Noncommutative effects on holographic superconductors with power Maxwell electrodynamics

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Conference on Current Developments in Quantum Field Theory and Gravity
S.N. Bose National Centre for Basic Sciences

06/12/2018

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Acknowledgment

Basics of superconductivity

- Electrical resistivity of some metals suddenly drops to zero when the temperature is lowered below a certain critical temeperature T_c - 1911.
- Magnetic field is expelled when $T < T_c$ - Meissner effect.
- BCS theory successfully described low temperature superconductors-Cooper pair formation-1957.
- High T_c superconductors were discovered in 1986 - J.G.Bednorz and K.A.Muller -Nobel Prize - 1987.
- Example- cuprates (T_c=134k)
- Pairing mechanism is not well understood for such strongly coupled system.
- Gauge/gravity duality provides a new tool to understand high T_c superconductors.

Holographic principle

- A weakly coupled gravity theory in AdS_{n+1} spacetime is equivalent to a strongly coupled conformally invariant quantum field theory CFT_n in one less dimension.(Maldacena 1998)
- also known as gauge/gravity duality.
- The name suggests that one can look at a two (spatial) dimensional superconductor and see a 3-dimensional image.

Constructing gravity dual

- To construct a proper gravitational dual of a superconductor we need to incorporate the following ideas in our theory
  - In the superconductor, we need a notion of temperature. On the gravity side, that role is played by a black hole.
  - In the superconductor, we also need a condensate. In the bulk, this is described by some field coupled to gravity.
- Static non-zero field outside a black hole corresponds to a non-zero condensate. This is usually called black hole hair.
- Hence, to describe a superconductor, we need a black hole that has hair at low temperatures, but no hair at high temperatures.
- consider

$$S = \int d^d x \sqrt{-g}[R - 2\Lambda - \beta(F_{\mu\nu}F^{\mu\nu})^q - \\ |\nabla_\mu \psi - ieA_\mu \psi|^2 - m^2|\psi|^2] \quad (1)$$

Constructing gravity dual

- This is just general relativity with a, coupled to a power Maxwell field and charged scalar with mass m and charge e.
- It is easy to see why black holes in this theory might be unstable to forming scalar hair:
  - For an electrically charged black hole, the effective mass of ψ is $m_{eff}^2 = m^2 + e^2 g^{tt} A_t^2$
  - But the last term is negative, so there is a chance that m_{eff}^2 becomes sufficiently negative near the horizon to destabilize the scalar field.
- The above mechanism does not work for asymptotically flat black holes. One has to choose a background metric that is asymptotically AdS.
- If one rescales $A_\mu = \tilde{A}_\mu / e$ and $\psi = \tilde{\psi}/e$, then the matter action has a $1/e^2$ in front, so the backreaction of the matter fields on the metric is suppressed when e is large. The limit $e \to \infty$ with $\tilde{A}_\mu$ and $\tilde{\psi}$ fixed is called the probe limit.

Basic set-up

- The metric of a d-dimensional planar Schwarzschild-AdS black hole reads

$$ds^2 = -f\left(r\right)dt^2 + \frac{1}{f\left(r\right)}dr^2 + r^2 dx_i dx^i \qquad (2)$$

- where, $f(r) = r^2\left(1 - \frac{r_+^{d-1}}{r^{d-1}}\frac{\gamma(\frac{d-1}{2},\frac{r^2}{4\theta})}{\gamma(\frac{d-1}{2},\frac{r_+^2}{4\theta})}\right)$.
  (ref:10.1016/j.physletb.2005.11.004,Piero Nicolini)
- $\gamma(s,x) = \int_0^x t^{s-1}e^{-t}dt$ - lower incomplete gamma function
- r_+ - horizon radius
- θ - noncommutative parameter
- The Hawking temperature is given by
  $T = \frac{f'(r_+)}{4\pi} = \frac{r_+}{4\pi}\left[d - 1 - \frac{4MG_d}{\Gamma(\frac{d-1}{2})}\frac{e^{-\frac{r_+^2}{4\theta}}}{(4\theta)^{\frac{d-1}{2}}}\right]$.

Equations of motion

- The equations of motion for the Maxwell and the scalar field obtained by varying the action mentioned previously :

$$\partial_z^2\phi + \frac{1}{z}\left(2 - \frac{d-2}{2q-1}\right)\partial_z\phi + \frac{2\phi(z)\psi^2(z)r_+^{2q}(\partial_z\phi)^{2(1-q)}}{z^{4q}2^{q+1}(-1)^{3q}\beta q(2q-1)f(z)} = 0 \quad (3)$$

$$\partial_z^2\psi + \left(\frac{f^{'}(z)}{f(z)} - \frac{d-4}{z}\right)\partial_z\psi + \frac{\phi^2\psi r_+^2}{z^4 f^2(z)} - \frac{m^2\psi r_+^2}{z^4 f(z)} = 0\ . \quad (4)$$

- where we have chosen the ansatz $A_t = \phi(r)$ and $\psi = \psi(r)$ and made a variable change $z = \frac{r_+}{r}$.
- $f(z) = \frac{r_+^2}{z^2}g_0(z)\ ;\ g_0(z) = \left(1 - z^{d-1}\frac{\gamma(\frac{d-1}{2}, \frac{r_+^2}{4\theta z^2})}{\gamma(\frac{d-1}{2}, \frac{r_+^2}{4\theta})}\right).$

Boundary conditions

- The boundary condition $\phi(r = r_+) = 0$ now becomes $\phi(z = 1) = 0$.
- Near the boundary $(z \to 0, r \to \infty)$, the scalar potential $\phi(z)$ and the scalar field $\psi(z)$ can be approximated as :
$\phi(z) = \mu - \frac{\rho^{\frac{1}{2q-1}}}{r_+^{\frac{d-2}{2q-1}-1}} z^{\frac{d-2}{2q-1}-1}$; $\psi(z) = \frac{\psi_-}{r_+^{\lambda_-}} z^{\lambda_-} + \frac{\psi_+}{r_+^{\lambda_+}} z^{\lambda_+}$
where, λ is the conformal dimension of the condensation operator in the boundary field theory.
$\lambda_\pm = \frac{1}{2}\left[d - 1 \pm \sqrt{(d-1)^2 + 4m^2}\right]$. (ref: Phys. Lett. B, 718 (2013), p. 1089, D. Roychowdhury)
- The coefficients ψ_- and ψ_+ correspond to the vacuum expectation values of the condensation operator.
- μ and ρ are interpreted as the chemical potential and the charge density of the dual theory on the boundary.

Matching Method

- In this method we expand the $\phi(z)$ and $\psi(z)$ field near the horizon $(r = r_+, z = 1)$ and equate that to the asymptotic $(r \to \infty, z \to 0)$ solution at some point $z = z_m$.

$$\left[\mu - \frac{\rho^{\frac{1}{2q-1}}}{r_+^{\frac{d-2}{2q-1}-1}} z^{\frac{d-2}{2q-1}-1}\right]_{z=z_m} =$$

$$\left[\phi(1) - \phi'(1)(1-z) + \frac{1}{2}\phi''(1)(1-z)^2 + O((1-z)^3)\right]_{z=z_m}$$

- By doing so for $\phi(z)$ field we get:

$$\mu - \frac{\rho^{\frac{1}{2q-1}} z_m^{\frac{d-2}{2q-1}-1}}{(r_+)^{\frac{d-2}{2q-1}-1}} = v(1-z_m) + \frac{1}{2}(1-z_m)^2 \times \left[\left(2 - \frac{d-2}{2q-1}\right) v - \frac{2 r_+^{2(q-1)} \alpha^2 (-v)^{3-2q}}{(-1)^{3q} 2^{q+1} (\beta q)(2q-1)(g_0')}\right] \quad (5)$$

Matching Method

- and for $\psi(z)$ field we get :

$$\frac{\langle O_+\rangle z_m^{\lambda_+}}{r_+^{\lambda_+}} = \alpha - (1-z_m)\alpha\left(\frac{m^2}{g_0^{'}(1)}\right) + \frac{1}{2}\alpha(1-z_m)^2\times$$
$$\left[\frac{1}{2}\left(\frac{m^2}{g_0^{'}(1)}\right)\left(d-4-\frac{g_0^{''}(1)}{g_0^{'}(1)}+\frac{m^2}{g_0^{'}(1)}\right)-\frac{\tilde{v}^2}{2g_0^{'}(1)^2}\right] \quad . \quad (6)$$

- where $v=-\phi^{'}(1), \alpha=\psi(1)$ and $\tilde{v}=\frac{v}{r_+}$. and replaced $\phi^{''}(1)$ and $\psi^{'}(1)$, $\psi^{''}(1)$ respectively form eq. (3) eq. and (4)
- taking derivative on both side of eq (5) and (6) yields:

$$-\frac{\rho^{\frac{1}{2q-1}} z_m^{\frac{d-2}{2q-1}-2}}{(r_+)^{\frac{d-2}{2q-1}-1}}\left(\frac{d-2}{2q-1}-1\right) = -v-(1-z_m)\times$$
$$\left[\left(2-\frac{d-2}{2q-1}\right)v - \frac{2r_+^{2(q-1)}\alpha^2(-v)^{3-2q}}{(-1)^{3q}2^{q+1}(\beta q)(2q-1)g_0^{'}(1)}\right] \quad (7)$$

Matching method

- and

$$\lambda_+ \frac{\langle O_+ \rangle \, z_m^{\lambda_+ - 1}}{r_+^{\lambda_+}} = \alpha \left(\frac{m^2}{g_0^{'}(1)} \right) - \alpha(1 - z_m) \times$$
$$\left[\frac{1}{2} \left(\frac{m^2}{g_0^{'}(1)} \right) \left(d - 4 - \frac{g_0^{''}(1)}{g_0^{'}(1)} + \frac{m^2}{g_0^{'}(1)} \right) - \frac{\tilde{v}^2}{2 g_0^{'}(1)^2} \right] . \quad (8)$$

Critical temperature-charge density relationship

- From the above set of equations and using the expression for Hawking temperature it is simple to obtain :
  $\alpha^2 \equiv \alpha^2_{NC} = -\frac{(-1)^{5q-3}2^q(\beta q)(2q-1)g_0^{'}(1)}{\tilde{v}_{NC}^{2(1-q)}(1-z_m)} \times \left[1+\left(2-\frac{d-2}{2q-1}\right)(1-z_m)\right] \times \left(\frac{(T_c)_{NC}}{T}\right)^{\frac{d-2}{2q-1}}\left[1-\left(\frac{T}{(T_c)_{NC}}\right)^{\frac{d-2}{2q-1}}\right]$
- where

$$(T_c)_{NC} = \xi_{NC}\rho^{\frac{1}{d-2}} \quad (9)$$

$$\xi_{NC} = -\frac{z_m^{(\frac{d-2}{2q-1}-2)(\frac{2q-1}{d-2})}}{\tilde{v}_{NC}^{\frac{2q-1}{d-2}}}\left(\frac{g_0^{'}(1)}{4\pi}\right) \times \frac{(\frac{d-2}{2q-1}-1)^{\frac{2q-1}{d-2}}}{[1+(2-\frac{d-2}{2q-1})(1-z_m)]^{\frac{2q-1}{d-2}}}\,. \quad (10)$$

Critical temperature-charge density relationship

- Analytical results from $eq.(10) \rightarrow$ the critical temperature decreases with increase in the noncommutative parameter $\theta \rightarrow$ condensate gets harder to form as the spacetime noncommutativity increases.

Table: Analytical values of ξ_{NC} for different values of M and θ [$q = 1$, $m^2 = 0$, $z_m = 0.5$ and $d = 5$]

θ	ξ_{NC}	
	$MG_d = 50$	$MG_d = 100$
0.3	0.16933	0.1702
0.5	0.16058	0.1678
0.7	0.1492	0.1608
0.9	0.1418	0.1525

Critical temperature-charge density relationship

- mass of the black hole increases → the critical temperature for a particular value of θ increases → the effects of spacetime noncommutativity becomes prominent for lower mass black holes.

Table: Analytical values of ξ_{NC} for different values of M and θ [$q = 1$, $m^2 = -3$, $z_m = 0.5$ and $d = 5$]

θ	ξ_{NC}	
	$MG_d = 50$	$MG_d = 100$
0.3	0.2003	0.2015
0.5	0.18798	0.1977
0.7	0.1744	0.1883
0.9	0.1669	0.1782

Critical temperature-charge density relationship

Table: Analytical values of ξ_{NC} for different values of M and θ [$q = 5/4$, $m^2 = -3$, $z_m = 0.5$ and $d = 5$]

θ	ξ_{NC}	
	$MG_d = 50$	$MG_d = 100$
0.3	0.1126	0.1134
0.5	0.1067	0.1114
0.7	0.1008	0.1069
0.9	0.0980	0.1024

Table: Analytical values of ξ_{NC} for different values of M and θ [$q = 7/4$, $m^2 = -3$, $z_m = 0.5$ and $d = 5$]

θ	ξ_{NC}	
	$MG_d = 50$	$MG_d = 100$
0.3	0.0177	0.0178
0.5	0.0171	0.0176
0.7	0.0167	0.0171
0.9	0.0168	0.0168

- the onset of power Maxwell electrodynamics (for a value of $q \neq 1$) makes the condensate difficult to form and the effect of the power Maxwell theory on the formation of the condensate decreases with increase in the mass of the black holes.

Introducing magnetic field

- we have chosen the following ansatz to introduce an external magnetic field B in the bulk theory and observe how the condensation behaves for noncommutative black hole background in the bulk.
  $A_t = \phi(z) \quad , \quad A_y = Bx \quad , \quad \psi = \psi(x,z) \ .$
- The intention is to find a critical magnetic filed B_c above which the condensation vanishes.
- The equation of motion for the complex scalar field ψ that follows from the above ansatz reads:
  $\partial_z^2\psi(x,z) + \left(\frac{f'(z)}{f(z)} - \frac{d-4}{z}\right)\partial_z\psi(x,z) + \frac{\phi^2(z)\psi(x,z)r_+^2}{z^4 f^2(z)} - \frac{m^2 r_+^2\psi(x,z)}{z^4 f(z)} + \frac{1}{z^2 f(z)}(\partial_x^2\psi - B^2x^2\psi) = 0 \ .$
- For solving the above equation, we write $\psi(x,z)$ as $\psi(x,z) = X(x)R(z) \ .$

Introducing magnetic field

- and we get:

$$R''(z) + \left(\frac{f'(z)}{f(z)} - \frac{d-4}{z}\right) R'(z) + \frac{\phi^2(z) r_+^2 R(z)}{z^4 f^2(z)} - \frac{m^2 r_+^2 R(z)}{z^4 f(z)} = \frac{BR(z)}{z^2 f(z)} \ . \quad (11)$$

- Using the matching method as earlier after few algebraic steps a quadratic equation for B can be obtain:

$$B^2 + p r_+^2 B + n r_+^4 - \phi'^2(1) r_+^2 = 0 \quad (12)$$

- At $B = B_c$, the condensate vanishes $\rightarrow \psi = 0$

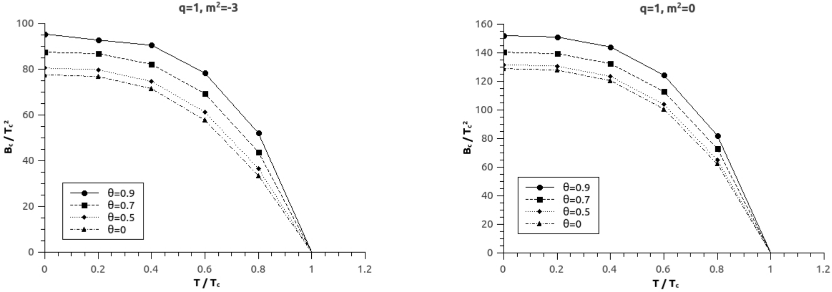
Introducing magnetic field

- Solving *eq*.(12) :

$$(B_c)_{NC} = \frac{(-g_0'(1))^{\frac{d-2}{2q-1}-2}}{2(4\pi)^{\frac{d-2}{2q-1}-2}\xi_{NC}^{\frac{d-2}{2q-1}}}(T_c)^2_{NC} \times \left[\Omega_{NC}(d,q,m) - p\left(-\frac{4\pi\xi_{NC}}{g_0'(1)}\right)^{\frac{d-2}{2q-1}}\left(\frac{T}{(T_c)_{NC}}\right)^{\frac{d-2}{2q-1}}\right] \tag{13}$$

- where, $p = 2m^2 + \left(d-4-\frac{g_0''(1)}{g_0'(1)}\right)g_0'(1) + 2g_0'(1) - \frac{4g_0'(1)(\lambda_+(1-z_m)+z_m)}{(1-z_m)(\lambda_+(1-z_m)+2z_m)}$
- $n = m^4 + m^2 g_0'(1)\left[\left(d-4-\frac{g_0''(1)}{g_0'(1)}\right) - \frac{4(z_m+\lambda_+(1-z_m))}{(1-z_m)(2z_m+\lambda_+(1-z_m))}\right] + \frac{4\lambda_+ g_0'^2(1)}{(1-z_m)(2z_m+\lambda_+(1-z_m))}$.

Summary and Conclusion



- Figure: B_c/T_c^2 vs T/T_c plot : $z_m = 0.5, MG_d = 100, d = 5$

Summary and Conclusion

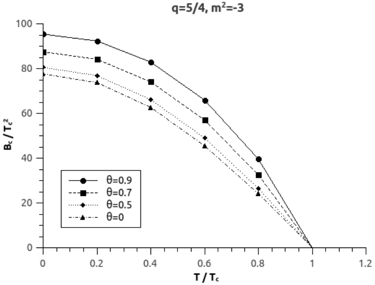


Figure: B_c/T_c^2 vs T/T_c plot : $z_m = 0.5, MG_d = 100, d = 5$

- there exists a critical magnetic field as well as a critical temperature above which the superconducting phase vanishes.
- critical magnetic field above which the condensate vanishes increases with increase in the noncommutative parameter θ.

Acknowledgment

I would like to acknowledge Dr. Sunanadan Gangopadhyay for his proper guidance, Prof. P.K. Panigrahi for his support , DPS IISER KOLKATA and CSIR for providing me with fellowship.

Thank You