Quantum Information Measures of Non-SUSY ‘black’ D3 brane

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Outline

- Basics
- HEE and Entanglement Thermodynamics
- Entropy Cross Over at High T
- Outlook

Basic Preliminaries

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- For a bipartite system consisting of A and B, EE of a subsystem A is the von Neumann entropy and is defined as $S_A = -\mathrm{Tr}(\rho_A \log \rho_A)$, where $\rho_A = \mathrm{Tr}_B(\rho_{\mathrm{tot}})$.

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- For a bipartite system consisting of A and B, EE of a subsystem A is the von Neumann entropy and is defined as $S_A = -\mathrm{Tr}(\rho_A \log \rho_A)$, where $\rho_A = \mathrm{Tr}_B(\rho_{\mathrm{tot}})$.
- Seminal work of Ryu-Takayanagi: the holographic version of this can be written as

$$S_E(A) = \frac{\mathrm{Area}(\gamma_A^{\mathrm{min}})}{4G_N}$$

γ_A^{min} is the minimal codimension 2 surface in AdS_{d+2} space with $\partial\gamma_A^{\mathrm{min}} = \partial A$ and G_N is the $(d+2)$-dimensional Newton's constant.

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- Susskind proposed something holographically in the gravity side which is different from EE and he terms it as complexity.
- Various motivations (e.g; Complexity = Volume, Subregion Duality, RT formula) led Alishahiha to propose another idea, namely Subregion Holographic Complexity, by which one can quantify the complexity associated with a subsystem of a bipartite system, using the bulk volume $(V(\gamma))$ dual to a RT surface.

$$C_V = \frac{V(\gamma)}{8\pi R G_N}$$

- The solution we study here is of a Non-supersymmetric D3 brane with finite temperature. The metric is of the form

$$ds^2 = F_1(\rho)^{-\frac{1}{2}} G(\rho)^{-\frac{\delta_2}{8}} \left[-G(\rho)^{\frac{\delta_2}{2}} dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right] + F_1(\rho)^{\frac{1}{2}} G(\rho)^{\frac{1}{4}} \left[\frac{d\rho^2}{G(\rho)} + \rho^2 d\Omega_5^2 \right]$$

$$e^{2\phi} = G(\rho)^{-\frac{3\delta_2}{2} + \frac{7\delta_1}{4}}, \qquad F_{[5]} = \frac{1}{\sqrt{2}} (1 + *) Q \mathrm{Vol}(\Omega_5).$$

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- The functions $G(\rho)$ and $F_1(\rho)$ are defined as,

$$G(\rho) = 1 + \frac{\rho_0^4}{\rho^4}, \qquad F_1(\rho) = G(\rho)^{\frac{\alpha_1}{2}} \cosh^2\theta - G(\rho)^{-\frac{\beta_1}{2}} \sinh^2\theta$$

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- The solution has two interesting limits.
  1. In parameter values $\delta_2 = -2 \;\; \delta_1 = -\frac{12}{7}$ and choice $\alpha_1 + \beta_1 = 2$, the solution reduces to standard black brane solution.
  2. When, $\delta_2 = 0$, it reduces to the zero temperature nonsusy D3 brane solution.

- We find the temperature to be

$$T_{\text{nonsusy}} = \left(\frac{-2\delta_2}{(\alpha_1 + \beta_1)^2} \right)^{\frac{1}{4}} \frac{1}{\pi \rho_0 \cosh \theta}$$

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- The Einstein frame metric after a few reparametrization, looks like

$$ds^2 = H(\rho)^{-\frac{1}{2}} \left[-G(\rho)^{\frac{2+3\delta_2}{8}} dt^2 + G(\rho)^{\frac{2-\delta_2}{8}} \sum_{i=1}^{3} (dx^i)^2 \right] +$$

$$\mathrm{H}(\rho)^{\frac{1}{2}} \left[\frac{d\rho^2}{G(\rho)} + \rho^2 d\Omega_5^2 \right]$$

- The throat geometry of the Einstein frame metric is as follows,

$$\rho \sim \rho_0 \ll \rho_0 \cosh^{\frac{1}{2}} \theta$$

In this limit, $\theta \to \infty$ and the function $H(\rho)$ can be approximated as $H(\rho) \approx \rho_1^4/\rho^4$, but $G(\rho)$ remains unchanged.

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- The metric then reduces to

$$ds^2 = \frac{\rho^2}{\rho_1^2} G(\rho)^{\frac{1}{4}-\frac{\delta_2}{8}} \left[-G(\rho)^{\frac{\delta_2}{2}} dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right] + \frac{\rho_1^2}{\rho^2} \frac{d\rho^2}{G(\rho)} + \rho_1^2 d\Omega_5^2$$

where $\rho_1 = \rho_0 \cosh^{\frac{1}{2}} \theta$ is the radius of the transverse 5-sphere which decouples from the five dimensional asymptotically AdS_5 geometry.

HEE and Entanglement Thermodynamics

- The form of metric we deal with here is an asymptotically AdS_5 metric of the form

$$ds^2 = \frac{\rho^2}{\rho_1^2} G(\rho)^{\frac{1}{4} - \frac{\delta_2}{8}} \left[-G(\rho)^{\frac{\delta_2}{2}} dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right] + \frac{\rho_1^2}{\rho^2} \frac{d\rho^2}{G(\rho)}$$

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- The asymptotic limit is $\rho \to \infty$, $G(\rho) \to 1$ where the metric reduces to AdS_5 form.
- After reducing our metric to FG form, a simple coordinate transformation, and choosing the embedding $x^1 = x^1(z)$ the spacelike part takes the form

$$ds^2 = \frac{\rho_1^2}{z^2} \left[\left(1 - \frac{\delta_2}{8} \frac{z^4}{z_0^4}\right) \sum_{i=2,3} (dx^i)^2 + dz^2 \left\{ 1 + \left(1 - \frac{\delta_2}{8} \frac{z^4}{z_0^4}\right) x_1'^2(z) \right\} \right]$$

where $z_0^4 = \rho_1^8 / \rho_0^4$.

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- We can now write the area integral as

$$A = \int\int\int dx_2 dx_3 dz \frac{\rho_1^3}{z^3}\left(1 - \frac{\delta_2}{8}\frac{z^4}{z_0^4}\right)\sqrt{1 + \left(1 - \frac{\delta_2}{8}\frac{z^4}{z_0^4}\right)x_1'^2(z)}$$

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and minimize the area integral using the Euler-Lagrange equations.

- Now using the minimized area integral, we get the entanglement entropy to be

$$S_E = S_{E(0)} + \frac{\rho_1^3 L^2}{4G_{(5)}}\int_0^{z_*} dz\left[\frac{\frac{(-3\delta_2)z^4}{8z_0^4}}{z^3\sqrt{1-\frac{z^6}{z_*^6}}} + \frac{\frac{\delta_2 z^4}{8z_0^4}\sqrt{1-\frac{z^6}{z_*^6}}}{z^3}\right]$$

- Using the turning point (z_*) value,

$$z_* = \frac{\ell \Gamma\left(\frac{1}{6}\right)}{2\sqrt{\pi}\Gamma\left(\frac{2}{3}\right)}$$

the change in entanglement entropy is found to be

$$\Delta S_E = \frac{(-\delta_2)\rho_1^3 L^2 \ell^2}{320\sqrt{\pi}G_{(5)}z_0^4} \frac{\Gamma^2\left(\frac{1}{6}\right)\Gamma\left(\frac{1}{3}\right)}{\Gamma^2\left(\frac{2}{3}\right)\Gamma\left(\frac{5}{6}\right)}.$$

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- Putting the AdS black hole limit, we get the change of entanglement entropy to match with the black hole result exactly.

$$\Delta S_E = \frac{\rho_1^3 L^2 \ell^2}{160\sqrt{\pi} G_{(5)} z_0^4} \frac{\Gamma^2\left(\frac{1}{6}\right)\Gamma\left(\frac{1}{3}\right)}{\Gamma^2\left(\frac{2}{3}\right)\Gamma\left(\frac{5}{6}\right)}$$

Entanglement thermodynamics

- By looking at the Fefferman-Graham form of the metric, we extract the boundary stress tensor components using the relation,

$$\langle T^{(d+1)}_{\mu\nu}\rangle = \frac{(d+1)\rho_1^d}{16\pi G_{(d+2)}} h^{(d+1)}_{\mu\nu}$$

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- The stress tensor for the boundary theory of 'black' non-susy D3 brane is,

$$\langle T_{tt} \rangle = \frac{-3\rho_1^3\delta_2}{32\pi G_{(5)}}, \qquad \langle T_{x_i x_j} \rangle = \frac{-\rho_1^3\delta_2}{32\pi G_{(5)}}\delta_{ij}$$

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- Using these values, we can write the change of HEE as,

$$\Delta S_E = \frac{L^2 \ell^2 \sqrt{\pi}}{24} \frac{\Gamma^2\left(\frac{1}{6}\right)\Gamma\left(\frac{1}{3}\right)}{\Gamma^2\left(\frac{2}{3}\right)\Gamma\left(\frac{5}{6}\right)} \left[\langle T_{tt} \rangle - \frac{3}{5} \langle T_{x_1 x_1} \rangle \right]$$

- The change of HEE can then be written as

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- Final form of entanglement thermodynamics

$$\Delta E = T_E \Delta S_E + \frac{3}{5}\Delta P_{x_1x_1} V_3$$

where, entanglement temperature T_E is,

$$T_E = \frac{24\Gamma\left(\frac{5}{6}\right)\Gamma^2\left(\frac{2}{3}\right)}{\ell\sqrt{\pi}\Gamma\left(\frac{1}{3}\right)\Gamma^2\left(\frac{1}{6}\right)}$$

Entropy Cross Over at high temperature

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- In this case, there is no such Bekenstein-Hawking kind of entropy defined a priori as there isn't any event horizon present for the general solution. We take the high temperature limit of the HEE and try to get a feel of whether the high temperature HEE can give us some notion of thermal entropy.
- The high temperature limit of HEE is the limit $z_* \to z_0$. This corresponds to increasing z_* and thus ℓ to bigger value where it covers most part of the system and normally converges with thermal entropy.

- The subsystem size integral can be written in general as,

$$\frac{\ell}{2} = z_* \int_0^1 \frac{x^3}{\sqrt{1-x^6}} \left[1 + \frac{-\frac{3\delta_2 z_*^4}{8z_0^4} + \frac{3\delta_2 z_*^4 x^4}{8z_0^4}}{1-(\frac{z}{z_*})^6} + \frac{\delta_2 z_*^4 x^4}{8z_0^4} \right] dx = z_* \mathcal{I}\left(\frac{z_*}{z_0}\right)$$

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- Similarly, the area integral can be written in the form,

$$A_{min} = \frac{2\rho_1^3 L^2}{z_*^2} \int_0^1 \frac{dx}{x^3} \sqrt{\frac{\left(1 - \frac{5\delta_2}{8}\frac{z_*^4}{z_0^4}x^4\right)}{\left(1-\frac{3\delta_2}{8}\frac{z_*^4}{z_0^4}x^4\right) - \left(1-\frac{3\delta_2}{8}\frac{z_*^4}{z_0^4}\right)x^6}}$$

$$= \frac{2\rho_1^3 L^2}{z_*^2} \tilde{\mathcal{I}}\left(\frac{z_*}{z_0}\right)$$

- In the high temperature limit $z_* \to z_0$, both the integrals $\mathcal{I}$ and $\tilde{\mathcal{I}}$ are dominated by the pole at $x = 1$, i.e., in this limit

$$\mathcal{I}\left(\frac{z_*}{z_0}\right) \approx \tilde{\mathcal{I}}\left(\frac{z_*}{z_0}\right)$$

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- After replacing $\tilde{\mathcal{I}}\left(\frac{z_*}{z_0}\right)$ by $\mathcal{I}\left(\frac{z_*}{z_0}\right)$, the entanglement entropy in high temperature limit reduces to

$$S_E = \frac{\mathrm{Area}(\gamma_A^{\mathrm{min}})}{4G_{(5)}} = \frac{\rho_1^3 L^2 \ell}{4G_{(5)} z_*^3} = \frac{\pi^3 \rho_1^3 V_3}{4G_{(5)} (\pi z_0)^3}$$

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- The HEE of the nonsusy "black" D3 brane in high temperature is found to have a cross over with the thermal entropy of the standard AdS5 BH.

Conclusion and Outlook

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- The form of the Entanglement thermodynamics is unchanged upto first order in case of Nonsupersymmetric solution.
- The zero temperature nonsusy D3 brane carries the same amount of information as the pure AdS case as per as quantified by HEE.
- The entropy cross over between ‘black’ nonsusy D3 brane and the standard black brane in high temperature hints a possible crossover between the physics of the two.

- We have also computed the 2nd order change in subregion complexity (leading order), from which we can comment on the quantum Fisher information of the dual nonconformal, nonsupersymmetric QFT. It also seems that the Fisher information metric is quite a robust and universal quantity independent of the supersymmetry of the underlying theory,

Based on:

- 1. Holographic Entanglement Entropy and Entanglement Thermodynamics of ‘black’ Non-Susy D3 brane
Aranya Bhattacharya, Shibaji Roy.
arXiv: 1712.03740
Published in Phys.Lett. B781 (2018) 232-237.

- 2. Holographic Entanglement Entropy, Subregion Complexity and Fisher Information Metric of "black" Non-Susy D3 Brane
Aranya Bhattacharya, Shibaji Roy.
arXiv:1807.06361.

Thank you for your attention !!