Faux anomalies in QCD

with and without axions

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Abstract

While regularizations break the classical chiral symmetry which occurs for massless fermions, some symmetries which break naïve regularizations can be consistently regularized with care. These classical symmetries are therefore *not* anomalous, unlike the usual chiral symmetry; they only appear so. As example, we review the case of the Peccei-Quinn symmetry and introduce a new non-abelian generalization of axions and Peccei-Quinn symmetry. Pure QCD with a twisted mass term but without axions also has a parity symmetry which is only apparently anomalous.

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Anomaly

- Known for over half a century that the chiral symmetry which holds in classical Dirac theory with massless fermions interacting with gauge fields is broken upon quantization (Adler, Bell-Jackiw).
- The transformation

$$\psi \to e^{i\alpha\gamma_5}\psi, \quad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5},$$

  is a symmetry of the kinetic term $\bar{\psi}[i\partial\!\!\!/]\psi$ and also of the interaction term $\bar{\psi}[iA\!\!\!/]\psi$ with the gauge field A_μ.
- But the axial current $\bar{\psi}\gamma_\mu\gamma_5\psi$, which appears to be conserved from the equations of motion is found to violate this conservation when the fermion triangle diagram is carefully regularized and evaluated. Called the chiral anomaly.
- In the early literature, only the one-loop diagram was consid-

ered, but subsequently it became clear that higher order loops do not cause any further damage. Instead of considering individual diagrams it is more natural to consider the action and regularize it by one of the many available methods.

- A lattice regularization is an example. Here the spacetime itself is temporarily imagined to be discrete and fields appropriately defined on the lattice sites or links.
- The discrete Lagrangian density takes the form $\bar{\psi}[\frac{1}{2}(\not{D}^L + \not{D}^{L*}) - aD_\mu^{L*} D_\mu^L]\psi$. Here a is the lattice spacing, D^L is the covariant forward difference, divided by a and D^{L*} the covariant backward difference, again divided by a. While the kinetic term is chirally invariant as in the continuum, there is the double derivative term which breaks this invariance on the lattice (Wilson).

  The anomaly can be calculated from this lack of invariance.
- Another example is the regularization introduced by Pauli and

Villars, where some fictitious species are temporarily introduced. In both cases, calculations are done with the temporary regularized action, after which the realistic limit is taken. This means making the lattice spacing vanish, or the masses of the fictitious species go to infinity.

- The simple Pauli-Villars regularized Lagrangian density is

$$\bar{\psi}[i\partial\!\!\!/]\psi + \bar{\chi}[i\partial\!\!\!/ - M]\chi,$$

where χ is a fictitious field which takes the form of a spinor like ψ but has to be assigned Bose statistics if loop diagrams are to be regularized by this combination. The mass M of χ is taken to infinity at the end of calculations.

The chiral symmetry of the ψ part is broken only when the mass $m \neq 0$. However, the χ part has to break chiral symmetry because M, which has to be taken to infinity at the end, cannot be taken to vanish.

Thus the simple Pauli-Villars regularization already shows that chiral symmetry does not survive regularization and has an anomaly which can be calculated by taking the $M \to \infty$ limit of the divergence of the full axial current.

- It is found to be finite and proportional to $\mathrm{tr} F\tilde{F}$.
- In general, a classical symmetry may or may not survive quantization. The simple phase symmetry, whereby ψ is multiplied by a phase factor, does survive quantization for all values of m. To see whether a symmetry survives quantization, the action has to be regularized. If the regularized action still has the symmetry, the symmetry has no anomaly. If the regularized action does not possess the symmetry, one tends to think that the symmetry has an anomaly, but it is necessary to be careful. There are different ways of regularizing fermion field theories and one must check whether a different regularization can preserve the symmetry.

- We shall review the example of the Peccei-Quinn symmetry which occurs in the presence of a hypothetical field called the axion and was introduced by these authors in an attempt to solve the strong CP problem. It appeared to be anomalous at that time because it was thought to violate regularizations. However, it has recently been shown to survive quantization.
- After that, a nonabelian analogue of Peccei-Quinn theory is introduced. The new symmetry too is not anomalous.
- After that we go back to axionless chromodynamics and discuss strong parity in the presence of a twisted mass term.
- Finally there are discussions involving lattices and the measure approach to anomalies.
- Before that, let us look at what has been called the strong CP problem.

Strong CP

- Recall that a Dirac field can be decomposed into a left part and a right part: $\psi = \psi_L + \psi_R$.
- Weak interactions involve one handedness and break parity
- Vector interactions involve both and respect parity
- What about the strong and electromagnetic interactions? They are rather similar in the sense that both are vector gauge theories, the gauge symmetry being abelian in the electrodynamic case and non-abelian in the strong interactions. So again there is no violation of parity or time-reversal to begin with.
- Topological term $F\tilde{F}$ involving gluons and "vacuum angle" θ, if present, violates P,T:
  The Lagrangian of the standard model for the strong interactions, viz., quantum chromodynamics, theoretically allows an

adjustable

$$\theta F^{\mu\nu}\tilde{F}_{\mu\nu} \sim \theta\vec{E}\cdot\vec{B}$$

term involving the gluon fields. This violates both parity ($\vec{B}$ even, $\vec{E}$ odd) and time reversal ($\vec{E}$ even, $\vec{B}$ odd).

- Worse, the quark mass terms, which are related to time-reversal violations in the weak interactions through phase factors, can be an unknown complex mixture of scalar and pseudoscalar structures,

$$\exp[i\gamma_5\theta']$$

- Quark mass term has this chiral phase θ' from symmetry breaking in electroweak sector
  May be large ≈ 1, may violate P and T
- $\bar{\psi}_L m e^{i\theta'}\psi_R + hc = \bar{\psi}m e^{i\theta'\gamma_5}\psi = \cos\theta'\bar{\psi}m\psi + i\sin\theta'\bar{\psi}m\gamma_5\psi.$
  Looks like scalar $(\bar{\psi}\psi)$ + pseudoscalar $(\bar{\psi}\gamma^5\psi)$
  $\Rightarrow$ suggests parity violation

- Phase factor $e^{i\theta'} \to e^{-i\theta'}$ under antilinear operation $\Rightarrow$ suggests time-reversal violation
- Chiral transformation believed to combine θ, θ'
- May define an effective parameter $\bar{\theta} \equiv \theta - \theta'$
- But there is no experimental evidence of such violations. A possible signature would be an electric dipole moment of the neutron, which is a bound state of quarks formed by the strong interactions. An electric dipole moment satisfies an equation like

$$\vec{d} = \epsilon \vec{s},$$

  violating both parity and time-reversal because $\vec{d}$ is odd under parity and even under time- reversal, whereas the spin $\vec{s}$ is just the other way round.
- No such moment has been experimentally observed.
- Electric dipole moment of neutron $< 10^{-26}$ e-cm

- One could imagine the gluon effect parametrized by θ to cancel effects of the unknown phase θ' in the mass terms.
  One asks: **Is $\theta = \theta'$?**
- But this would involve something like a conspiracy between the strong interactions and the mass terms and is considered unnatural.
- Increasing a symmetry of an action by a choice of a parameter is **natural**: 't Hooft's criterion of naturalness
- Making $\theta = \theta'$ **unnatural**:
  symmetry of effective action increases, not of classical action
- Why then no experimental observation?
  This was the **Strong CP Problem**
- Banerjee, Chatterjee and I showed over 15 years back that even in the standard theory, the θ'-term does *not* cause parity or time-reversal violation.

- The combination of a scalar and a pseudoscalar involving θ' does not really lead to P or T violation in classical field theory: they just get redefined.
- Although it may appear that the θ'-term violates parity and time-reversal classically, a classically conserved parity and a classically conserved time-reversal can be defined: violations can then occur only if this parity or this time-reversal has an *anomaly*, *i.e.*, a quantum breaking of the classical symmetry. However, this parity symmetry is not afflicted with any anomaly and is conserved in the quantum theory.
- The result holds in quantum field theory *as the regularization or measure of fermion integration can be adjusted to take this redefinition into account.*
- Remaining CP violation can be removed by $\theta = 0$.
- The gluon θ-term does violate both symmetries, but this coef-

ficient can be set equal to zero without invoking any magical cancellation with a non-trivial term.

This is **natural**.

However, let us go on to axions.

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Chiral symmetry is explicitly broken by the mass term $m\bar{\psi}\psi$ and also by quantum effects, *i.e.* the anomaly. However, an artificial chiral symmetry for massive fermions works by letting a new field φ absorb the chiral transformation. The mass term is replaced by

$$\bar{\psi} m e^{i\varphi\gamma_5}\psi,$$

which is invariant if the field φ transforms under

$$\psi \to e^{i\alpha\gamma_5}\psi, \quad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}, \quad \varphi \to \varphi - 2\alpha.$$

This transformation leaves the action invariant provided the new field φ is massless. This is the Peccei-Quinn symmetry. The particle corresponding to the new field φ introduced by them is called the axion, but it has not been seen in any experiment.

To see whether this symmetry survives quantization, we may follow Pauli and Villars. The Lagrangian density

$$\bar{\psi}[i\partial\!\!\!/ - m e^{i\varphi\gamma_5}]\psi + \bar{\chi}[i\partial\!\!\!/ - M]\chi$$

is invariant. But the fictitious field χ has to be treated in the same way as ψ. If χ is transformed as

$$\chi \to e^{i\alpha\gamma_5/2}\chi, \quad \bar{\chi} \to \bar{\chi}e^{i\alpha\gamma_5/2},$$

the second term does not stay invariant. That is why it was believed that the Peccei-Quinn symmetry does not survive quantization.

However, the basic idea of the Pauli-Villars regularization is that the diagrams must be duplicated with the physical particle replaced by the fictitious one, so that the regulator field must behave in the same way as the physical field. Thus the field χ should be minimally coupled to the gauge field A_μ. This yields

$$\bar{\psi}[i\not{D} - me^{i\varphi\gamma_5}]\psi + \bar{\chi}[i\not{D} - M]\chi.$$

This coupling makes the Lagrangian density gauge invariant under identical gauge transformations of ψ and χ. But the interactions with φ are still different. To rectify this, one has to introduce an axion interaction with the regulator in the same way as in the Peccei-Quinn term:

$$\bar{\psi}[i\not{D} - me^{i\varphi\gamma_5}]\psi + \bar{\chi}[i\not{D} - Me^{i\varphi\gamma_5}]\chi.$$

With this correction, the action, including the kinetic term of the axion, is invariant under the Peccei-Quinn transformation extended to χ:

$$\psi \to e^{i\alpha\gamma_5}\psi, \quad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}$$

$$\chi \to e^{i\alpha\gamma_5}\chi, \quad \bar{\chi} \to \bar{\chi}e^{i\alpha\gamma_5}$$

$$\varphi \to \varphi - 2\alpha.$$

Thus this Pauli-Villars regularization respects the Peccei-Quinn symmetry, which accordingly is not anomalous but survives quantization.

The conserved Noether current for this symmetry is

$$\bar{\psi}\gamma_\mu\gamma_5\psi + \bar{\chi}\gamma_\mu\gamma_5\chi + 2F^2\partial_\mu\varphi,$$

where F is a constant of mass dimension such that the axion kinetic term is $\frac{1}{2}F^2\partial_\mu\varphi\partial^\mu\varphi$.

For simplicity, we have considered so far the simple version of Pauli-Villars regularization with only one regulator field χ. But this is not sufficient to remove all divergences. There is a generalized Pauli-Villars regularization

$$\bar{\psi}[i\not{D} - m]\psi + \sum_j \sum_{k=1}^{|c_j|} \bar{\chi}_{jk}[i\not{D} - M_j]\chi_{jk}.$$

Here c_j are integers whose signs are related to the statistics assigned to the spinor fields χ_{jk}. They are positive for Fermi statistics and negative for Bose statistics. They have to satisfy some conditions to ensure regularization of the divergences:

$$1+\sum_j c_j = 0, \quad m^2+\sum_j c_j M_j^2 = 0.$$

The consistency with the symmetry of the unregularized action, which has been achieved above by coupling the axion to a single χ, is to be attained by applying the same coupling to all the fields χ_{jk} here:

$$\bar\psi[i\not{D} - me^{i\varphi\gamma_5}]\psi + \sum_j \sum_{k=1}^{|c_j|} \bar\chi_{jk}[i\not{D} - M_j e^{i\varphi\gamma_5}]\chi_{jk}.$$

The axial symmetry can be made local by introducing an extra gauge field B_μ for this purpose:

$$\bar\psi[i\not{D} + \not{B}\gamma_5 - me^{i\varphi\gamma_5}]\psi + \frac{1}{2}F^2(\partial_\mu\varphi + 2B_\mu)(\partial^\mu\varphi + 2B^\mu),$$

with additional kinetic terms of the gauge fields. Regularization is as before.

We have considered an ordinary abelian or nonabelian gauge field above. The spacetime too can be taken to be curved. Then

$$\not{D} = \gamma^l e_l^\mu (\partial_\mu - iA_\mu - \frac{i}{2} A_\mu^{mn} \sigma_{mn})$$

involves a tetrad e_l^μ and a spin connection A_μ^{mn} in addition to the gauge field A_μ, but nevertheless it continues to anticommute with γ_5. Hence the symmetry survives quantization even in curved spacetime.

The usual chiral symmetry is under a transformation of the fermion in spinor space. If the fermion is an $SU(N)$ multiplet, there exist nonabelian chiral symmetries. The kinetic piece

$$\bar{\psi} i \partial\!\!\!/ \psi = \bar{\psi}_L i \partial\!\!\!/ \psi_L + \bar{\psi}_R i \partial\!\!\!/ \psi_R$$

is invariant under the chiral transformations

$$\psi_L \rightarrow U_L \psi_L, \quad \psi_R \rightarrow U_L \psi_R,$$

where U_L, U_R are spacetime independent $SU(N)$ matrices acting on the two chiral projections of ψ. The gauge interactions will also be invariant under these provided a different factor group is gauged and the matrix A_μ commutes with U_L, U_R. For instance, the $SU(N)$ could be a flavour group and the colour $SU(3)$ or the $U(1)$ could be gauged.

The usual mass term $m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$ is not invariant unless $U_L = U_R$, in which case of course the transformation is a *vector* transformation. The analogue of the Peccei-Quinn mass term is $m(\bar{\psi}_L W \psi_R + \bar{\psi}_R W^\dagger \psi_L)$. Here W has to be a hypothetical $SU(N)$ matrix field similar to the axion. Considering that the original axion has not been detected, we certainly do not intend to suggest that such an object should exist, but the mathematical construction may be useful for calculations because of the symmetry. This term is invariant under the joint transformation if W transforms as

$$W \to U_L W U_R^\dagger.$$

The kinetic term for this matrix field has to be of the form

$$Tr[\partial_\mu W \partial^\mu W^\dagger].$$

This is invariant when W changes as above. Thus the full action is invariant under the generalized Peccei-Quinn symmetry

$$\psi_L \to U_L \psi_L, \quad \psi_R \to U_L \psi_R, \quad W \to U_L W U_R^\dagger.$$

The question we have to ask is whether this nonabelian chiral symmetry survives quantization. The naïve way of regularization would be to introduce a Pauli-Villars $SU(N)$ multiplet with the usual kinetic term and a mass-like term $M(\bar{\chi}_L \chi_R + \bar{\chi}_R \chi_L)$. This term will not be invariant under chiral transformations of χ. But as noted above, Pauli-Villars requires a coupling of χ to W: $M(\bar{\chi}_L W \chi_R + \bar{\chi}_R W^\dagger \chi_L)$. Adding the interaction with the W field ensures invariance of the regularized action. For the full Pauli-Villars regularization one has

$$\bar{\psi}[i\not{D} - m\bar{W}]\psi + \sum_j \sum_{k=1}^{|c_j|} \bar{\chi}_{jk}[i\not{D} - M_j \bar{W}]\chi_{jk},$$

where P_L, P_R are the projection operators for left and right chirality respectively and

$$\bar{W} = W P_R + W^\dagger P_L.$$

This is invariant under the transformations

$$\psi_L \to U_L\psi_L, \psi_R \to U_L\psi_R, \chi_L \to U_L\chi_L, \chi_R \to U_L\chi_R, W \to U_LWU_R^\dagger.$$

Hence like the Peccei-Quinn symmetry, this classical symmetry too survives quantization and is not anomalous.

Again, gauge fields can be used to extend the global chiral symmetries to local ones. For example, for the left handed chiral symmetry, one needs an $SU(N)$ gauge field matrix B_μ:

$$\bar\psi[i\not{D} + \not{B}P_L - m\bar{W}]\psi + \frac{1}{2}F^2Tr[(\partial_\mu - iB_\mu)W(\partial^\mu W^\dagger + iW^\dagger B^\mu)].$$

Gauge field kinetic terms have to be added.

As in the abelian case, the generalization to curved spacetime can be carried out here too using the Dirac operator of the earlier section.

Now the axion is only a hypothetical field which was introduced for some historical reasons. Let us consider the situation of strong CP without axions. First we consider a single flavour. The Lagrangian density

$$\bar{\psi}[i\not{D} - me^{i\theta\gamma_5}]\psi$$

represents QCD with a twisted mass term as may arise from spontaneous symmetry breaking and the Higgs mechanism. Note that this action may also be reached from the theory including the axion by setting the field φ equal to the constant θ.

This action no longer has the Peccei-Quinn symmetry. It is usually believed that this action breaks parity. This is because the mass term is not invariant under the usual parity transformation

$$\psi(\vec{x}) \to \gamma^0\psi(-\vec{x}), \quad \bar{\psi}(\vec{x}) \to \bar{\psi}(-\vec{x})\gamma^0.$$

In fact this term is believed to cause strong CP violation and has been at the origin of the so-called strong CP problem.

However, as was pointed out earlier, the action is invariant under a twisted parity transformation

$$\psi(\vec{x}) \to \gamma^0 e^{i\theta\gamma_5}\psi(-\vec{x}), \quad \bar{\psi}(\vec{x}) \to \bar{\psi}(-\vec{x})\gamma^0 e^{-i\theta\gamma_5}.$$

This is equally admissible as a parity transformation and is an actual symmetry of the action. The main reason why it looks unfamiliar is that few people are used to twisted mass terms. There is also the fact that this transformation involves γ_5 and may be suspected to be anomalous. To see whether that is the case, one has to consider regularizing the action. The naïve regularization

$$\bar{\psi}[i\not{D} - me^{i\theta\gamma_5}]\psi + \bar{\chi}[i\not{D} - M]\chi$$

seems to respect it if χ is transformed under parity the usual way, but χ has to be treated exactly like ψ and thus has the twisted parity transformation above:

$$\psi(\vec{x}) \to \gamma^0 e^{i\theta\gamma_5}\psi(-\vec{x}), \quad \bar{\psi}(\vec{x}) \to \bar{\psi}(-\vec{x})\gamma^0 e^{-i\theta\gamma_5}$$

$$\chi(\vec{x}) \to \gamma^0 e^{i\theta\gamma_5}\chi(-\vec{x}), \quad \bar{\chi}(\vec{x}) \to \bar{\chi}(-\vec{x})\gamma^0 e^{-i\theta\gamma_5}.$$

This symmetry is respected if the mass term of χ is twisted:

$$\bar{\psi}[i\not{D} - me^{i\theta\gamma_5}]\psi + \bar{\chi}[i\not{D} - Me^{i\theta\gamma_5}]\chi$$

For the full Pauli-Villars regularization,

$$\bar{\psi}[i\not{D} - me^{i\theta\gamma_5}]\psi + \sum_j \sum_{k=1}^{|c_j|} \bar{\chi}_{jk}[i\not{D} - M_j e^{i\theta\gamma_5}]\chi_{jk}.$$

Such a phase with a γ_5 is consistent with Pauli-Villars theory. Thus there does exist a regularization which respects the unfamiliar parity symmetry of the action with twisted mass term.

In other words, this parity is not anomalous and is a genuine symmetry of the theory. The theory conserves P and CP. This was pointed out over a decade back, but is still not quite commonly understood. Of course, a tr$F\tilde{F}$ term in the action violates CP, but it is *natural* to set its coefficient equal to zero. This is apparently not as dramatic as imagining a new particle to force it to vanish.

Like the action we started with, this regularized action too can be derived from the regularized action with the axion by replacing the field φ by the constant θ.

In QCD with $SU(N)$ flavour, the fields W can be replaced by a constant $SU(N)$ matrix w. In this case, the mass term takes the form $m(\bar{\psi}_L w \psi_R + \bar{\psi}_R w^\dagger \psi_L)$ which is not invariant under the generalized Peccei-Quinn symmetry, but is invariant under a twisted parity

$$\psi_L(\vec{x}) \to w\gamma_0\psi_R(-\vec{x}), \quad \psi_R(\vec{x}) \to w^\dagger\gamma_0\psi_L(-\vec{x}),$$

$$\bar{\psi}_L(\vec{x}) \to \bar{\psi}_R(-\vec{x})\gamma_0 w^\dagger, \quad \bar{\psi}_R(\vec{x}) \to \bar{\psi}_L(-\vec{x})\gamma_0 w.$$

This twisted parity is maintained by putting w in all mass terms, *i.e.*,

$$\bar{\psi}[i\not{D} - m(wP_R + w^\dagger P_L)]\psi + \sum_j \sum_{k=1}^{|c_j|} \bar{\chi}_{jk}[i\not{D} - M_j(wP_R + w^\dagger P_L)]\chi_{jk}$$

and is therefore not anomalous.

Although the lattice regularization has been mentioned in the introduction, it has not been indicated how this preserves symmetries in the examples we study. We proceed to present the details. In the axion case, the axion coupling has to be introduced at two places in the action because there are two terms which are chirally non-invariant:

$$\bar{\psi}[\frac{1}{2}(\not{D}^L + \not{D}^{L*}) - ae^{i\varphi\gamma_5/2}(D_\mu^{L*}D_\mu^L)e^{i\varphi\gamma_5/2} - me^{i\varphi\gamma_5}]\psi.$$

In the $SU(N)$ case, one needs

$$\bar{\psi}[\frac{1}{2}(\not{D}^L + \not{D}^{L*}) - a(D_\mu^{L*}D_\mu^L)WP_R - aW^\dagger P_L(D_\mu^{L*}D_\mu^L) - m\bar{W}]\psi.$$

In the case of QCD, the field φ gets replaced by the constant θ:

$$\bar{\psi}[\frac{1}{2}(\not{D}^L + \not{D}^{L*}) - ae^{i\theta\gamma_5}(D_\mu^{L*}D_\mu^L) - me^{i\theta\gamma_5}]\psi.$$

In the $SU(N)$ flavour QCD case, one has

$$\bar{\psi}[\frac{1}{2}(\not{D}^L + \not{D}^{L*}) - a(D_\mu^{L*}D_\mu^L)wP_R - aw^\dagger P_L(D_\mu^{L*}D_\mu^L) - m\bar{w}]\psi,$$

where

$$\bar{w} = wP_R + w^\dagger P_L.$$

All these lattice actions preserve the symmetry of the continuum action.

Anomalies arise because regularizations may break some symmetries of classical actions. In the functional integral approach, Fujikawa suggested that the action has a symmetry which is broken by the measure. It may be interesting to look at our three examples and the respective measures.

To formulate the fermion measure, it is customary to expand the fermion field in eigenfunctions of some operator. To maintain gauge invariance, the covariant Dirac operator is considered. The eigenvalue equation is

$$i\not{D} f_n = \lambda_n f_n,$$

where the subscript labels the eigenvalue and the eigenfunction. Under a gauge transformation,

$$D_\mu \to U D_\mu U^{-1}, \quad \psi \to U\psi, \quad \bar{\psi} \to \bar{\psi} U^{-1},$$

so that

$$f \to Uf.$$

The field is expanded as

$$\psi = \sum_n a_n f_n, \quad \bar{\psi} = \sum_n \bar{a}_n f_n^\dagger.$$

Each $a, \bar{a}$ is gauge invariant because ψ and f transform the same way under gauge transformations and $\bar{\psi}$ and $f^\dagger$ also transform like each other. The gauge invariant measure $\prod_n da_n d\bar{a}_n$ is used for the fermion integration. It is well known that the measure is not chirally invariant: chiral transformations alter $a, \bar{a}$ and the change of the measure is a Jacobian which can be evaluated after some regularization and yields the chiral anomaly. One needs measures for other fields too, but these do not break symmetries.

Given this situation, it would appear that the Peccei-Quinn transformation would also alter the measure. The above measure would certainly be altered, but remembering that the requirement of gauge invariance led to the use of a fermion measure involving the eigen-functions of the Dirac operator which contains the gauge field, we can invoke the axion field now. Consider the new expansion

$$\psi = e^{-i\varphi\gamma_5/2}\sum_n b_n f_n, \quad \bar{\psi} = \sum_n \bar{b}_n f_n^\dagger e^{-i\varphi\gamma_5/2}.$$

Although the fermion field changes under the Peccei-Quinn transformation, the exponential factor too changes and cancels it, leaving $b, \bar{b}$ invariant. Hence the measure $\prod_n db_n d\bar{b}_n$ is invariant under the transformation. In other words, although the naïve fermion measure is altered by the Peccei-Quinn transformation, there does exist a fermion measure which is left invariant. This is very similar to what happens with regularizations. It may be added that the measure for φ is translation invariant.

In the case of QCD without axions, the field φ is replaced by θ, so that the measure $\prod_n db_n d\bar{b}_n$ with

$$\psi = e^{-i\theta\gamma_5/2}\sum_n b_n f_n, \quad \bar{\psi} = \sum_n \bar{b}_n f_n^\dagger e^{-i\theta\gamma_5/2}$$

comes into consideration. Of course this is not chirally invariant, but the question is whether it is invariant under the parity of the action with the twisted mass term. Now the parity transformation of f follows from the eigenvalue equation:

$$f(\vec{x}) \to \gamma^0 f(-\vec{x}).$$

It follows that

$$e^{-i\theta\gamma_5/2} f(\vec{x}) \to \gamma^0 e^{i\theta\gamma_5/2} f(-\vec{x}) = [\gamma^0 e^{i\theta\gamma_5}] e^{-i\theta\gamma_5/2} f(-\vec{x}).$$

Thus the combination $e^{-i\theta\gamma_5/2} f$ transforms exactly the same way as ψ under parity. Consequently, the variables b are parity invariant.

Similarly $\bar{b}$ is parity invariant because

$$f^\dagger(\vec{x})e^{-i\theta\gamma_5/2} \to f^\dagger(-\vec{x})\gamma^0 e^{-i\theta\gamma_5/2} = f^\dagger(-\vec{x})e^{-i\theta\gamma_5/2}[\gamma^0 e^{-i\theta\gamma_5}]$$

and accordingly the measure too is. Thus the redefined measure including the phase preserves the new parity symmetry. This is exactly as in the case with axions.

For the $SU(N)$ version of axions, the construction of the measure is more complicated. First, note that eigenvalues and eigenfunctions of $i\not{D}$ come in pairs:

$$i\not{D} f_n = \lambda_n f_n, \quad i\not{D}\gamma_5 f_n = -\lambda_n \gamma_5 f_n.$$

Hence it is possible to consider expansions in f_{nL}, f_{nR}, which are chiral combinations of the $f_n, \gamma_5 f_n$, though they are not eigenfunctions of $i\not{D}$. We expand

$$\psi_L = \sum_n a_n^L f_{nL}, \quad \psi_R = \sum_n a_n^R f_{nR},$$

$$\bar{\psi}_L = \sum_n \bar{a}_n^L f_{nL}^\dagger, \quad \bar{\psi}_R = \sum_n \bar{a}_n^R f_{nR}^\dagger.$$

Of course, the range of n is implicitly altered here. The measure $\prod_n da_n^L da_n^R d\bar{a}_n^L d\bar{a}_n^R$ is not invariant under an $SU(N)$ chiral transformation because $a, \bar{a}$ have to change unless $U_L = U_R$, in which case the common *vector* transformation may be absorbed in f.

However, a new measure can be constructed using the generalized axion field. Consider the expansions

$$W^\dagger \psi_L = \sum_n b_n^L f_{nL}, \quad \psi_R = \sum_n b_n^R f_{nR},$$

$$\bar{\psi}_L W = \sum_n \bar{b}_n^L f_{nL}^\dagger, \quad \bar{\psi}_R = \sum_n \bar{b}_n^R f_{nR}^\dagger.$$

AS W is invertible, it may also be taken to the right if desired. This construction is not unique, but serves the purpose. Note the asymmetric use of W here. Because of this asymmetry, the left hand sides of both equations in the first line acquire U_R and the left hand sides in the second line acquire $U_R^\dagger$, so that the common *vector* factor U_R may be absorbed in f, leaving the $b, \bar{b}$ invariant. This means that there exists a measure $\prod_n db_n^L db_n^R d\bar{b}_n^L d\bar{b}_n^R$ invariant under the $SU(N)$ version of the Peccei-Quinn transformation, exactly as is the case with regularizations. As regards the measure for W, it can be chosen to be $SU(N)$ invariant.

In the case of $SU(N)$ flavour QCD without axions, the nonabelian version of Peccei-Quinn symmetry cannot of course be maintained. But the action has a twisted parity. Let us consider the measure $\prod_n db_n^L db_n^R d\bar{b}_n^L d\bar{b}_n^R$ corresponding to the expansion

$$w^\dagger \psi_L = \sum_n b_n^L f_{nL}, \quad \psi_R = \sum_n b_n^R f_{nR},$$

$$\bar{\psi}_L w = \sum_n \bar{b}_n^L f_{nL}^\dagger, \quad \bar{\psi}_R = \sum_n \bar{b}_n^R f_{nR}^\dagger.$$

As w is invertible, it may also be taken to the right if desired. This construction too is not unique. Noting that

$$f_{nL}(\vec{x}) \to \gamma^0 f_{nR}(-\vec{x}), \quad f_{nR}(\vec{x}) \to \gamma^0 f_{nL}(-\vec{x}),$$

we find that the occurrence of w in the parity transformation of $\psi, \bar{\psi}$ causes b_n^L, b_n^R to be interchanged under parity and similarly there is an interchange of $\bar{b}_n^L, \bar{b}_n^R$ too. This means that the product measure is invariant under the parity symmetry of the action.

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- Pure QCD with twisted mass term actually has parity symmetry with no anomaly, so that axions are not needed to solve strong CP problem.

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THANK YOU!