Strings, Schwarzian, Maximal Chaos

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Current Developments in QFT and Gravity

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Outline

Motivation & Introduction

many-body chaos

Strings & OTOCs

maximal chaos on the world sheet

Schwarzian Action on the Strings, D-brane Horizons

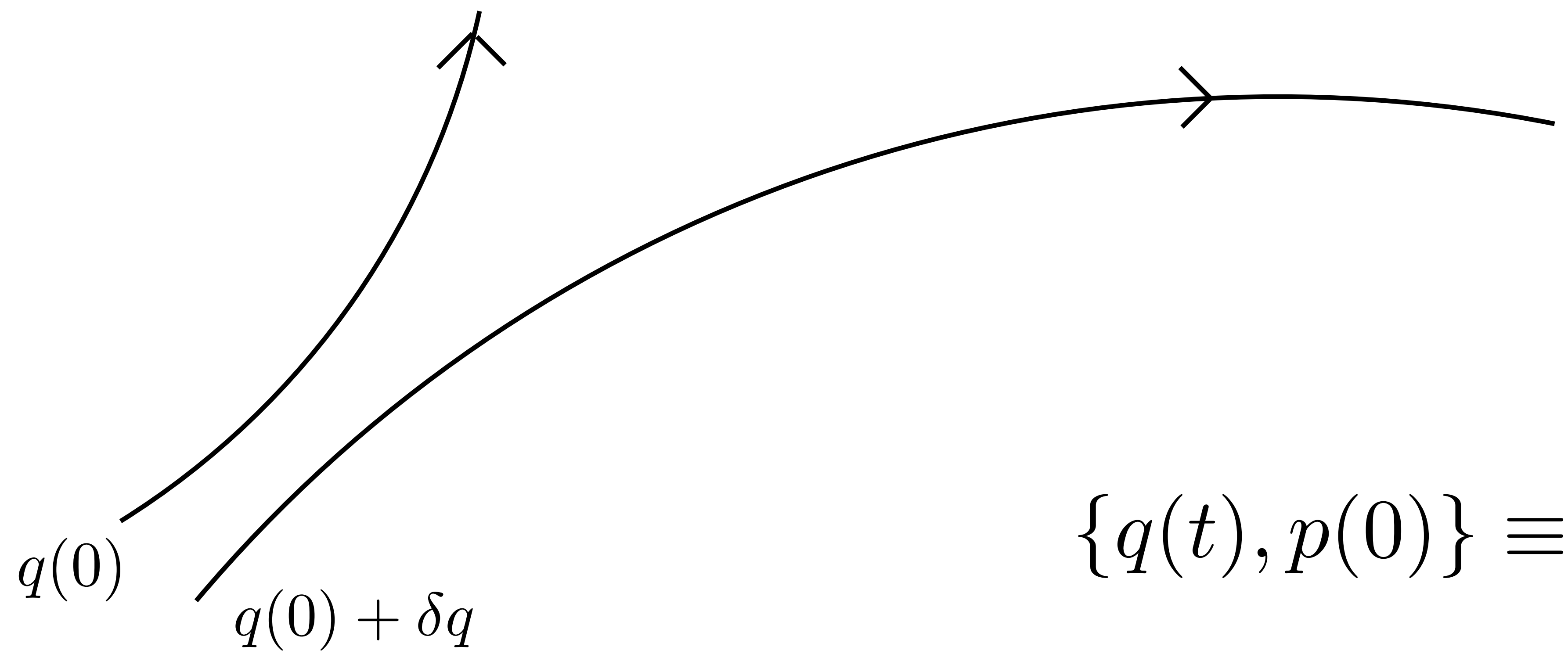
and how it couples to fluctuations

Conclusions and Outlook

General lessons, future work etc.

Motivation & Introduction

Classical dynamics: trajectories



$$\{q(t), p(0)\} \equiv \frac{\partial q(t)}{\partial q(0)} \sim e^{\lambda_{\mathrm{L}} t}$$

initial perturbation

large separation @ late time behaviour

Lyapunov Exponent: λ_{L}

Motivation & Introduction

Quantum dynamics: operators

Larkin, Ovchinnikov

(semi-classical)

$$\{q(t), p(0)\} \to \frac{1}{i\hbar}\,[q(t), p(0)]$$

Generic Diagnostic Function: $C_n(t) = -\langle [W(t), V(0)]^n \rangle_\beta$

a standard choice is: $n = 2$

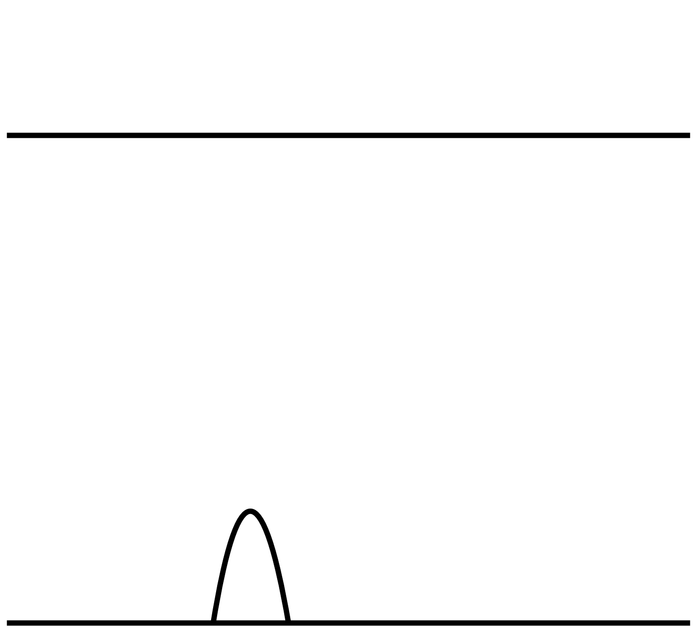
Two kinds of correlates: Time-Ordered, Out-of-Time-Ordered

$\langle V(0)V(0)W(t)W(t)\rangle$ $\langle V(0)W(t)V(0)W(t)\rangle$

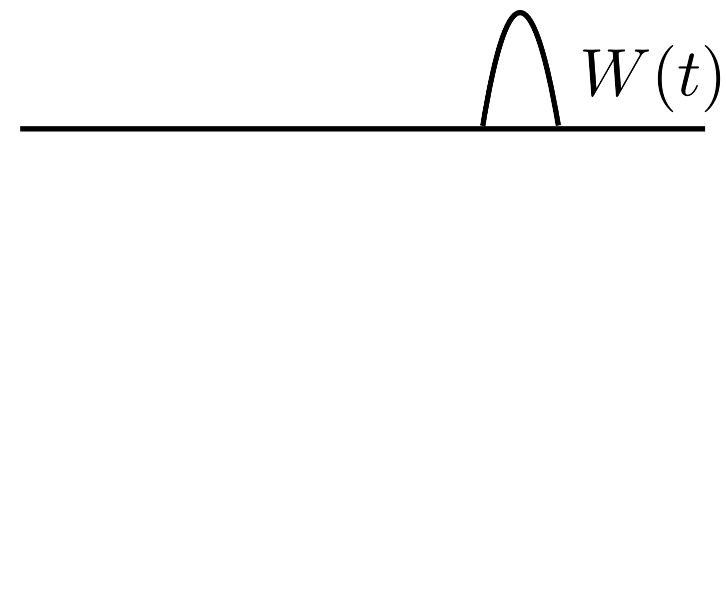
Diagnostic Functions

Shenker, Stanford

An Amplitude Perspective



$$V(0)\,|\beta\rangle$$

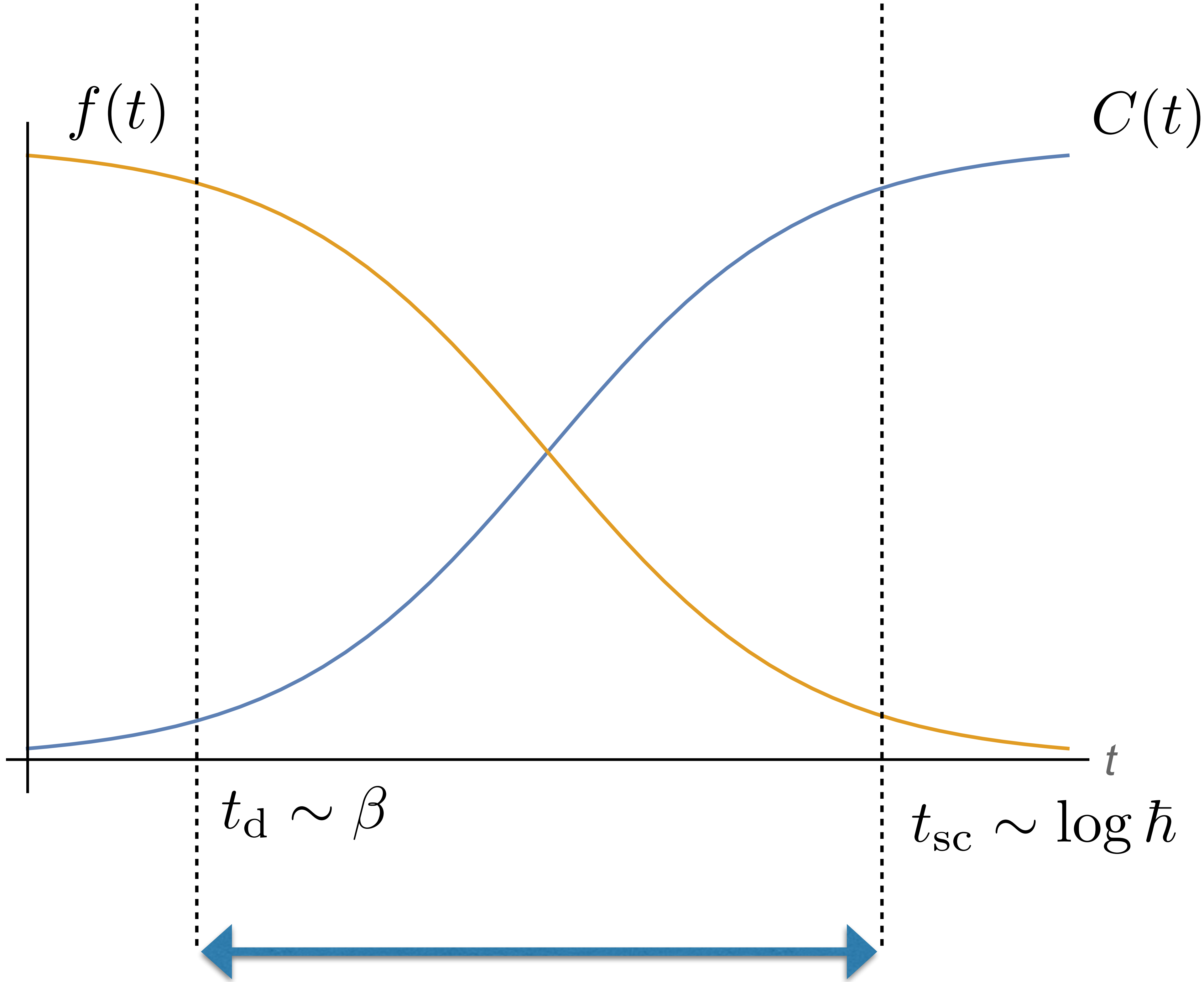


$$W(t)V(0)\,|\beta\rangle$$

$$W(t)V(0)\,|\beta\rangle \equiv |\text{in}\rangle \qquad V(0)W(t)\,|\beta\rangle \equiv |\text{out}\rangle$$

An equivalent diagnostic: $$f(t) \equiv \langle \text{in}|\text{out}\rangle$$

Diagnostic Behaviour



semi-classical regime ensures this separation

The Bound

Typical Behaviour of the Diagnostic Function:

$$C(t,x) \sim \exp\left(\lambda_{\mathrm{L}}\left(t - \frac{|x|}{v_{\mathrm{B}}}\right)\right)$$

(spin chains, SYK-type model, CFTs)

Butterfly velocity

Maldacena-Shenker-Stanford Bound:

$$\lambda_{\mathrm{L}} \leq \frac{2\pi k_{\mathrm{B}} T}{\hbar}$$

(fairly minimal assumptions)

Semi-Classical Systems for Holographers

$\mathrm{O}(N)$ vector theory

small anomalous dimension

Higher Spin Theories

SYK-type

O(1) anomalous dimension

Something in-between

$\mathrm{SU}(N)$ gauge theory

large anomalous dimension

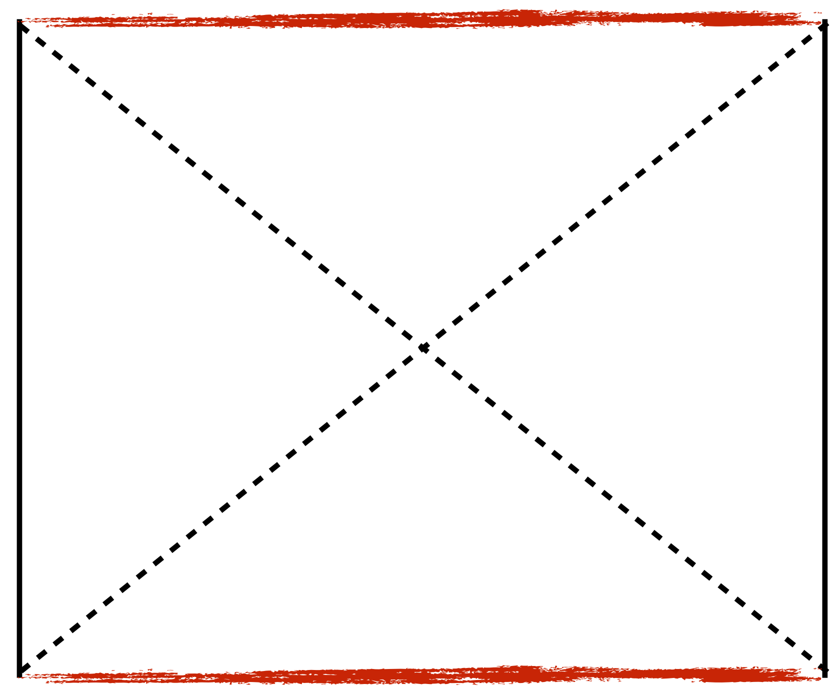
Einstein gravity

semi-classical—ness $\sim \dfrac{1}{N_{\mathrm{d.o.f.}}}$

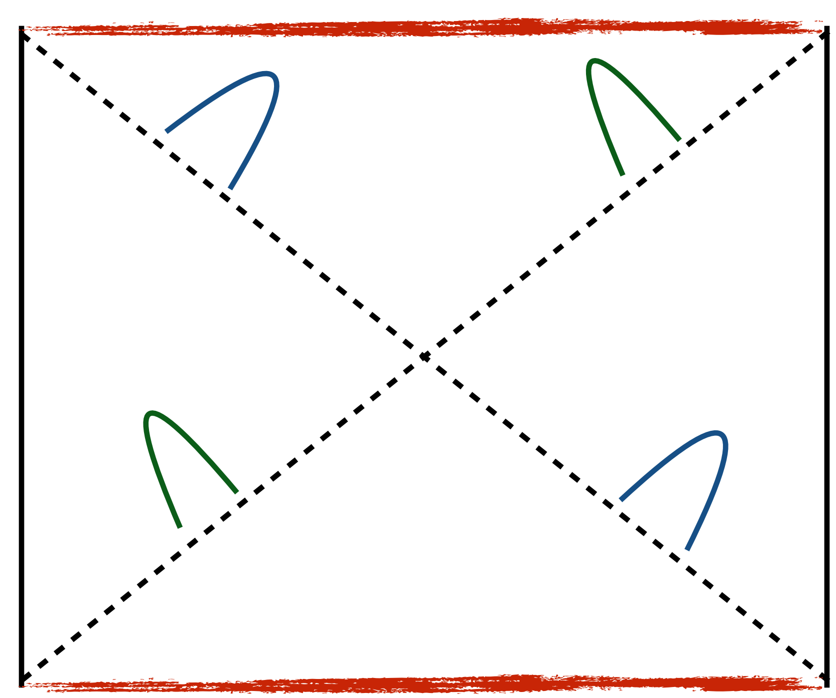
Lyapunov Exponent from Event Horizon

Shenker, Stanford

$$|\beta\rangle \quad \equiv$$



$$\langle V(t_1)W(t_2)V(t_3)W(t_4)\rangle_\beta \equiv$$



Lyapunov Exponent from Event Horizon

Elastic Eikonal 2-2 Scattering:

$$\langle \mathrm{out} | \mathrm{in} \rangle \sim e^{i\delta(s)}$$

Pure phase, a function of the Mandelstam variable

OTOC: $\sim \int dp \, p e^{i\delta} \Psi(p)$

carries the information of bulk-to-boundary propagators

only a handful of explicit calculations: $\lambda_{\mathrm{L}} = 2\pi T$

Forming a Question

What results in maximal chaos?

Example survey: BHs in gravity, SYK-model type Hamiltonian

IR effective Schwarzian theory

Jackiw-Teitelboim gravity yields an effective Schwarzian description

Natural Questions: Is gravity necessary, or a non-linear theory would do?

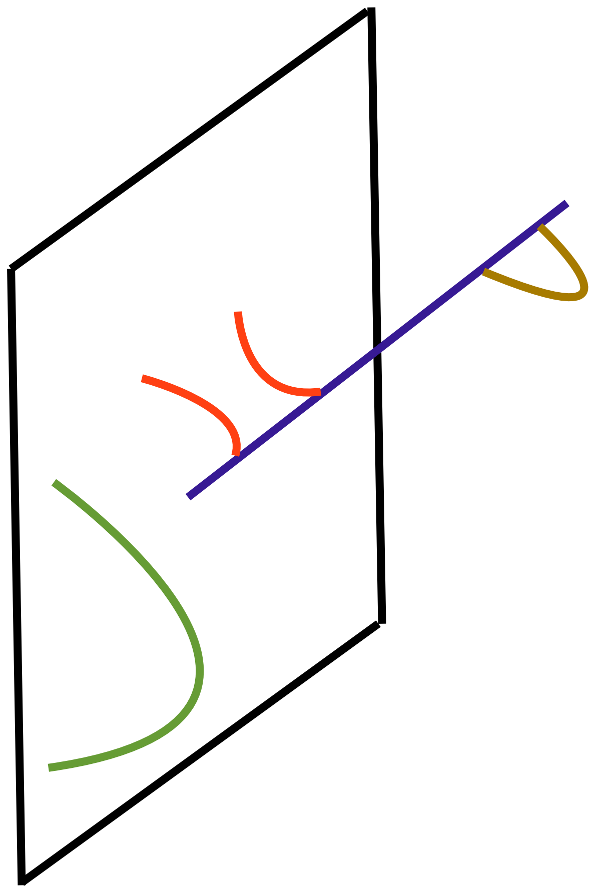
If yes, is there an effective IR description for such systems?

Introducing Open Strings

A typical arrangement of d.o.f.

Background geometry is made of N D3-branes

Add N_f D7-branes



3-3 strings: adjoint sector

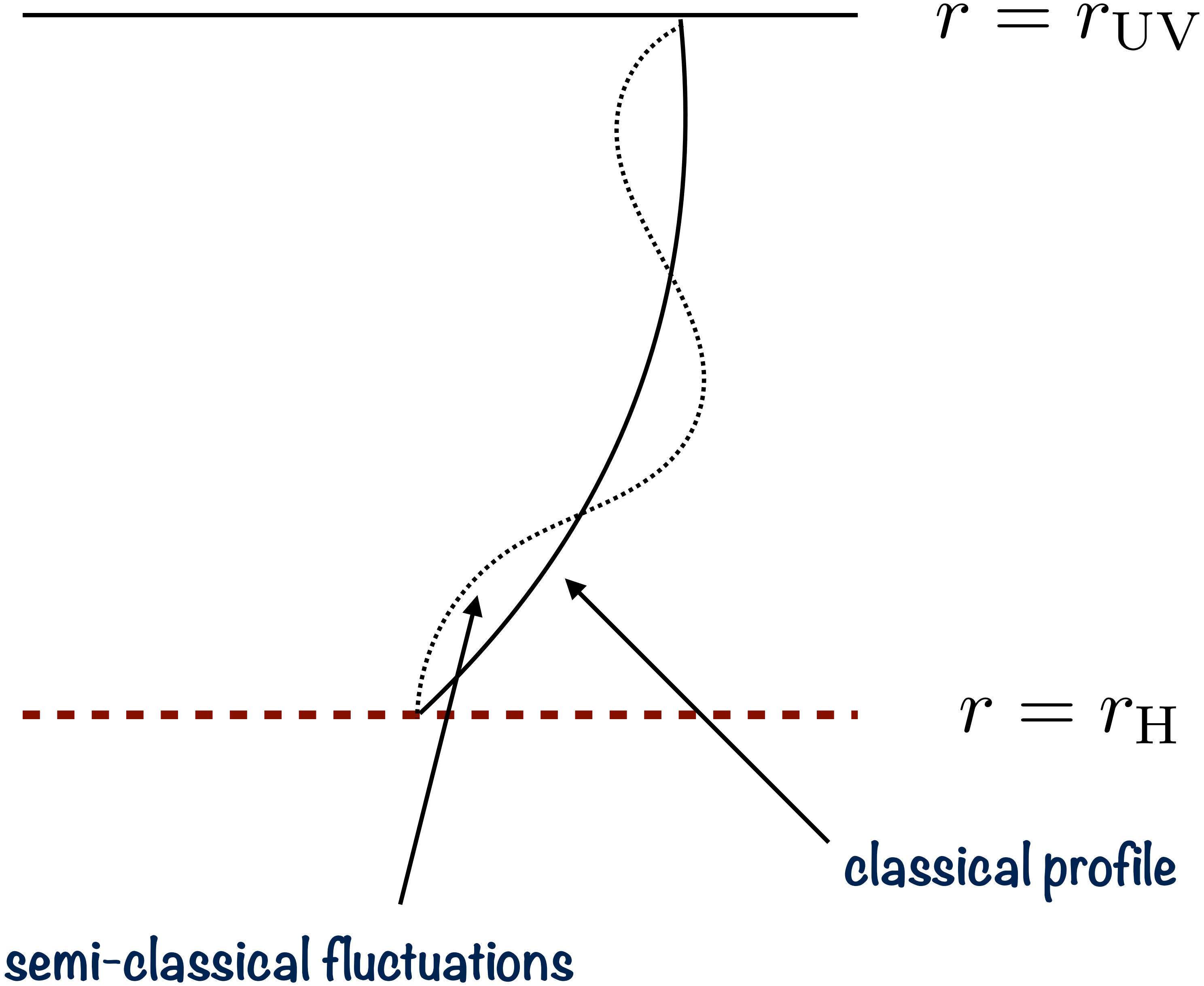
3-7 strings: fundamental matter

7-7 strings: global symmetry

$U(N_f)$

A Simple Example

Take a BTZ-background



$$m_{\mathrm{q}} \sim r_{\mathrm{UV}} \frac{\beta}{\alpha'}$$

the UV cut-off is the physical mass

the UV cut-off is where the brane is located

Bit More Explicitly

Standard Poincare patch

$$ds^2 = \left(r^2 - r_{\mathrm{H}}^2\right) dt^2 + \frac{dr^2}{r^2 - r_{\mathrm{H}}^2} + r^2 dX^2$$

Simplest classical solution: $X\left(t, r\right) = 0 \quad \Longrightarrow \quad \mathrm{AdS}_2$ worldsheet

(manifestly static gauge)

Semi-classical modes: $\delta X\left(t, r\right)$

solves the linearised NG-eom

OTOC is computed by a quartic interaction of $\delta X\left(t, r\right)$

Maximal chaos results from world sheet event horizon

de Boer et al

Is there a Schwarzian

Generic nature of the argument:

Nambu-Goto theory has a gauge symmetry = world sheet diffeomorphism

Embedding space has a (conformal) boundary

there are "large diffeo" symmetries of this system

The soft modes associated to these symmetries are responsible for maximal chaos

The effective Schwarzian describes the soft modes

Similar to arguments made in JT gravity

How to Obtain the Schwarzian

Carry out a purely world sheet analyses

this is hard

We “cheat”

exploit large diffeo structure of the embedding AdS

“project” the large diffeos on the worldsheet

$$\xi_a = \frac{\partial X^\mu}{\partial \sigma^a} \xi_\mu$$

Information about the world sheet

Brown-Henneaux large diffeo

Schwarzian & its Coupling

A special case:

Impose rigid condition on the semi-classical world sheet AdS_2

The projection of embedding space yields the complete non-linear form

The on-shell Nambu-Goto is given by the Schwarzian action:

$$S_{\mathrm{NG}} \sim \frac{\epsilon_{\mathrm{IR}}}{\alpha'} \int d\tau \, \{f(\tau), \tau\} \qquad \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$$

Schwarzian & its Coupling

A more general case:

Fluctuations may not preserve the rigid world sheet

The soft modes due to large diffeos exist

Evaluate the Nambu-Goto with:

$$\tilde{\gamma}_{ab} = \gamma_{ab} + \mathcal{L}_\xi \gamma_{ab}$$

Perturbative calculation yields: $\partial_t^3 \epsilon(t)$ $\partial_t \delta X(t) \epsilon(t) \partial_t^2 \delta X(t)$

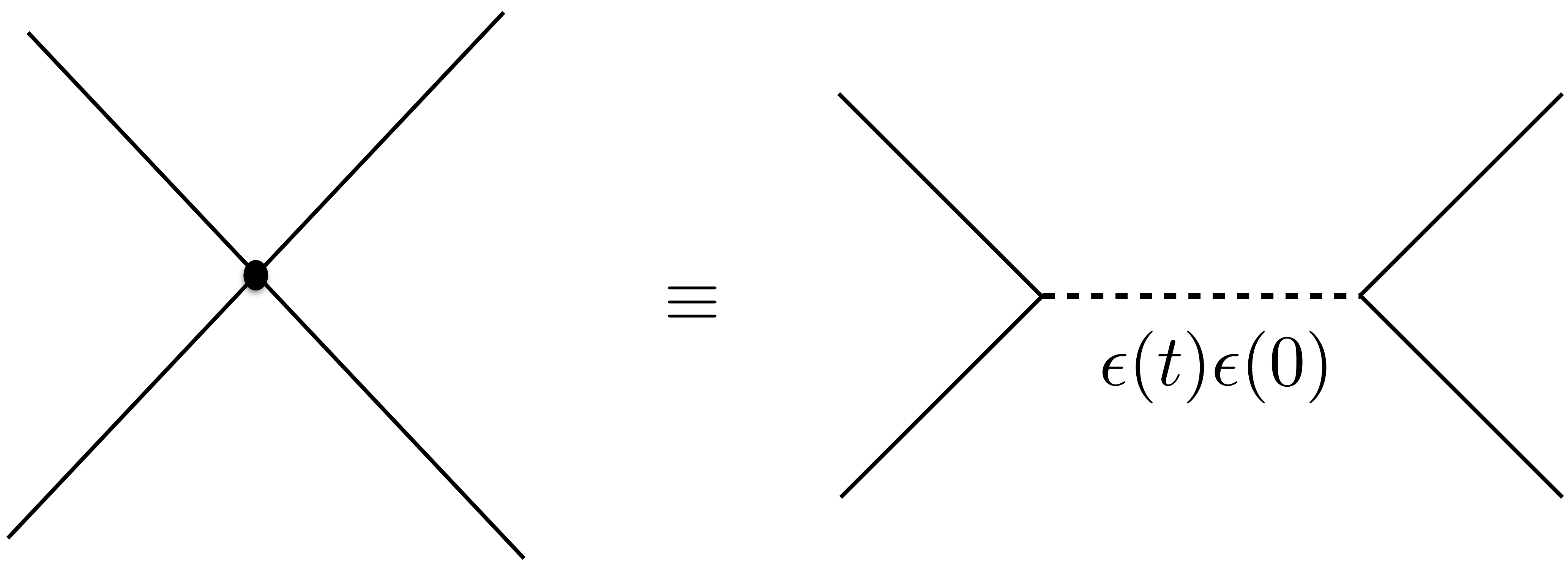
linearized Schwarzian action

interaction term

A Brief Summary

This demonstrates a direct coupling between the soft modes and the fluctuation modes

This resolves a four point vertex into a product of three point vertices, connected by the soft sector propagator



Maximal chaos results from this, from purely world sheet horizon

The D-brane Story

Take a background: $\mathrm{AdS}_5 \times S^5$ (no event horizon)

Introduce a probe brane, turn on a world volume flux: An Electric Field

All open string d.o.f. couple to the Open String Metric

A horizon appears due to the Electric Field

An OTOC computation yields:

$$\lambda_{\mathrm{L}} = 2\pi \left(T^4 + E^2\right)^{1/4}$$

The "gravity temperature"

Multiple Interpretations

Summary

Understanding of chaos, in & away from the semi-classical limit

OTOC for arbitrary states, beyond thermal?

beyond local operators?

"maximal chaos" for Lieb-Robinson bound saturating operators?

Role of the Schwarzian effective theory for strings

both closed & open strings have them, in what sense we learn about the quantum strings & interactions

Is there always an effective description, near an event horizon?

Thank You!

Towards D-branes

Similar maximal chaos expected on a brane horizon

A simple estimate of the scrambling time:

on the string world sheet: $$t_{\rm sc} \sim \beta \log(\sqrt{\lambda})$$

on a D1-brane world volume: $$t_{\rm sc} \sim \beta \log\left(\frac{\sqrt{\lambda}}{N_c}\right)$$

on a Dp-brane world volume: $$t_{\rm sc} \sim \beta \log\left(\frac{\lambda^{(3-p)/4}}{N_c}\right)$$

semi-classical regime is expected

D-brane Picture

Simple background: $N_c \quad D3$ branes $\Longrightarrow$ $\underbrace{\mathrm{AdS}_5 \times S^5}_{\text{background}}$

Introduce: $N_f \quad D5$ branes $\Longrightarrow$ $\underbrace{\mathrm{AdS}_4 \times S^2}_{\text{D5-brane world volume}}$

no event horizon in the background

D-brane Horizon

Excite a gauge field on the world volume: $A = (-Et + a_x(r))\,dx^1$

electric field

contains info about current

The dynamics is now governed by a Dirac-Born-Infeld action

$$\mathcal{L}\left[\phi(r), a_x'(r), r\right]$$

embedding function

current $\sim \dfrac{\partial \mathcal{L}}{\partial a_x'}$

D-brane Horizon

Perform a semi-classical analyses around the classical profile

various fluctuations: (i) scalar, (ii) vector, (iii) fermion

The corresponding actions are:

$$S_{\rm scalar} \sim S^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \ldots$$

$$S_{\rm vector} \sim S^{\mu\nu}S^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} + \ldots$$

$$S_{\rm fermion} \sim \bar{\psi}S^{\mu\nu}\gamma_\mu\nabla_\nu\psi + \ldots$$

Fluctuations couple to open string metric: $S_{\mu\nu} = g_{\mu\nu} - \left(F\cdot g^{-1}\cdot F\right)_{\mu\nu}$

D-brane Horizon to Maximal Chaos

The electric field results in an event horizon in the open string metric:

$$r_{\rm eh}^4 \sim E^2$$

Find a suitable fluctuation sector with analytic control

$$\delta A = \delta A_1(t,r)dx^1 + \delta A_2(t,r)dx^2$$

The vector fluctuations do

All steps can be explicitly calculated: 2-2 scattering

Maximal chaos results on the brane horizon

Another Interim Summary

The general result for Lyapunov exponent:

$$\lambda_{\mathrm{L}} = 2\pi \left(T^4 + E^2\right)^{1/4}$$

The BH temperature

Is there an effective IR description?

For D1-brane, a Schwarzian description is expected

What about a general description at the Rindler horizon?

More complicated fluctuations?

NSSTV, MTV

maximal chaos seems robust, an explicit check is useful

A similar statement for cosmological horizons?

A Phenomenologist's Hat

Demand only a global symmetry $\mathrm{SL}(2, R)$

Schwarzian effective action is certainly not unique

A general IR-effective theory: $$S_{\mathrm{IR}} \sim \int d\tau \mathcal{F}\left[\{f(\tau), \tau\}\right]$$

some functional

A class of choice: $$\mathcal{F}[x] = x^N \ , \quad N \in \mathbb{R}$$

The thermal saddle exists for the entire class

Possible to carry out a semi-classical expansion near this saddle

Towards the Lyapunov Exponent

Standard Schwarzian theory: $$\langle \epsilon(u)\epsilon(0)\rangle \sim \sum \frac{e^{inu}}{n^2\left(n^2-1\right)}$$

zero modes at: $$\underbrace{n = 0, \pm 1}_{\mathrm{SL}(2,R)}$$

For the general class of theories: additional zero modes

zero modes at: $$n = 0, \pm 1, \pm\sqrt{\frac{2N-1}{2N-2}}$$

a naive calculation: $$\lambda_{\mathrm{L}} = \frac{2\pi}{\beta}\max\left(1, k_N\right)$$

Some Remarks

Naively, seems violation of the bound can be engineered

similarity with a higher spin d.o.f. result

if physical, what is the additional symmetry?

may give a handle to track the possible pathology behind the violation

For large class still, maximal chaos is observed

some of these are special

what is the UV-completion?