Non-supersymmetric D3-brane: complexity and information metric

(Based on: arXiv:1807.06361 with Aranya Bhattacharya)

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Outline

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- Apparently there should not be any connection. Here one side is purely quantum mechanical without gravity and the other side is a gravitational theory.
- The bigger picture is to understand the black holes better. Particularly, the information loss paradox. Necessary to get a consistent theory of quantum gravity.
- This paradox appeared to have been resolved by the idea of stretched horizon and black hole complementarity proposed by Susskind (also by Preskill and 't Hooft).

- However, a few years ago it has been pointed out by AMPS [A. Almheiri, D. Marolf, J. Polchinski, J. Sully, JHEP02 (2013) 62] that Susskind idea does not resolve the paradox. It leads to a dramatic event (another paradox) the so-called firewall on the horizon which is against the principle of general relativity.

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- The firewall argument depends on an observer's ability to decode information from the outgoing Hawking radiation near the horizon. Using complexity Harlow and Hayden showed that the number of stepts required to decode the outgoing information increases exponentially as e^S (S is the entropy of the black hole).

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- Therefore, there is a need to understand various concepts in information theory, such as complexity, information metric (information geometry) to better understand the quantum nature of our space-time.
- More formally the quantum state complexity tells us how difficult it is to go from a reference state $|\psi_0\rangle$ to a final state $|\psi\rangle$ by some series of unitary transformation.

$$|\psi\rangle = U|\psi_0\rangle$$

Complexity is the minimum number of unitary transformations (or gates) to obtain U

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- Moreover, when this analysis was applied to a certain tensor network (called Multiscale Entanglement Renormalization Ansatz or MERA) used to obtain ground state of certain critical physical theories, it gives a tensor network structure of AdS time slice.

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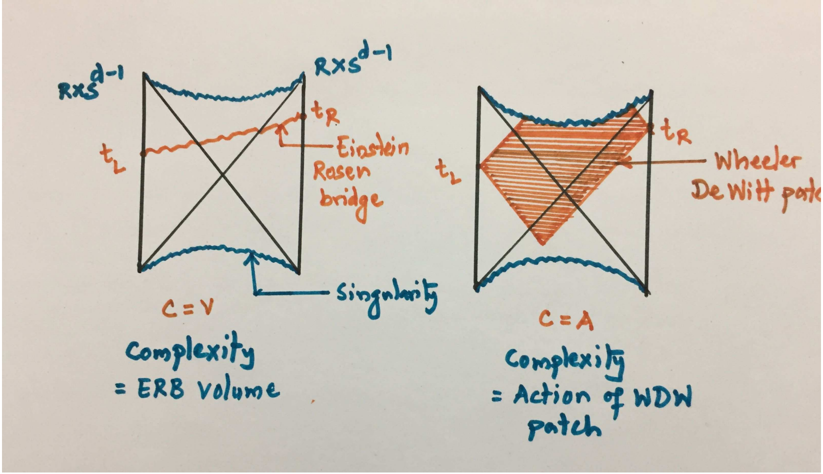
- This was the primary motivation for the Complexity = Volume (CV) and Complexity = Action (CA) proposal of Susskind et. al. (for AdS-Schwarzschild black hole) [A. R. Brown, D. A. Roberts, L. Susskind and B. Swingle, PRL116 (2016) 191301]

- According to Susskind,
  1. CV conjecture: Holographic Complexity (of a CFT state) = Maximum volume of Einstein Rosen Bridge connecting the two black holes (two sided black holes).

$$C_V(\Sigma) = max_{\Sigma=\partial\mathcal{B}}[\frac{V_{\mathcal{B}}}{G_N R}]$$

  where $\mathcal{B}$ is the codimension-one bulk surface, $\partial\mathcal{B}$ is its boundary $= \Sigma$ and $V_{\mathcal{B}}$ is the volume. G_N, Newton's constant and R is the AdS radius.
  2. CA conjecture: Holographic Complexity = Classical action of the certain region of the bulk called the Wheeler-De Witt Patch

$$C_A(\Sigma) = \frac{I_{WDW}}{\pi\hbar}$$


R×S^{d-1}
R×S^{d-1}
t_R
t_L
Einstein Rosen bridge
Singularity
C = V
Complexity = ERB volume
t_R
t_L
Wheeler De Witt patc
C = A
Complexity = Action of WDW patch

- We will be computing complexity (Subregion Complexity) using another definition proposed by Alishahiha [M. Alishahiha, Phys. Rev. D92 (2015) 126009] motivated by Ryu-Takayanagi [S. Ryu and T. Takayanagi, PRL 96 (2006) 181602] formula.

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- Ryu and Takayanagi proposed how to compute another information theoretic quantity called the entanglement entropy for a bipartite system holographically.

- For a system consisting of two parts A and B (complement of A), EE of a subsystem A is the von Neumann entropy and is defined as

$$S_A = -\mathrm{Tr}(\rho_A \log \rho_A)$$

where $\rho_A = \mathrm{Tr}_B(\rho_{\mathrm{tot}})$.

- According to Ryu-Takayanagi: the EE can be calculated holographically as

$$S_E(A) = \frac{\mathrm{Area}(\gamma_A^{\min})}{4G_N}$$

$\gamma_A^{\min}$ is the minimal codimension 2 surface in AdS_{d+2} space with $\partial\gamma_A^{\min} = \partial A$ and G_N is the $(d+2)$-dimensional Newton's constant.

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- Alishahiha made an observation: The Ryu-Takayanagi minimal codimension two area naturally gives rise to another unique quantity on the gravity side. It is the regularized volume enclosed by the area. By dimensionality it must be some form of complexity. So, he defined:

$$\mathrm{Subregion\ \ complexity}\,(C_V) = \frac{V(\gamma)}{8\pi R G_N}$$

γ is the same Ryu-Takayanagi surface as before.

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- To introduce the notion of Fisher information metric, we mention that if ρ_λ is a density matrix parametrized by a parameter λ then a new density matrix $\rho_{\lambda+\delta\lambda}$ can be obtained by perturbing it by $\delta\lambda$. Thus,

$$\rho_{\lambda+\delta\lambda} = \rho_\lambda + (\delta\lambda)\rho_1 + \frac{1}{2}(\delta\lambda)^2\rho_2 + \cdots$$

Then a notion of distance metric may be defined as,

$$G_{\lambda\lambda} = \langle\delta\rho\delta\rho\rangle_{\lambda\lambda} = \frac{1}{2}\mathrm{Tr}\left(\delta\rho\frac{d}{d(\delta\lambda)}\log(\rho_\lambda + \delta\lambda\delta\rho)\Big|_{\delta\lambda=0}\right)$$

- This is obtained by calculating the relative entropy between the two adjacent density matrices as [N. Lashkari and M. van Raamsdonk, JHEP 04 (2016) 153]

$$S(\rho_{\lambda+\delta\lambda}||\rho_\lambda)_{\lambda^2} = (\delta\lambda)^2\langle\delta\rho\delta\rho\rangle_{\lambda\lambda} = (\delta\lambda)^2 G_{\lambda\lambda}$$

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- For the two adjacent density matrices one can define a quantity called the Fidelity defined as,

$$\mathcal{F} = \mathrm{Tr}\sqrt{\sqrt{\rho_\lambda}\rho_{\lambda+\delta\lambda}\sqrt{\rho_\lambda}}$$

Again a quantum distance between the same two states can be defined as (also known as Fidelity Susceptibility) [S. -J. Gu, IJMPB 24 (2010) 4371]

$$\tilde{G}_{\lambda\lambda} = \partial_\lambda^2\mathcal{F}$$

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- The Fidelity can be calculated holographically essentially from the subregion complexity proposed by Alishahiha $\Rightarrow$ the regularized volume of the Ryu-Takayanagi surface [S. Banerjee, J. Erdmenger and D. Sarkar, JHEP 08 (2018) 001] .

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- This is the difference of volumes of the excited state (perturbed state upto second order) and the ground state (pure AdS_5) i.e.,

$$\mathcal{F} = C_d \left(V^{(m^2)} - V^{(0)} \right)$$

where m is the perturbation parameter and C_d is a d-dimensional constant. The Fisher information metric then can be obtained as,

$$G_{mm} = \partial_m^2 \mathcal{F}$$

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- Standard D3 brane is supersymmetric (BPS). Here we consider a non-supersymmetric version of D3 brane. This is also a solution of low energy Type IIB string theory as BPS D3 brane. [B. Zhou and C.J. Zhu, hep-th/9905146; P. Brax, G. Mandal and Y. Oz, PRD 63 (2001) 064008; J. X. Lu and S. Roy, JHEP 02 (2005) 001.]

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- The solution (with metric in Einstein frame) in some suitable coordinates take the form,

$$ds^2 = F_1(\rho)^{-\frac{1}{2}} G(\rho)^{-\frac{\delta_2}{8}} \left[-G(\rho)^{\frac{\delta_2}{2}} dt^2 + \sum_{i=1}^{3} (dx^i)^2\right] + F_1(\rho)^{\frac{1}{2}} G(\rho)^{\frac{1}{4}} \left[\frac{d\rho^2}{G(\rho)} + \rho^2 d\Omega_5^2\right]$$

$$e^{2\phi} = G(\rho)^{-\frac{3\delta_2}{2} + \frac{7\delta_1}{4}}, \qquad F_{[5]} = \frac{1}{\sqrt{2}}(1 + *) Q \mathrm{Vol}(\Omega_5).$$

- The functions $G(\rho)$ and $F_1(\rho)$ are defined as,

$$G(\rho) = 1 + \frac{\rho_0^4}{\rho^4}, \qquad F_1(\rho) = G(\rho)^{\frac{\alpha_1}{2}} \cosh^2\theta - G(\rho)^{-\frac{\beta_1}{2}} \sinh^2\theta$$

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- There are several parameters (δ_1, δ_2, α_1, β_1, θ, ρ_0, Q) in the solution. They satisfy certain relations. For simplicity we will choose $\alpha_1 = \beta_1 = 1$. The other relations are,

$$42\delta_2^2 + 49\delta_1^2 - 84\delta_1\delta_2 = 24, \qquad Q = 2\rho_0^4 \sinh 2\theta$$

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- So, there are three independent parameters δ_2, ρ_0 and θ. Q is the charge of D3 brane. F_1 now reduces to

$$F_1(\rho) = \left(1 + \frac{\rho_0^4 \cosh^2\theta}{\rho^4}\right) G(\rho)^{-\frac{1}{2}}$$

  In the double scaling limit $\rho_0 \to 0$, $\theta \to \infty$ such that $\rho_0^4 \cosh^2\theta = R_1^4$ = fixed, we recover the standard BPS D3 brane from the non-supersymmetric D3 brane solution given before. In this case $G(\rho) \to 1$.

- The solution

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- Also $\delta_2 = 0$ corresponds to zero temperature solution. That is also non-supersymmetric (unlike BPS D3 brane).

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- Decoupling limit is zooming into the region [K. Nayek and S. Roy, PLB 766 (2017) 192] :

$$\rho \sim \rho_0 \ll \rho_0 \cosh^{\frac{1}{2}} \theta \quad \Rightarrow \theta \to \infty$$

- The solution reduces to:

$$ds^2 = \frac{\rho^2}{R_1^2} G(\rho)^{\frac{1}{4}-\frac{\delta_2}{8}} \left[-G(\rho)^{\frac{\delta_2}{2}} dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right] + \frac{R_1^2}{\rho^2} \frac{d\rho^2}{G(\rho)} + R_1^2 d\Omega_5^2$$

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- For non-zero δ_2, the background has a temperature [Y. Kim, B. H. Lee and S. J. Sin, JHEP09 (2007) 105]

$$T_{\text{nonsusy}} = \left(\frac{-\delta_2}{2} \right)^{\frac{1}{4}} \frac{1}{\pi \rho_0 \cosh\theta}$$

Here $-2 \leq \delta_2 \leq 0$ and for $\delta_2 = -2$, it reduces to standard AdS_5 black hole temperature.

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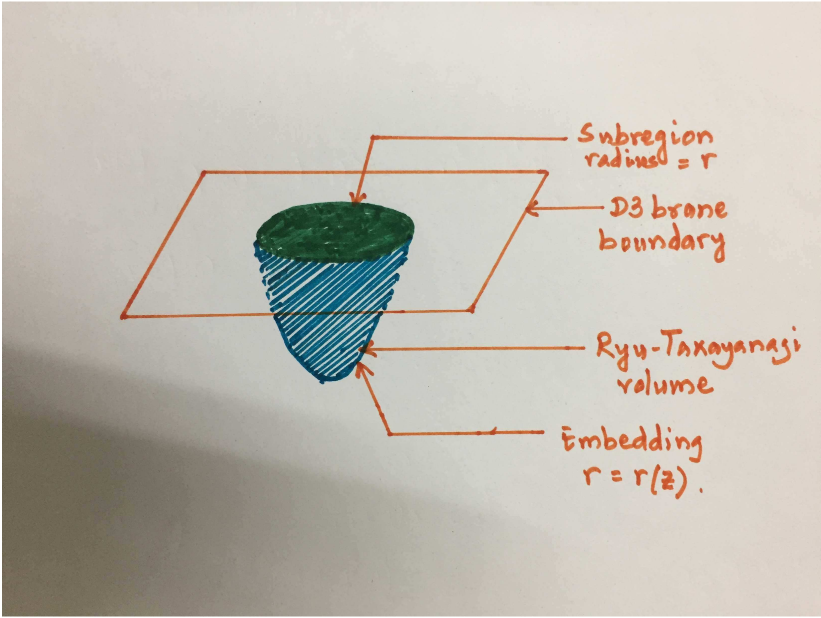
- The background in z coordinate takes the form,

$$ds^2 = \frac{R_1^2}{z^2}\left[-(1-mz^4)^{\frac{1}{4}-\frac{3\delta_2}{8}}dt^2 + (1-mz^4)^{\frac{1}{4}+\frac{\delta_2}{8}}\left(dr^2 + r^2 d\Omega_2^2\right) + \frac{dz^2}{(1-mz^4)}\right]$$

where $m = \rho_0^4/R_1^8$ and we have written $\sum_{i=1}^{3}(dx^i)^2 = dr^2 + r^2 d\Omega_2^2$. Note that for $\delta_2 = -2$, it reduces to the AdS_5 black hole solution.


Subregion
radius = r
D3 brane
boundary
Ryu-Takayanagi
volume
Embedding
r = r(z).

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$$A_{nsD3} = \int \sqrt{g} dz d\theta d\phi$$

$$= 4\pi R_1^3 \int_{z=\epsilon}^{L} dz \frac{r(z)^2 (1 - mz^4)^{\frac{\delta_2}{8} - \frac{1}{4}}}{z^3} \left[1 + (1 - mz^4)^{\frac{5}{4} + \frac{\delta_2}{8}} r'(z)^2\right]^{\frac{1}{2}}$$

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- We use the E-L equation of motion for r and solve it with suitable boundary condition.

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- By replacing m by η/L^4 and making an ansatz $r(z) = \sqrt{L^2 - z^2} + \eta R(z)$, we solve the equation for $R(z)$ with BC (i) $R(0) = L/5$ and (ii) $\mathrm{Lim}_{z \to L} R(z) = 0$. This way the embedding $r(z)$ can be obtained as

$$r(z) = \sqrt{L^2 - z^2}\left[1 + \eta \frac{4\delta_2 L^2 z^4 - (\delta_2 + 10) z^6 + 16 L^6}{80 L^4 (L^2 - z^2)}\right]$$

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- Once we have the embedding we can calculate the Ryu-Takayangi volume as,

$$V_{nsD3}=\frac{4\pi R_1^4}{3}\int_\epsilon^L\frac{dz}{z^4}r(z)^3\left(1-\eta\frac{z^4}{L^4}\right)^{\frac{3\delta_2}{16}-\frac{1}{8}}$$

- Now using the form of $r(z)$ in the volume integral we can evaluate it upto second order in η as,

$$V_{0(nsD3)} = \frac{4\pi R_1^4}{3}\int_\epsilon^L dz\left[\frac{(L^2-z^2)^{3/2}}{z^4}\right] = V_{AdS}$$

$$V_{1(nsD3)} = \eta\left(\frac{\pi R_1^4}{60L^4}\right)\int_\epsilon^L \frac{dz}{z^4}\left[\sqrt{(L^2-z^2)}\left((10-3\delta_2)L^2z^4+4(3\delta_2-10)z^6+48L^6\right)\right]$$

$$V_{2(nsD3)} = \eta^2\left(\frac{\pi R_1^4}{3200L^8}\right)\int_\epsilon^L \frac{dz}{z^4\sqrt{L^2-z^2}}\left[32(10-7\delta_2)L^8z^4+32(13\delta_2-30)L^6z^6\right.$$
$$+\left(300-\delta_2(13\delta_2+420)\right)L^4z^8-16\left((\delta_2-65)\delta_2+50\right)L^2z^{10}$$
$$\left.+\left(\delta_2(47\delta_2-740)+700\right)z^{12}+512L^{12}\right]$$

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- Evaluating the integral and sending $\epsilon \to 0$, we get the subregion complexity using the Alishahiha proposal as,

$$C_V^{(1)} = \frac{V_{1(nsD3)}-V_{0(nsD3)}}{8\pi R_1 G_5} = 0$$

$$C_V^{(2)} = \frac{V_{2(nsD3)}-V_{0(nsD3)}}{8\pi R_1 G_5} = \left(\frac{4\pi R_1^3}{24\pi G_5}\right)\left[\frac{3\pi\left(381\delta_2^2-6508\delta_2-13580\right)}{3276800}\right](m\ell^4)^2$$
$$= \frac{\pi R_1^3\left(381\delta_2^2-6508\delta_2-13580\right)}{6553600 G_5}(m\ell^4)^2$$

- The holographic fidelity between two states has been given before as the difference in the volume upto a d-dimensional constant as $\mathcal{F} = C_d \left(V^{(m^2)} - V^{(0)} \right)$. In our case we can identify the constant to be

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$$\mathcal{F}_{nsD3} = C_4\left(V_{2(nsD3)} - V_{0(nsD3)}\right) = \frac{32\pi^2}{4725 G_5} R_1^3 m^2 \ell^8$$

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- So, the corresponding Fisher Information metric can be obtained as,

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- To our surprise, this is independent of δ_2. Therefore, it has same value even for AdS_5 black hole. This is independent of underlying supersymmetry of the theory.

Conclusion

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- It will be interesting to find its relation (if any) to the complexity = volume or complexity = action proposal of Susskind.
- The dual field theory corresponding to decoupled geometry of non-supersymmetric D3 brane is a non-conformal, non-supersymmetric QFT. This can be thought of as a perturbation of $D = 4$, $\mathcal{N} = 4$ CFT. It will be very interesting to understand how to compute complexity or information metric on the field theory side.

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- Various connections of many other quantities of quantum information theory and space-time geometry are still being revealed and hopefully, they will clarify the mysteries of quantum gravity.

Thank You!!