Invariant Planck scale correction to photon dynamics, degenerate Fermi gas and its consequences.

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Outline of Talk

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Motivation of DSR

A new relativistic theory:

- Almost all the Quantum Gravity(QG) candidates agree on a scale(Planck scale) where QG effects become dominant.
- This scale is formed out of the fundamental constants in the theory.
- Planck scale(κ) enters the action, κ should be invariant to keep EOM covariant and for all observers to agree with the same QG scale.
- Many high energy experiments(like Gamma ray burst etc.) suggest that the dispersion relation at such energies gets modified.
- Either give up Lorentz symmetry and consider violation of relativistic principle ad hoc.(e.g SME).
- Or preserve the Lorentz symmetry(suggested by Amelino Camelia and furthered by Lee Smolin as well) but the Poincare algebra gets modified.

Basics of DSR

Where does it live?

- It is a modification of known relativistic theory incorporating two invariants c and κ.
- Lives on the boundary where QG effects start becoming dominant (this is Planck scale) and so has residual effects.
- Physical world lives in 0 to κ or sub-Planck regime. This is an effective theory.
- Since we are are looking for classical relativistic theory we neglect the Quantum dynamical effects(i.e DSR is independent of the actual underlying QG theory).
- The consequence of this modification is the ultraviolet cut-off (as an invariant scale) and the Modified Dispersion Relation (MDR).

Rewriting relativity:

- Relativity of inertial frames is still valid.
- Introduce an observer independent scale κ.
- Correspondence principle giving known relativistic theory in a limit $\kappa \to \infty$.

The deformed algebra:

- Lorentz sub-algebra should remain intact.
- Deform the generators in such a way that above is true but we get an invariant κ.
- So the Poincare algebra gets deformed to $\kappa-$Poincare algebra.
- Reduces to the known Poincare algebra in the limit $\kappa \to \infty$.
- Many ways to do this depending on the basis that we choose (Bicross-product, Magueijo-Smolin(MS basis), Classical etc.)

Basics of DSR

The choice of basis:

- DSR naturally formulates in momentum space.(Whole configuration space still unsettled?)
- We will here consider MS basis where we keep the rotation generators intact but modify the boost generators by adding a dilatation term.

Standard Lorentz generator: $L_{\alpha\beta} = P_\alpha \frac{\partial}{\partial P^\beta} - P_\beta \frac{\partial}{\partial P^\alpha}$
Deformed boost generator: $K^i := L_0^i + \frac{P^i}{\kappa} D$, where D is dilatation term given as $D = P_\alpha \frac{\partial}{\partial P^\alpha}$
Rotation generator: $J^i := \epsilon^{ijk} L_{ij}$

Expressions of Deformed algebra:

$$[J^i, K^j] = i\epsilon^{ijk} K_k; \tag{1}$$

$$[K^i, K^j] = -i\epsilon^{ijk} J_k; \tag{2}$$

$$[J^i, J^j] = i\epsilon^{ijk} J_k; \tag{3}$$

$$[K^i, P^j] = i\left(\delta^{ij} P^0 - \frac{P^i P^j}{\kappa}\right); \tag{4}$$

$$[K^i, P^0] = i\left(1 - \frac{P^0}{\kappa}\right) P^i \tag{5}$$

Dispersion Relation and cut-off: The quadratic Casimir of the above deformed algebra is $m^2 = \frac{P_0^2 - \vec{P}^2}{\left(1 - \frac{P_0}{\kappa}\right)^2}$ or $E^2 - P^2 = m^2 \left(1 - \frac{E}{\kappa}\right)^2$. For our purpose the energy of particle is always less than κ.

Defining the Problem (Based on D. K. Mishra, N. Chandra and V. Vaibhav, Annals of Physics 385 (2017) 605622) :

- We consider a grand canonical ensemble of photons obeying Bose-Einstein statistics.
- For a photon, the mass being zero, dispersion relation remains same as in the case of SR, i.e. $\epsilon = p$.
- The DSR effect for the equilibrium properties of blackbody radiation is basically due to the cut-off κ in energy.
- We can the calculate various thermodynamic quantities as energy density, pressure, specific heat etc.
- The thermodynamic quantities reduce to the usual one in the limit $\kappa \to \infty$

The energy density:

$$u \equiv \frac{U}{V_{ac}} = \int_0^\kappa \frac{1}{\pi^2} \frac{\omega^3 d\omega}{e^{\frac{\omega}{T}} - 1} = \frac{6T^4}{\pi^2}\left[Z_4(0) - Z_4\left(\frac{\kappa}{T}\right)\right]$$
$$= \frac{\pi^2 T^4}{15} - \left[\left(\frac{6T^4}{\pi^2}\right)Li_4(e^{-\frac{\kappa}{T}}) + \left(\frac{6\kappa T^3}{\pi^2}\right)Li_3(e^{-\frac{\kappa}{T}})\right.$$
$$\left. + \left(\frac{3\kappa^2 T^2}{\pi^2}\right)Li_2(e^{-\frac{\kappa}{T}}) - \left(\frac{\kappa^3 T}{\pi^2}\right)ln(1 - e^{-\frac{\kappa}{T}})\right] \quad (6)$$

- Here $Z_n(x)$ is the incomplete zeta function and $Li_n(z)$ is polylogarithm function.
- We have used $Z_n(z) = \sum_{k=0}^{n-1} Li_{n-k}(e^{-z})\frac{z^k}{k!}$.
- We get correction in Stefan-Bolzmann law.

The specific heat:

$$\begin{aligned} C_V &= \left(\frac{\partial U}{\partial T}\right)_{V_{ac}} \\ &= -\frac{\kappa^4 V_{ac}}{\pi^2 T}\frac{1}{(e^{\frac{\kappa}{T}}-1)} + \Bigg[\frac{4\pi^2 T^3 V_{ac}}{15} - (\frac{24 T^3 V_{ac}}{\pi^2}) Li_4(e^{-\frac{\kappa}{T}}) \\ &- (\frac{24\kappa T^2 V_{ac}}{\pi^2}) Li_3(e^{-\frac{\kappa}{T}}) - (\frac{12\kappa^2 T V_{ac}}{\pi^2}) Li_2(e^{-\frac{\kappa}{T}}) \\ &+ \frac{4 V_{ac}\kappa^3}{\pi^2} ln(1-e^{\frac{-\kappa}{T}})\Bigg] = -\frac{\kappa^4 V_{ac}}{\pi^2 T}\frac{1}{(e^{\frac{\kappa}{T}}-1)} + 4\left(\frac{U}{T}\right) \end{aligned} \quad (7)$$

- Here we have used $\frac{\partial}{\partial\mu}[Li_n(e^{\mu})] = Li_{n-1}(e^{\mu})$.
- This is same as the expression for acoustic phonons in Debye theory with appropriate replacements.

Thermodynamics of photons in DSR(unmodified measure)

The radiation pressure:

$$\left(\frac{PV_{ac}}{T}\right) = \frac{-V_{ac}}{\pi^2}\int_0^{\kappa} ln(1 - e^{-\frac{\epsilon}{T}})\epsilon^2 d\epsilon \tag{8}$$

So,

$$P = -\frac{T\kappa^3}{3\pi^2} ln(1 - e^{-\frac{\kappa}{T}}) + \frac{1}{3}u \tag{9}$$

- The equation of state gets modification.

The entropy:

The Helmholtz free energy is given by (the chemical potential $\mu = 0$),

$$A = \mu N - PV_{ac} = -PV_{ac} = \frac{T\kappa^3 V_{ac}}{3\pi^2} ln(1 - e^{-\frac{\kappa}{T}}) - \left(\frac{U}{3}\right). \quad (10)$$

The entropy therefore becomes,

$$S = \frac{U - A}{T} = -\frac{\kappa^3 V_{ac}}{3\pi^2} ln(1 - e^{-\frac{\kappa}{T}}) + \frac{4}{3}\left(\frac{U}{T}\right) \quad (11)$$

The equilibrium number of photons:

$$\bar{N} = \int_0^{\kappa} \frac{V_{ac}}{\pi^2} \frac{\omega^2 d\omega}{e^{\frac{\omega}{T}} - 1} = \frac{2V_{ac}T^3}{\pi^2} \left[Z_3(0) - Z_3\left(\frac{\kappa}{T}\right)\right] \quad (12)$$

Change in phase-space measure for exotic spacetimes

- For exotic spacetimes while taking the large volume limit we expect the phase space to modify.
- The limit is expected to be

$$\sum_{\epsilon} \to \frac{1}{(2\pi)^3} \int \int d^3x d^3p f(\vec{x}, \vec{p}) \tag{13}$$

- Assuming the spacetime to be isotropic and that f is taylor expandable in powers of $\left(\frac{1}{r\kappa}\right)$ and $\left(\frac{\epsilon}{\kappa}\right)$.
- κ acts as highest energy cut-off while $\frac{1}{\kappa}$ acts as the lowest length cut-off.

Change in phase-space measure for exotic spacetimes

The integral of the form $\frac{1}{(2\pi)^3}\int\int d^3x d^3p F(\varepsilon)$ changes to

$$\frac{1}{(2\pi)^3}\int\int d^3x d^3p f(r,p) F(\varepsilon)$$

$$= \frac{1}{(2\pi)^3}\int_{r=\frac{1}{\kappa}}^{R}\int_{p=0}^{\kappa} d^3x d^3p \sum_{n=0,n'=0}^{\infty} \frac{a_{n,n'}}{n!n'!}\left(\frac{\varepsilon}{\kappa}\right)^n \left(\frac{1}{r\kappa}\right)^{n'} F(\varepsilon)$$

$$= \frac{1}{(2\pi)^3}\sum_{n=0,n'=0}^{\infty} \frac{a_{n,n'}}{n!n'!\kappa^{n+n'}} \int_{r=\frac{1}{\kappa}}^{R}\int_{p=0}^{\kappa} d^3x d^3p\ \varepsilon^n \left(\frac{1}{r}\right)^{n'} F(\varepsilon)$$

Here we have interchanged the double summation and the integration which is allowed if,

$$\sum_{n=0,n'=0}^{\infty} \frac{|a_{n,n'}|}{n!n'!\kappa^{n+n'}} \int_{r=\frac{1}{\kappa}}^{R}\int_{p=0}^{\kappa} d^3x d^3p\ \varepsilon^n \left(\frac{1}{r}\right)^{n'} |F(\varepsilon)| < \infty. \quad (14)$$

Change in phase-space measure for exotic spacetimes

Finally we have

$$\frac{1}{(2\pi)^3}\int\int d^3x d^3p f(r,p)F(\varepsilon)$$

$$= \frac{1}{(2\pi)^3}\sum_{\substack{n=0,n'=0\\ n'\neq 3}}^{\infty}\frac{a_{n,n'}}{n!n'!\kappa^{n+3}}\frac{4\pi}{(3-n')}\left[\left(\frac{3V\kappa^3}{4\pi}\right)^{\frac{3-n'}{3}}-1\right]\int_{p=0}^{\kappa} d^3p\,(\varepsilon)^n F(\varepsilon$$

$$+\frac{1}{(2\pi)^3}\sum_{n=0}^{\infty}\frac{a_{n,3}}{n!3!\kappa^{n+3}}\left(\frac{4\pi}{3}\right)\ln\left(\frac{3V\kappa^3}{4\pi}\right)\int_{p=0}^{\kappa} d^3p\ (\varepsilon)^n F(\varepsilon)$$

- $V = \frac{4}{3}\pi R^3$ is the volume of the spherical ball of radius R.
- The accessible part of the volume for the particle is $V_{ac} = V - \frac{4\pi}{3\kappa^3}$.
- For the large volume limit the minimum length $\frac{1}{\kappa} << R$ implying $V\kappa^3 >> 1$ which in turn implies $V_{ac} \approx V$.

Modified Planck's energy distribution and Wein's Law:

$$u(\omega) = \sum_{n=0}^{\infty} A_n \frac{\omega^{n+3}}{e^{\frac{\omega}{T}} - 1} \tag{16}$$

where

$$A_n = \frac{1}{\pi^2} \sum_{n'=0, n' \neq 3}^{\infty} \frac{a_{n,n'}}{n! n'! \kappa^n} \left(\frac{4\pi}{3-n'}\right) \frac{1}{\kappa^3 V_{ac}} \left[\left(\frac{3V\kappa^3}{4\pi}\right)^{\frac{3-n'}{3}} - 1\right] + \frac{1}{\pi^2} \frac{a_{n,3}}{n! 3! \kappa^n} \left(\frac{4\pi}{3\kappa^3 V_{ac}}\right) \ln\left(\frac{3V\kappa^3}{4\pi}\right) \tag{17}$$

- This is a constant and is independent of both ω and T.
- We can express energy density distribution in terms of wavelength λ and using extremum condition $\left.\frac{du(\lambda)}{d\lambda}\right|_{\lambda_{max}} = 0$ we get $u(\lambda)$ maximum at $\lambda = \lambda_{max}$.

Thermodynamics of photons in DSR(modified measure)

Modified Planck's energy distribution and Wein's Law:
The extremum condition is

$$\sum_{n=0}^{\infty} T^n x_{max}^n A_n \left[n + 5 - \frac{x_{max}}{1 - e^{-x_{max}}} \right] = 0 \qquad (18)$$

- Here $x_{max} = \frac{2\pi}{\lambda_{max} T}$ is no more constant, but a function of T.
- Keeping the leading order terms in $\frac{T}{\kappa}$ and $\frac{1}{V^{1/3}\kappa}$ and neglect all the higher order terms we have

$$\frac{T}{\kappa} = -\frac{1}{x_{max} a_{1,0}} \left(\frac{5 - \frac{x_{max}}{1-e^{-x_{max}}}}{6 - \frac{x_{max}}{1-e^{-x_{max}}}} \right) = f(x_{max}) \qquad (19)$$

- We have $\lambda_{max} = \frac{2\pi}{T f^{-1}(\frac{T}{\kappa})}$.

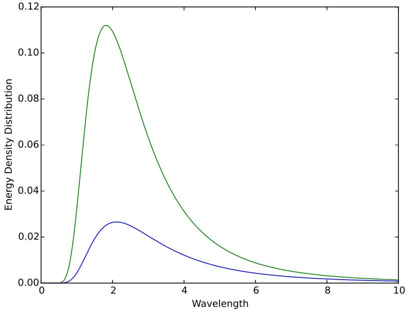


Figure: The plot showing the Planck energy distribution as a function of wavelength λ for both modified and unmodified measure. Here the blue and the green color correspond to the unmodified and the modified measure respectively. We have taken $a_{0,0} = 1.0$, $a_{0,1} = a_{1,0} = 0.2$ and all other a's are zero, temperature is $T = 0.6$, volume is 10^{35} and $\kappa = 1$ in Planck units.

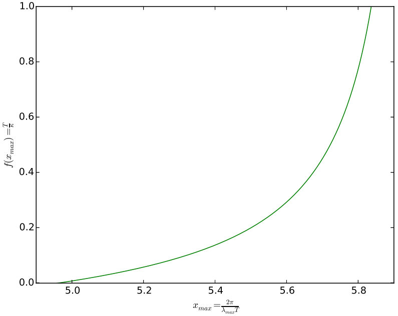


Figure: The plot showing the modification in the Wien's displacement law. The usual Wien's law would have given us a value of x_{max} corresponding to the point where $f(x_{max}) = 0$. We can clearly see from the plot that this value is $x_{max} \approx 4.965$. The coefficient $a_{1,0}$ has been taken as 1. We have chosen the range such that $\frac{T}{\kappa}$ lies between 0 and 1.

Energy density:

$$u = \frac{1}{(\pi)^2} \sum_{\substack{n=0, n'=0 \\ n' \neq 3}}^{\infty} \frac{a_{n,n'}}{n! n'! \kappa^n} \frac{4\pi}{(3-n')} \left(\frac{T^{n+4}}{V_{ac} \kappa^3} \right)$$

$$\left[\left(\frac{3 V \kappa^3}{4\pi} \right)^{\frac{3-n'}{3}} - 1 \right] \Gamma(n+4) \left[Z_{n+4}(0) - Z_{n+4}\left(\frac{\kappa}{T} \right) \right]$$

$$+ \frac{1}{(\pi)^2} \sum_{n=0}^{\infty} \frac{a_{n,3}}{n! 3! \kappa^n} \left(\frac{4\pi T^{n+4}}{3 V_{ac} \kappa^3} \right)$$

$$\ln \left(\frac{3\kappa^3 V}{4\pi} \right) \Gamma(n+4) \left[Z_{n+4}(0) - Z_{n+4}\left(\frac{\kappa}{T} \right) \right]$$

$$= \sum_{n=0, n'=0}^{\infty} u_{n,n'} \tag{20}$$

Thermodynamics of photons in DSR(modified measure)

Specific heat:

$$C_V = \left(\frac{\partial U}{\partial T}\right)_{V_{ac}} = \sum_{\substack{n=0,n'=0\\ n'\neq 3}}^{\infty}\left[\frac{1}{(\pi)^2}\frac{a_{n,n'}}{n!n'!\kappa^{n+3}}\frac{4\pi}{(3-n')}\right.$$

$$\left[\left(\frac{3V\kappa^3}{4\pi}\right)^{\frac{3-n'}{3}}-1\right]\left\{\frac{1}{(1-e^{\frac{\kappa}{T}})}\frac{\kappa^{n+4}}{T}\right\}+(n+4)\left.\left(\frac{u_{n,n'}V_{ac}}{T}\right)\right]$$

$$+\sum_{n=0}^{\infty}\left[\frac{1}{(\pi)^2}\frac{a_{n,3}}{n!3!\kappa^{n+3}}\left(\frac{4\pi}{3}\right)\ln\left(\frac{3\kappa^3 V}{4\pi}\right)\right.$$

$$\left\{\frac{1}{(1-e^{\frac{\kappa}{T}})}\frac{\kappa^{n+4}}{T}\right\}+(n+4)\left.\left(\frac{u_{n,3}V_{ac}}{T}\right)\right] \qquad (21)$$

Radiation pressure:

$$
\begin{aligned}
P = & \sum_{\substack{n=0,n'=0\\ n'\neq 3}}^{\infty}\left[\frac{1}{(\pi)^2}\frac{a_{n,n'}}{n!n'!\kappa^n}\frac{4\pi}{(3-n')}\left(\frac{T}{V_{ac}\kappa^3}\right)\right.\\
& \left[\left(\frac{3V\kappa^3}{4\pi}\right)^{\frac{3-n'}{3}}-1\right]\left\{-\ln(1-e^{-\frac{\kappa}{T}})\frac{\kappa^{n+3}}{n+3}\right\}+\left.\frac{u_{n,n'}}{(n+3)}\right]\\
& +\sum_{n=0}^{\infty}\left[\frac{1}{(\pi)^2}\frac{a_{n,3}}{n!3!\kappa^n}\left(\frac{4\pi T}{3V_{ac}\kappa^3}\right)\right.\\
& \ln\left(\frac{3\kappa^3 V}{4\pi}\right)\left\{-\ln(1-e^{-\frac{\kappa}{T}})\frac{\kappa^{n+3}}{n+3}\right\}+\left.\frac{u_{n,3}}{(n+3)}\right] \qquad (22)
\end{aligned}
$$

Helmholtz free energy:

$$
\begin{aligned}
A &= \mu N - PV_{ac} = -PV_{ac} \\
&= \sum_{\substack{n=0, n'=0 \\ n' \neq 3}}^{\infty} \Bigg[\frac{1}{(\pi)^2} \frac{a_{n,n'}}{n! n'! \kappa^{n+3}} \frac{4\pi T}{(3-n')} \\
&\left[\left(\frac{3V\kappa^3}{4\pi} \right)^{\frac{3-n'}{3}} - 1 \right] \left\{ \ln(1 - e^{-\frac{\kappa}{T}}) \frac{\kappa^{n+3}}{n+3} \right\} - \frac{u_{n,n'} V_{ac}}{(n+3)} \Bigg] \\
&+ \sum_{n=0}^{\infty} \frac{a_{n,3}}{n! 3! \kappa^{n+3}} \Bigg[\frac{1}{(\pi)^2} \left(\frac{4\pi T}{3} \right) \\
&\ln \left(\frac{3\kappa^3 V}{4\pi} \right) \left\{ \ln(1 - e^{-\frac{\kappa}{T}}) \frac{\kappa^{n+3}}{n+3} \right\} - \frac{u_{n,3} V_{ac}}{(n+3)} \Bigg] \qquad (23)
\end{aligned}
$$

Entropy:

And the entropy becomes,

$$S = \frac{U-A}{T} = \sum_{\substack{n=0,n'=0\\ n'\neq 3}}^{\infty} \left[\frac{1}{(\pi)^2} \frac{a_{n,n'}}{n!n'!\kappa^{n+3}} \frac{4\pi}{(3-n')} \right.$$
$$\left[\left(\frac{3V\kappa^3}{4\pi} \right)^{\frac{3-n'}{3}} - 1 \right] \left\{ -\ln(1-e^{-\frac{\kappa}{T}}) \frac{\kappa^{n+3}}{n+3} \right\} + \frac{(n+4)}{(n+3)} \left(\frac{u_{n,n'}V_{ac}}{T} \right) \Bigg]$$
$$+ \sum_{n=0}^{\infty} \left[\frac{1}{(\pi)^2} \frac{a_{n,3}}{n!3!\kappa^{n+3}} \left(\frac{4\pi}{3} \right) \right.$$
$$\left. \ln \left(\frac{3\kappa^3 V}{4\pi} \right) \left\{ -\ln(1-e^{-\frac{\kappa}{T}}) \frac{\kappa^{n+3}}{n+3} \right\} + \frac{(n+4)}{(n+3)} \left(\frac{u_{n,3}V_{ac}}{T} \right) \right] \tag{24}$$

Thermodynamics of photons in DSR(modified measure)

Equilibrium number of photons:

$$\bar{N} = \frac{1}{(\pi)^2} \sum_{\substack{n=0,n'=0 \\ n'\neq 3}}^{\infty} \frac{a_{n,n'}}{n!n'!k^{n+3}} \frac{4\pi T^{n+3}}{(3-n')}$$
$$\left[\left(\frac{3V\kappa^3}{4\pi}\right)^{\frac{3-n'}{3}} - 1\right] \Gamma(n+3)\left[Z_{n+3}(0) - Z_{n+3}\left(\frac{\kappa}{T}\right)\right]$$
$$+ \frac{1}{(\pi)^2}\sum_{n=0}^{\infty}\frac{a_{n,3}}{n!3!\kappa^{n+3}}\left(\frac{4\pi T^{n+3}}{3}\right)$$
$$\ln\left(\frac{3\kappa^3 V}{4\pi}\right)\Gamma(n+3)\left[Z_{n+3}(0) - Z_{n+3}\left(\frac{\kappa}{T}\right)\right] \qquad (25)$$

Low temperature limit(unmodified measure) $T \to 0$:

- We take $\frac{T}{\kappa} = \epsilon << 1$ and we note $Li_n(z) \to z$ as $z \to 0$ to get

$$u \approx \frac{\pi^2 \kappa^4 \epsilon^4}{15} - \frac{\kappa^2}{\pi^2} \epsilon e^{-\frac{1}{\epsilon}} \approx \frac{\pi^2 T^4}{15} \tag{26}$$

- We have neglected the second term with respect to the first putting $x = \frac{1}{\epsilon}$ and as $x \to \infty$ the ratio of second term to the first in the above equation goes to zero.
- Other quantities similarly are,

$$P \approx \frac{\pi^2 T^4}{45} \tag{27}$$

$$S \approx \frac{4 V_{ac} \pi^2 T^3}{45} \tag{28}$$

$$C_V \approx \frac{4 V_{ac} \pi^2 T^3}{15} \tag{29}$$

$$\bar{N} \approx \frac{2 V_{ac} \zeta(3) T^3}{\pi^2} \tag{30}$$

Low temperature limit(modified measure) $T \to 0$:

$$u \approx \frac{1}{(\pi)^2} \sum_{\substack{n=0, n'=0 \\ n' \neq 3}}^{\infty} \frac{a_{n,n'}}{n! n'! \kappa^n} \frac{4\pi}{(3-n')} \left(\frac{T^{n+4}}{V_{ac}\kappa^3} \right)$$

$$\left[\left(\frac{3V\kappa^3}{4\pi} \right)^{\frac{3-n'}{3}} - 1 \right] \Gamma(n+4) Z_{n+4}(0)$$

$$+ \frac{1}{(\pi)^2} \sum_{n=0}^{\infty} \frac{a_{n,3}}{n! 3! \kappa^n} \left(\frac{4\pi T^{n+4}}{3V_{ac}\kappa^3} \right) \ln \left(\frac{3\kappa^3 V}{4\pi} \right) \Gamma(n+4) Z_{n+4}(0) \quad (31)$$

Low temperature limit(modified measure) $T \to 0$:

$$P \approx \frac{1}{(\pi)^2} \sum_{\substack{n=0,n'=0 \\ n' \neq 3}}^{\infty} \frac{a_{n,n'}}{n!n'!\kappa^n} \frac{4\pi}{(3-n')} \left(\frac{T^{n+4}}{V_{ac}\kappa^3} \right)$$

$$\left[\left(\frac{3V\kappa^3}{4\pi} \right)^{\frac{3-n'}{3}} - 1 \right] \frac{\Gamma(n+4)}{(n+3)} Z_{n+4}(0)$$

$$+ \frac{1}{(\pi)^2} \sum_{n=0}^{\infty} \frac{a_{n,3}}{n!3!\kappa^n} \left(\frac{4\pi T^{n+4}}{3V_{ac}\kappa^3} \right) \ln \left(\frac{3\kappa^3 V}{4\pi} \right) \frac{\Gamma(n+4)}{(n+3)} Z_{n+4}(0) \tag{32}$$

Low temperature limit(modified measure) $T \to 0$:

$$S \approx \frac{1}{(\pi)^2} \sum_{\substack{n=0,n'=0\\ n'\neq 3}}^{\infty} \frac{a_{n,n'}}{n!n'!\kappa^n} \frac{4\pi}{(3-n')} \left(\frac{T^{n+3}}{\kappa^3} \right) \left[\left(\frac{3V\kappa^3}{4\pi} \right)^{\frac{3-n'}{3}} - 1 \right] \frac{\Gamma(n+4)(n+4)}{(n+3)} Z_{n+4}(0) + \frac{1}{(\pi)^2} \sum_{n=0}^{\infty} \frac{a_{n,3}}{n!3!\kappa^n} \left(\frac{4\pi T^{n+3}}{3\kappa^3} \right) \ln \left(\frac{3\kappa^3 V}{4\pi} \right) \frac{\Gamma(n+4)(n+4)}{(n+3)} Z_{n+4}(0) \quad (33)$$

Low temperature limit(modified measure) $T \to 0$:

$$C_V \approx \frac{1}{(\pi)^2} \sum_{\substack{n=0,n'=0 \\ n'\neq 3}}^{\infty} \frac{a_{n,n'}}{n!n'!\kappa^n} \frac{4\pi}{(3-n')} \left(\frac{T^{n+3}}{\kappa^3} \right)$$

$$\left[\left(\frac{3V\kappa^3}{4\pi} \right)^{\frac{3-n'}{3}} - 1 \right] \Gamma(n+4)(n+4)Z_{n+4}(0)$$

$$+ \frac{1}{(\pi)^2} \sum_{n=0}^{\infty} \frac{a_{n,3}}{n!3!\kappa^n} \left(\frac{4\pi T^{n+3}}{3\kappa^3} \right) \ln \left(\frac{3\kappa^3 V}{4\pi} \right) \Gamma(n+4)(n+4)Z_{n+4}(0) \tag{34}$$

Low temperature limit(modified measure) $T \to 0$:

$$\bar{N} \approx \frac{1}{(\pi)^2} \sum_{\substack{n=0,n'=0 \\ n'\neq 3}}^{\infty} \frac{a_{n,n'}}{n!n'!k^{n+3}} \frac{4\pi T^{n+3}}{(3-n')} \left[\left(\frac{3V\kappa^3}{4\pi} \right)^{\frac{3-n'}{3}} - 1 \right] \Gamma(n+3) Z_{n+3}(0)$$

$$+ \frac{1}{(\pi)^2} \sum_{n=0}^{\infty} \frac{a_{n,3}}{n!3!\kappa^{n+3}} \left(\frac{4\pi T^{n+3}}{3} \right) \ln \left(\frac{3\kappa^3 V}{4\pi} \right) \Gamma(n+3) Z_{n+3}(0) \tag{35}$$

- Energy density u and radiation pressure P follow $\sim T^4$ behaviour.
- The entropy S, the specific heat C_V and the equilibrium number of photons follow $\sim T^3$ behaviour.
- The similar behaviour is seen in case of modified measure.

High temperature limit(unmodified measure) $T \rightarrow \kappa$:

- $T \approx \kappa(1-\epsilon)$ such that $\epsilon << 1$ which gives $\frac{\kappa}{T} \approx 1+\epsilon$.
- Various thermodynamic quantities are,

$$u \approx -\frac{18\kappa^4}{\pi^2}\left[Z_4(0) - Z_4(1) - \frac{1}{18(e-1)}\right] + \frac{24\kappa^4}{\pi^2}\left(\frac{T}{\kappa}\right)\left[Z_4(0) - Z_4(1) - \frac{1}{24(e-1)}\right] \quad (36)$$

$$P \approx -\frac{6\kappa^4}{\pi^2}\left[Z_4(0) - Z_4(1)\right] + \frac{8\kappa^4}{\pi^2}\left(\frac{T}{\kappa}\right)\left[Z_4(0) - Z_4(1)\right] - \frac{\kappa^4}{3\pi^2}\left(\frac{T}{\kappa}\right)\ln\left(1-\frac{1}{e}\right) \quad (37)$$

$$S \approx -\frac{\kappa^3 V_{ac}}{3\pi^2}\ln(1-\frac{1}{e}) - \frac{16\kappa^3 V_{ac}}{\pi^2}\left[Z_4(0) - Z_4(1) - \frac{1}{48(e-1)}\right] + \frac{24\kappa^3 V_{ac}}{\pi^2}\left(\frac{T}{\kappa}\right)\left[Z_4(0) - Z_4(1) - \frac{1}{24(e-1)}\right] \quad (38)$$

High temperature limit(unmodified measure) $T \to \kappa$:

$$C_V \approx -\frac{48\kappa^3 V_{ac}}{\pi^2}\left[Z_4(0) - Z_4(1) - \frac{(3e-2)}{48(e-1)^2}\right] + \frac{72\kappa^3 V_{ac}}{\pi^2}\left(\frac{T}{\kappa}\right)\left[Z_4(0) - Z_4(1) - \frac{(4e-3)}{72(e-1)^2}\right] \quad (39)$$

- To get the above linear dependence of C_V on T by differentiating the high T behaviour of U we need to expand U upto T^2 order.

$$\bar{N} \approx -\frac{4\kappa^3 V_{ac}}{\pi^2}\left[Z_3(0) - Z_3(1) - \frac{1}{4(e-1)}\right] + \frac{6\kappa^3 V_{ac}}{\pi^2}\left(\frac{T}{\kappa}\right)\left[Z_3(0) - Z_3(1) - \frac{1}{6(e-1)}\right] \quad (40)$$

High temperature limit(modified measure) $T \to \kappa$:

The behaviour is linear and is very involved so presenting only the short expressions

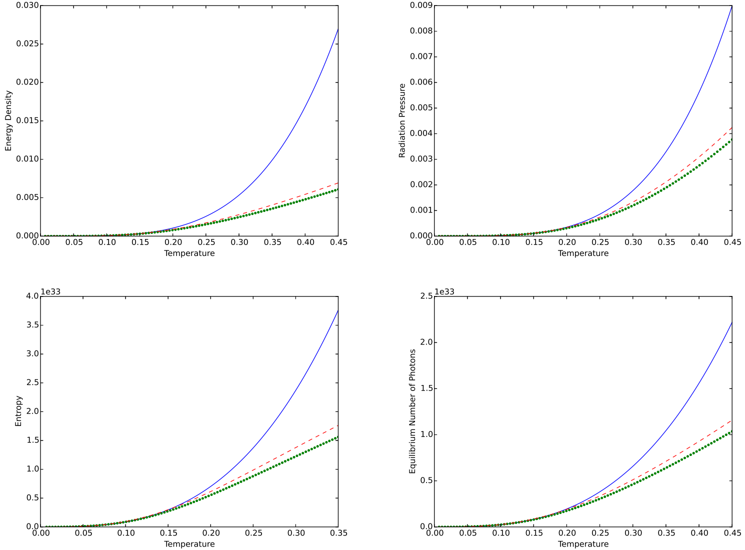
$$u \approx u_a + u_b T \tag{41}$$

$$P \approx P_a + P_b T \tag{42}$$

$$S \approx S_a + S_b T \tag{43}$$

$$C_v \approx C_{V\,a} + C_{V\,b} T \tag{44}$$

$$\bar{N} \approx \bar{N}_a + \bar{N}_b T \tag{45}$$



Energy Density
Temperature
Radiation Pressure
Temperature
Entropy
1e33
Temperature
Equilibrium Number of Photons
1e33
Temperature

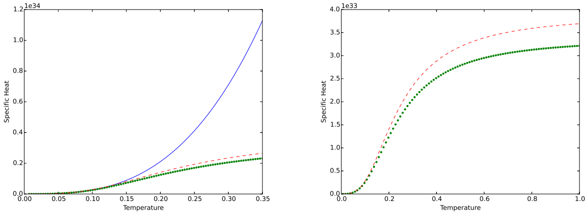


Figure: The plots show the variation of energy density, radiation pressure, entropy, specific heat and equilibrium number of photons with temperature for both unmodified and modified measure. The blue solid, the green dotted and the red dashed lines correspond to the SR, the DSR with unmodified measure and the DSR with modified measure respectively. As is visible from the plots it matches with SR at low T and with increasing T it deviates from SR significantly. Here $\kappa = 1$, $V = 10^{35}$ and $a_{0,0} = 1$, $a_{0,1} = a_{1,0} = 0.2$ and all other a's are taken to be zero. Note that all the quantities above become approximately linear near $T \to \kappa$. The specific heat however goes as T^3 for lower T. The behaviour of C_V for the full range of $\frac{T}{\kappa} \in [0, 1]$ is shown in the figure at bottom right corner, which certainly mimics the Debye theory. In the Debye theory however T may go upto infinity in which case the specific heat goes to a constant value.

Three scales of possible realization

- Can be effectively realized at three scales.
- Low energy realization is proposed by a table top experiment.
- Effective intermediate energy realization by studying compact stars such as white dwarfs and neutron stars.
- Possible high energy realization in case of the Big Bang and cosmology.

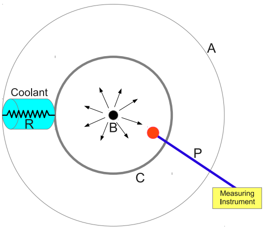
Three scales of possible realization

Low energy realization: A table top Experiment

- We have put an effective cut-off for the photons such that they behave as acoustic phonons. This cut-off may not be invariant.
- Consider a perfect blackbody B surrounded by a spherical cathode C which is enclosed by a spherical anode A and the circuit completed using a high resistance R.
- Suppose the cathode has the photoelectric threshold ν_{th} such that all the radiations above frequency ν_{th} gets absorbed by the cathode.
- Joule heating of the resistor is then cooled by an appropriate coolant.
- We obtain photons in the cathode cavity, with energy less than $\kappa = \nu_{th}$ which is the desired cut-off in the theory.
- These photons inside the cathode which mimic acoustic phonons.

Three scales of possible realization

Low energy realization: A table top Experiment



- Suppose we take at least 1% deviation from the usual photon thermodynamics at room temperature $T = 293$ K.
- The material of the cathode can be selected with threshold $\nu_{th} = \frac{k_B T}{0.1 h} = 6.1 \times 10^{13} Hz$.
- Many commercially available materials fall into this category.

Three scales of possible realization

High energy realization: Big Bang and Cosmology

- The modified energy density u and Pressure P gives us a modified energy-momentum tensor $T_{\mu\nu}$.
- Consider the radiation dominated epoch where we have the above calculated modified energy density and pressure.
- The standard energy conservation gets modified to

$$\frac{\dot{u}}{u} = -\left(4 - \frac{\frac{T\kappa^3}{\pi^2} ln(1 - e^{-\frac{\kappa}{T}})}{u}\right)\frac{\dot{a}}{a}. \tag{46}$$

- The Hubble parameter H, which characterizes the rate of expansion of the Universe gets modified to

$$H = \frac{\dot{a}}{a} = -\frac{\dot{T}}{T}\left[\frac{24\left[Z_4(0) - Z_4\left(\frac{\kappa}{T}\right)\right] - \left(\frac{\kappa}{T}\right)^4 Li_0\left(e^{\frac{-\kappa}{T}}\right)}{24\left[Z_4(0) - Z_4\left(\frac{\kappa}{T}\right)\right] + \left(\frac{\kappa}{T}\right)^3 Li_1\left(e^{\frac{-\kappa}{T}}\right)}\right] \tag{47}$$

Three scales of possible realization

High energy realization: Big Bang and Cosmology

- The numerator is always less than the denominator i.e, $\frac{H}{H_{SR}} < 1$ always, with $H_{SR} = -\frac{\dot{T}}{T}$.
- The expansion of the Universe was at a slower rate in the radiation dominated era than the rate of expansion without such modifications.
- Because of the slower expansion, all the epochs would eventually get delayed resulting in the modification in the age of the known Universe.
- In the SR limit the modified Hubble parameter becomes nearly equal to the normal SR one, as $\frac{\kappa}{T}$ is very large, so the correction terms go to zero as expected.

Three scales of possible realization

High energy realization: Big Bang and Cosmology

- Application in case of "bouncing" loop quantum cosmology theories (Ashtekar et al.)
- In short, we consider specific modifications to the spacetime geometry which effectively puts a bound on the curvature and in this way the Big Bang singularity can be avoided.
- For such "bouncing" models, we cannot use the perturbation technique at the curvature saturation, as the energy density of the cosmic fluid diverges.
- Here we have obtained the energy density of the cosmic fluid which saturates to the Planck energy which of course is finite.
- We can combine both the results obtained in this paper and the "bouncing" loop quantum cosmology to study the possible way out to avoid the Big Bang singularity.

Three scales of possible realization

Intermediate energy realization: White Dwarfs (based on the the work by D. K. Mishra and Nitin Chandra, arXiv:1803.06640 [gr-qc])

- We will consider a typical model of a white dwarf star with N free electrons and $\frac{N}{2}$ helium nuclei such that the mass of the star given as $M = N(m_e + 2m_p)$.
- The internal temperature of such star is $\sim 10^7$ K, not enough to hold the star against the huge gravitational pressure. But surprisingly they are stable
- Fowler and Chandrasekhar suggested that degeneracy (quantum nature of electrons) can be used to explain the stability of white dwarfs.
- The fermi energy is higher than average KE of the electrons and so the degeneracy condition holds.
- The gas of electrons can be approximated as a zero-temperature Fermi gas.
- The finite temperature will be discussed in Luminosity calculations.

Three scales of possible realization

Intermediate energy realization: White Dwarfs

- Consider a grand canonical ensemble of a degenerate Fermi gas composed of N relativistic electrons obeying Fermi-Dirac statistics.
- The degeneracy pressure is given as

$$P_{th} = \frac{1}{3\pi^2} \int_{m_0}^{E_F} dE \ \ [f(E)]^{3/2} \tag{48}$$

  where $f(E)$ is $f(E) = E^2 - m^2 \left(1 - \frac{E}{\kappa}\right)^2$
- Because of such a dispersion relation we have three different cases:
  1. $m_0 < \frac{\kappa}{2} \Leftrightarrow m < \kappa$
  2. $m_0 = \frac{\kappa}{2} \Leftrightarrow m = \kappa$
  3. $m_0 > \frac{\kappa}{2} \Leftrightarrow m > \kappa$

Intermediate energy realization: White Dwarfs
Case I: $m_0 < \frac{\kappa}{2} \Leftrightarrow m < \kappa$:

- The pressure for this case comes out to be

$$P_{th} = \frac{m'^4}{24\pi^2}\left(1 - \frac{m^2}{\kappa^2}\right)^{3/2}[x\sqrt{1+x^2}\left(2x^2-3\right)+3\ln[x+\sqrt{1+x^2}]] \tag{49}$$

  Here $x = A\frac{M^{1/3}}{R}$ and $A = \frac{1}{\sqrt{mm'}}\left(\frac{9\pi}{4m_e+8m_p}\right)^{1/3} = \frac{\left(1-\frac{m_e}{\kappa}\right)\left[1-\left(\frac{m_e}{\kappa-m_e}\right)^2\right]^{1/2}}{m_e}\left(\frac{9\pi}{4m_e+8m_p}\right)^{1/3}$ and $m' = \frac{m}{1-\frac{m^2}{\kappa^2}}$.

- The star is assumed to be spherical in shape, the change in the total thermodynamic energy of the star (dE_{th}) is $dE_{th} = P_{th}dV = 4\pi R^2 P_{th}dR$.
- The change in the gravitational energy is $dE_g = \frac{GM^2}{R^2}dR$.

Intermediate energy realization: White Dwarfs

Case I: $m_0 < \frac{\kappa}{2} \Leftrightarrow m < \kappa$**:**

- At equilibrium we have $dE_{th} = dE_g$ giving us $P_{th} = \frac{\alpha G M^2}{4\pi R^4}$
- Equating we get the modified mass energy relation.
- Taking the ultrarelativistic limit we get

$$P_{th} \simeq \frac{m'^4}{12\pi^2}\left(1 - \frac{m^2}{\kappa^2}\right)^{3/2}\left[A^4\frac{M^{4/3}}{R^4} - A^2\frac{M^{2/3}}{R^2}\right], \qquad (50)$$

and the relationship gives

$$R \simeq AM^{1/3}\left[1 - \left(\frac{M}{M_0}\right)^{2/3}\right]^{1/2} \qquad (51)$$

where

$$M_0 = \frac{(m'A)^6}{(3\pi\alpha G)^{3/2}}\left(1 - \frac{m^2}{\kappa^2}\right)^{9/4} \qquad (52)$$

which is the modified Chandrashekhar mass limit.

Intermediate energy realization: White Dwarfs

Case I: $m_0 < \frac{\kappa}{2} \Leftrightarrow m < \kappa$**:**

- The relation between SR and DSR case is,

$$M_0 = \frac{M_0^{SR}}{\left[1 - \frac{m^2}{\kappa^2}\right]^{\frac{3}{4}}} \tag{53}$$

Intermediate energy realization: White Dwarfs

Case I: $m_0 < \frac{\kappa}{2} \Leftrightarrow m < \kappa$**:**

The M-R Plot is:

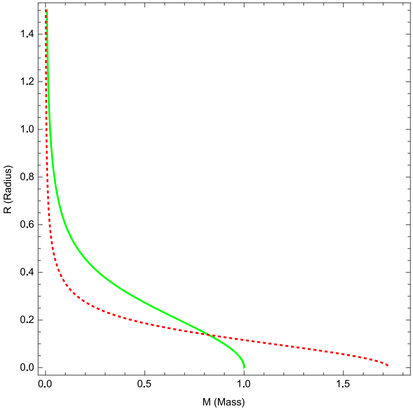


Figure: Figure showing the Mass-Radius relationship.

Intermediate energy realization: White Dwarfs

Case I: $m_0 < \frac{\kappa}{2} \Leftrightarrow m < \kappa$**:**

The Pressure Plot is:

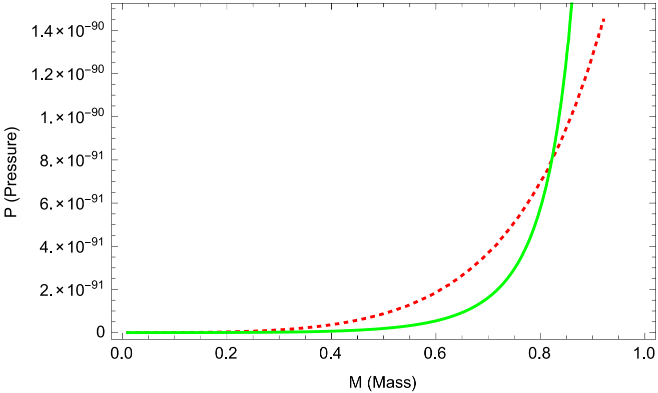


Figure: Figure showing the variation of degeneracy pressure (P) with radius (R).

Three scales of possible realization

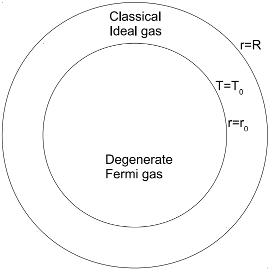
Intermediate energy realization: White Dwarfs
Case I: $m_0 < \frac{\kappa}{2} \Leftrightarrow m < \kappa$**:**

- This states that for a given mass we can observe the white dwarf stars with greater than and/or less than that given by the known SR theory.
- There has been some experimental observation where they have observed stars with smaller radius than that predicted by the SR theory for a given mass.
- Yet such stars are taken as anomalous cases but may be this work may pave way to explain such corrections.
- This preliminary calculation can be more refined by studying the modified Lane-Emden and structure equation.
- Other two cases are not that interesting as we do not obtain Chandrasekhar limit.

Intermediate energy realization: White Dwarfs

Luminosity

- So we assumed the fermions at 0 K. But white dwarfs have a finite temperature and they radiate as well. That is why the name "White Dwarf".
- Such a cooling can be explained by considering a model of star with a radiative envelope.



Three scales of possible realization

Intermediate energy realization: White Dwarfs Luminosity

- Model is that upto some radius $r = r_0$ the gas behaves as degenerate Fermi gas and above that it behaves as an nondegenerate Ideal gas.
- All the radiation is due to this envelope of nondegenerate gas.
- We will calculate the Ideal gas pressure on both sides of $r = r_0$ and equate at $T = T_0$ to get the expression of the Luminosity for both nonrelativistic and relativistic case.
- We find that the Luminosity gets a negative correction in both the cases.
- The correction can be observed more accurately in case of more compact stars like Neutron stars etc.

Summary & Future plan

- We recapitulated the previous obtained results.
- Suggested a table top experiment to possible realization of theory in low energy.
- Saw the effects of the theory near big bang and in cosmology.
- Saw possible modification in Chandrasekhar limit.
- Looked into the correction to the luminosity of the white dwarf.
- Similar correction can be studied in case of bosons and fermions.
- Understanding the above theory from the measurement theory point of view is another interesting regime.

Thank You All