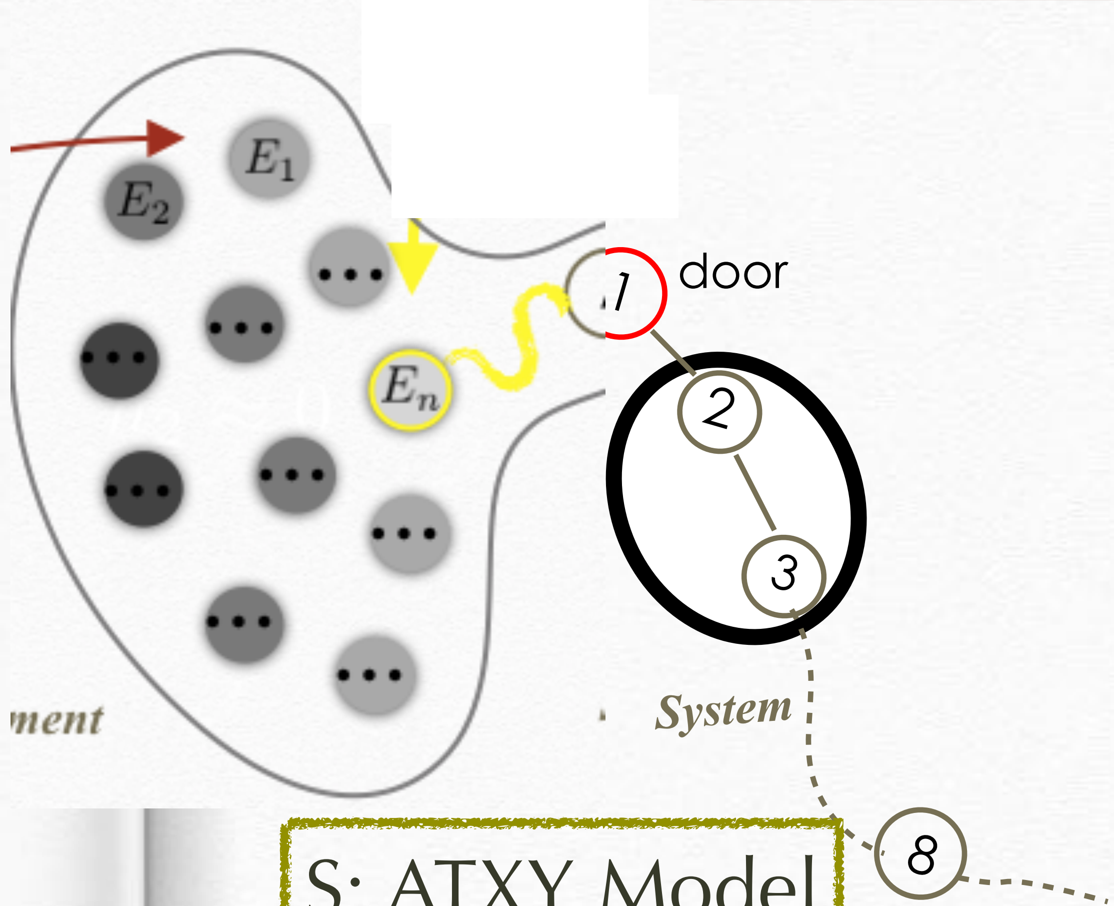


Nonmonotonic map of ent with temperature

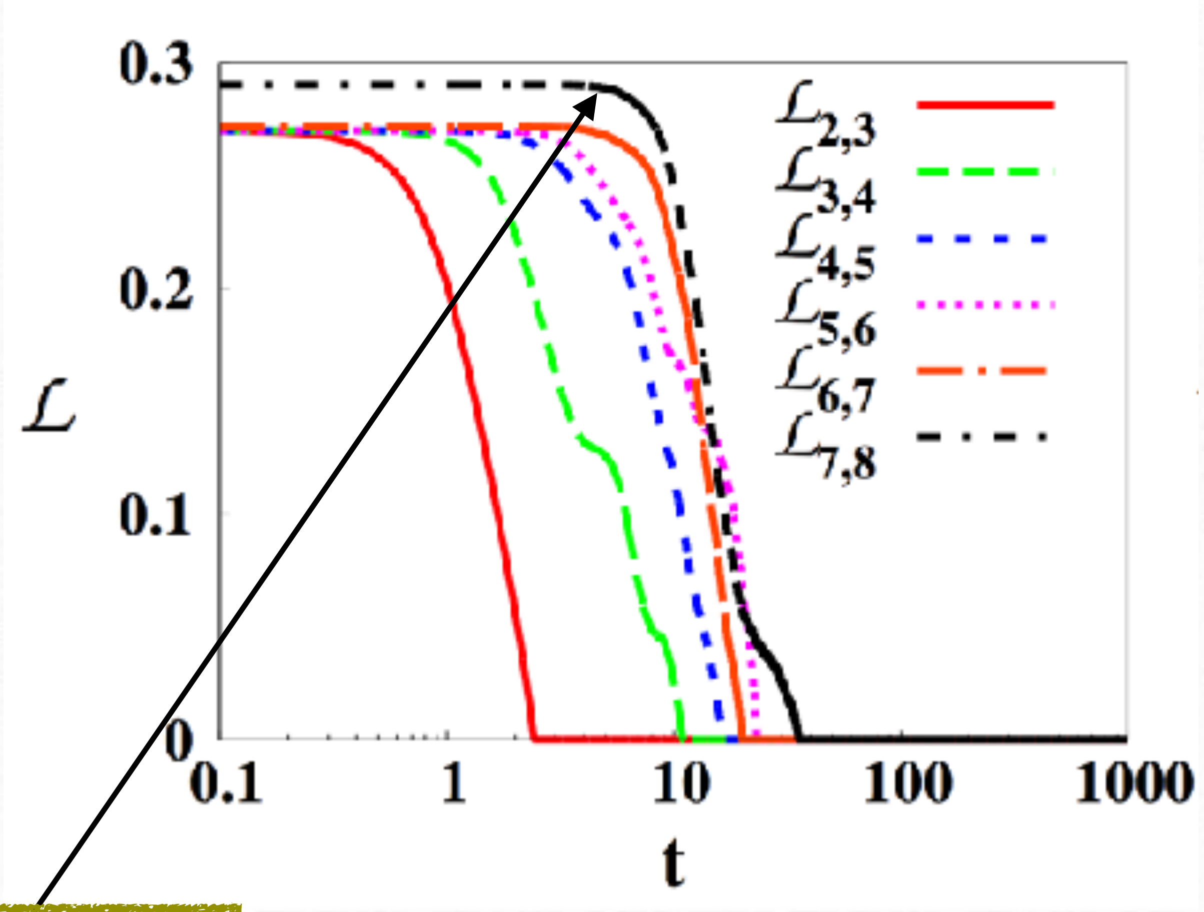
Open Quantum System: Effects of environment

Freezing of entanglement?

Freezing of Entanglement



S: ATXY Model



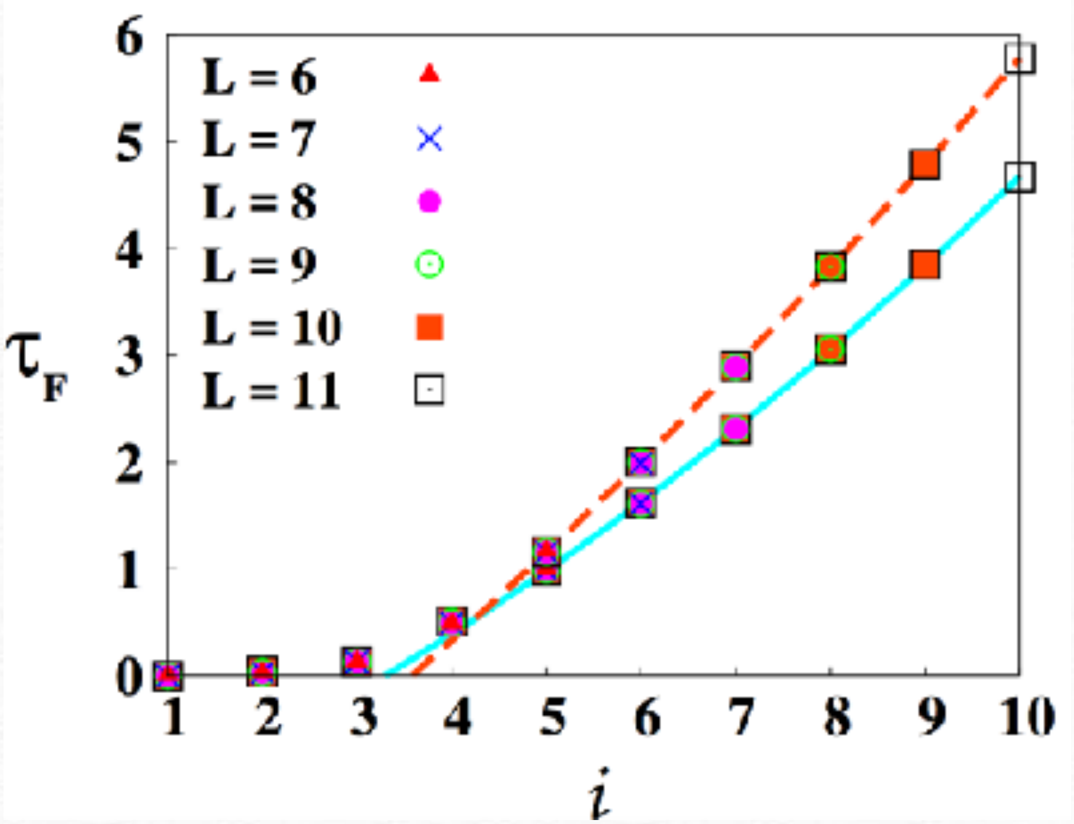
Freezing terminal

PM-II phase

T. Chanda, T. Das, D.Sadhukhan, AK Pal, ASD, & U. Sen, arXiv: 1610.00730

Freezing Terminal: Scale invariant

$\tau_F^{(i,i+1)}$: Freezing terminal for spin pair $(i,i+1)$



PM-II phase

T. Chanda, T. Das, D.Sadhukhan, AK Pal, ASD, & U. Sen, arXiv: 1610.00730

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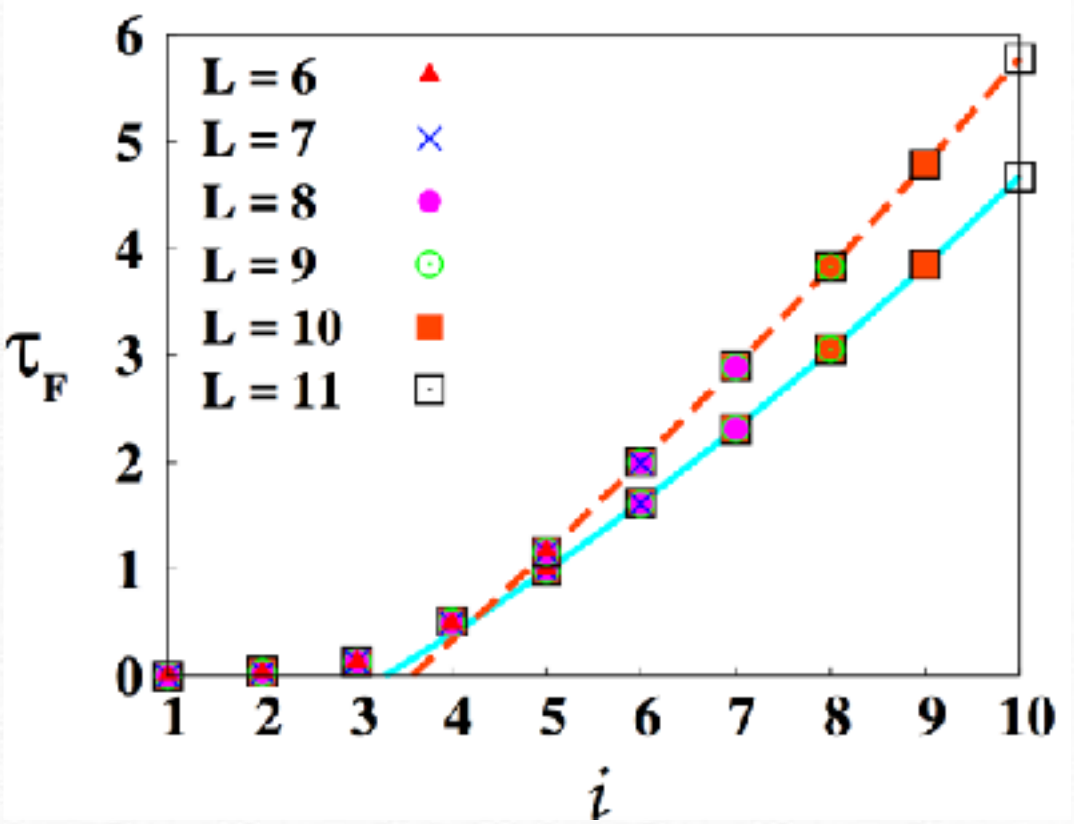
$$\boxed{\tau_F^{i,i+1} = ai^2 + bi + c \quad \forall L}$$



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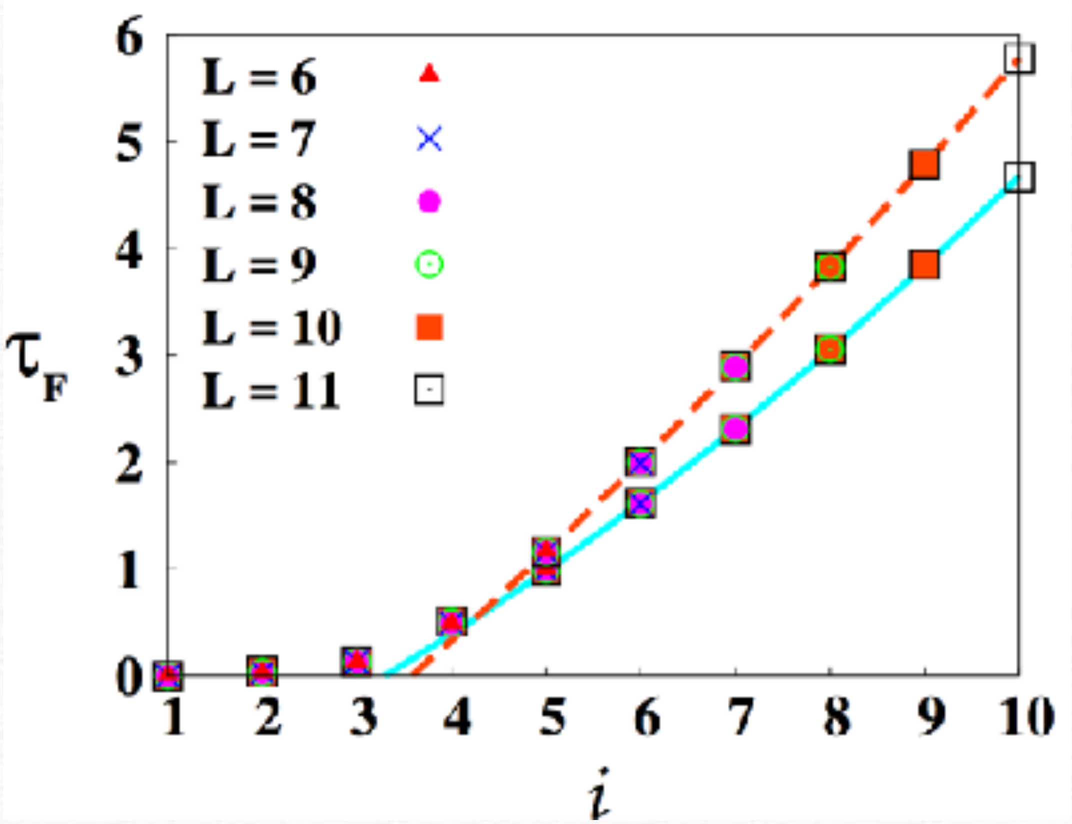
Both in PM-I and PM-II phase



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for spin pair $(i,i+1)$

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Scale invariance

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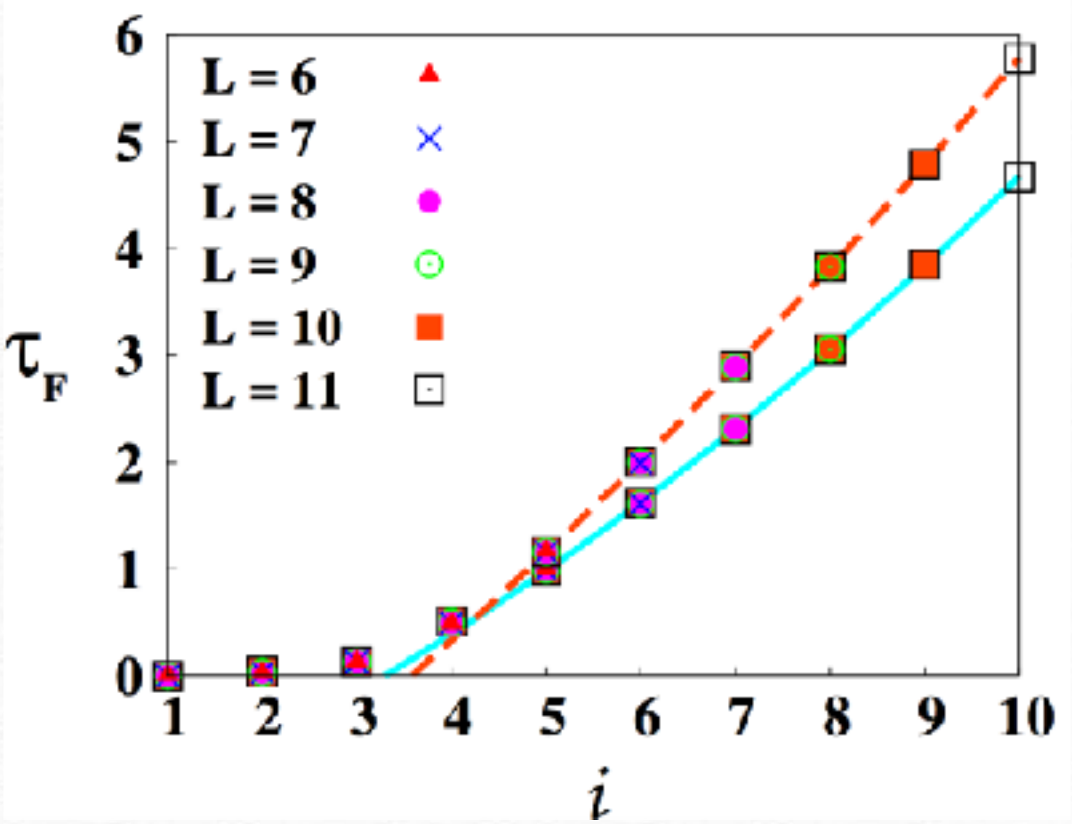

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Summary

Quantum discord freezes:
Necessary and sufficient condition
Discord and one-way WD: Different freezing behavior

Ent can detect QPT in ATXY model

Ent dynamics: Freezing of ent

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Thank you!!