Interacting Abelian 1-Form Gauge Theory: (Anti-)Chiral Superfield Approach

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This Talk is Based on

(Anti-)Chiral Superfield Approach to Interacting Abelian 1-Form Gauge Theories: Nilpotent and Absolutely Anticommuting Charges

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Outline of the Talk

- Introduction
- Lagrangian Density and It's Symmetries
- (Anti-)Chiral Superfield Formalism
- Conserved (anti-)BRST Charges: Nilpotency and Absolute Anticommutativity Properties
- Summary

Interacting Abelian 1-Form Gauge Theory with Dirac Fields: Lagrangian Density

We begin with the *interacting* Abelian 1-form gauge theory where there is a coupling between the $U(1)$ gauge field (A_μ) and the Dirac fields $(\bar{\psi}, \psi)$. The Lagrangian density for this system is as follow:

$$\mathcal{L}_B = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}\,(i\,\gamma^\mu D_\mu - m)\,\psi + B\,(\partial \cdot A) + \frac{B^2}{2} - i\,\partial_\mu \bar{C}\,\partial^\mu C.$$

Where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor and $D_\mu \psi = \partial_\mu \psi + i\,e\,A_\mu\,\psi$ is the covariant derivative on the Dirac field ψ.

(Anti-)BRST Symmetries

- BRST: Becchi-Rouet-Stora-Tyutin
- The (anti-)BRST symmetry transformations for the theory are:

$$
\begin{array}{llll}
s_b A_\mu = \partial_\mu C, & s_b C = 0, & s_b \bar{C} = i\, B, & s_b B = 0, \\
s_b \bar{\psi} = -\, i\, e\, \bar{\psi}\, C, & s_b \psi = -\, i\, e\, C\, \psi, & s_b F_{\mu\nu} = 0, & s_b (\partial \cdot A) = \Box C, \\
s_{ab} A_\mu = \partial_\mu \bar{C}, & s_{ab} \bar{C} = 0, & s_{ab} C = -\, i\, B, & s_{ab} B = 0, \\
s_{ab} \bar{\psi} = -\, i\, e\, \bar{\psi}\, \bar{C}, & s_{ab} \psi = -\, i\, e\, \bar{C}\, \psi, & s_{ab} F_{\mu\nu} = 0, & s_{ab} (\partial \cdot A) = \Box \bar{C}.
\end{array}
$$

- Under the above (anti-)BRST transformations, the kinetic term of the theory remains invariant i.e. $s_{(a)b}\, [-\, \frac{1}{4}\, F_{\mu\nu}\, F^{\mu\nu}] = 0.$

Invariance of the Lagrangian Density

- Under the (anti-)BRST symmetry transformations, the Lagrangian density transforms to the total spacetime derivatives, as:

$$s_{ab}\, \mathcal{L}_B = \partial_\mu[B\, \partial^\mu \bar{C}], \qquad s_b\, \mathcal{L}_B = \partial_\mu[B\, \partial^\mu C],$$

- Thus, the action integral $S = \int d^D x\, \mathcal{L}_B$ remains invariant.

- Symmetries are TRUE in any arbitrary dimension of spacetime.

Conserved (Anti-)BRST Currents

- The conserved currents, due to Noether's theorem, are:

$$J_b^\mu = -\, F^{\mu\nu}\partial_\nu C + B\,\partial^\mu C - e\,\bar\psi\,\gamma^\mu\,C\,\psi,$$
$$J_{ab}^\mu = -\, F^{\mu\nu}\partial_\nu \bar C + B\,\partial^\mu \bar C - e\,\bar\psi\,\gamma^\mu\,\bar C\,\psi.$$

- The Euler-Lagrange (EL) equations of motion (EOM):

$$\partial_\mu F^{\mu\nu} - \partial^\nu B = e\,\bar\psi\,\gamma^\nu\,\psi, \qquad \Box C = 0, \qquad \Box \bar C = 0,$$
$$(i\,\gamma^\mu\partial_\mu - m)\,\psi = e\,\gamma^\mu\,A_\mu\,\psi, \qquad B = -\,(\partial\cdot A),$$
$$i\,(\partial_\mu\bar\psi)\,\gamma^\mu + m\bar\psi = -\,e\,\bar\psi\,\gamma^\mu\,A_\mu.$$

- The conservation law $(\partial_\mu\,J^\mu_{(a)b} = 0)$ can be proven by using the EL-EOMs.

Conserved (Anti-)BRST Charges

- The conserved BRST and anti-BRST charges are:

$$\begin{aligned} Q_b &= \int d^{D-1}x\,[-F^{0i}\partial_i C + B\,\dot{C} - e\,\bar{\psi}\,\gamma^0 C\,\psi], \\ &\equiv \int d^{D-1}x\,[B\,\dot{C} - \dot{B}C], \\ Q_{ab} &= \int d^{D-1}x\,[-F^{0i}\partial_i \bar{C} + B\,\dot{\bar{C}} - e\,\bar{\psi}\,\gamma^0 \bar{C}\,\psi], \\ &\equiv \int d^{D-1}x\,[B\,\dot{\bar{C}} - \dot{B}\bar{C}], \end{aligned}$$

- The above conserved charges are nilpotent $(Q^2_{(a)b} = 0)$ of order two and absolutely anticommuting $(Q_b Q_{ab} + Q_{ab} Q_b = 0)$ in nature.

Motivation

Is there any way to derive these symmetries?

Yes: (Anti-)chiral Superfield Formalism !!

Basic Features of the Superfield Formalism

- The ordinary spacetime manifold is generalized to an appropriate supermanifold.
- The supermanifold is parametrized by the ordinary spacetime coordinates x^μ and the Grassmannian variables $(\theta, \bar{\theta})$.
- The basic fields defined on the ordinary spacetime are generalized onto the corresponding superfields defined the supermanifold.
- We use the (anti-)BRST invariant quantities by using superfield formalism to derive the (anti-)BRST symmetries.

Superfield Formalism

- $\Phi(x)$ $\longrightarrow$ $\tilde{\Phi}(x, \theta, \bar{\theta})$

  Field — Superfield

  e.g. (D): Minkowski Spacetime — (D, 2)-dimensional: Supermanifold

- Algebra of Grassmannian variables and their derivatives

$$\theta\,\bar{\theta} + \bar{\theta}\,\theta = 0, \qquad \theta^2 = 0, \qquad \bar{\theta}^2 = 0,$$
$$\partial_\theta\,\partial_{\bar{\theta}} + \partial_{\bar{\theta}}\,\partial_\theta = 0, \qquad {\partial_\theta}^2 = 0, \qquad {\partial_{\bar{\theta}}}^2 = 0. \qquad (1)$$

- Algebra of BRST (s_b) and anti-BRST (s_{ab}) symmetries

$$s_b\,s_{ab} + s_{ab}\,s_b = 0, \qquad {s_b}^2 = 0, \qquad {s_{ab}}^2 = 0. \qquad (2)$$

- Connection between (1) and (2) is established by superfield approach

$$s_b \longrightarrow \partial_{\bar{\theta}}\,|_{\theta=0}, \qquad s_{ab} \longrightarrow \partial_\theta\,|_{\bar{\theta}=0}\,.$$

Bonora and Tonin Phys. Lett. B. (1981)

Superfield Formalism

- Connection and Geometrical Interpretation

$$\tilde{\Phi}(x,\theta,\bar{\theta}) = \Phi(x) + \theta\left(\frac{\partial}{\partial\theta}\,\tilde{\Phi}(x,\theta,\bar{\theta})\right)\Big|_{\bar{\theta}=0} + \bar{\theta}\left(\frac{\partial}{\partial\bar{\theta}}\,\tilde{\Phi}(x,\theta,\bar{\theta})\right)\Big|_{\theta=0} + \theta\,\bar{\theta}\left(\frac{\partial^2}{\partial\theta\partial\bar{\theta}}\,\tilde{\Phi}(x,\theta,\bar{\theta})\right)$$

$$\equiv \Phi(x) + \theta\,(s_{ab}\,\Phi(x)) + \bar{\theta}\,(s_b\,\Phi(x)) + \theta\bar{\theta}\,(s_b s_{ab}\,\Phi(x)),$$

$$s_b \longleftrightarrow \frac{\partial}{\partial\bar{\theta}}\Big|_{\theta=0}, \qquad s_{ab} \longleftrightarrow \frac{\partial}{\partial\theta}\Big|_{\bar{\theta}=0}.$$

Bonora and Tonin Phys. Lett. B. (1981)

(Anti-)Chiral Superfield Formalism

- Limiting case of BT superfield approach

$$\Phi(x, \theta, \bar{\theta}) \mid_{\bar{\theta}=0} \quad \longrightarrow \quad \tilde{\Phi}(x, \theta)$$

Superfield → Chiral Superfield

$$\Phi(x, \theta, \bar{\theta}) \mid_{\theta=0} \quad \longrightarrow \quad \tilde{\Phi}(x, \bar{\theta})$$

Superfield → Anti-Chiral Superfield

e.g. (D, 2)-dimensional: Supermanifold → (D, 1)-dimensional: Super-submanifolds

Objective

To take the (anti-)chiral version of superfield formalism and to derive the (anti-)BRST symmetry transformations for the interacting Abelian 1-form gauge theory with Dirac fields.

Superfield Approach to BRST Formalism

- The usual superfield approach (USFA) to BRST formalism exploits the idea of **horizontality condition** which leads to the derivation of the (anti-)BRST symmetries associated with the gauge field and (anti-)ghost fields.
- The USFA sheds light on the **geometrical meaning** of the nilpotency and absolute anticommutativity properties of the (anti-)BRST symmetries and corresponding conserved charges.
- These observations are true for a given p-form $(p = 1, 2, 3, ...)$ gauge theory where there is **no** interaction with matter field(s).

Superfield Approach to BRST Formalism

- The USFA has been systematically generalized to derive the (anti-)BRST symmetry transformations for the **matter**, gauge and (anti-)ghost fields **together**. This generalized version of superfield approach to BRST formalism is known as the **augmented superfield approach** (ASFA) to BRST formalism **(BHU Group)**.

- In our present endeavor, we consider (anti-)chiral superfield approach (ACSA) to BRST formalism in which a given D-dimensional gauge theory is generalized onto the (D, 1)-dimensional (anti-)chiral super-submanifolds of the **general** (D, 2)-dimensional supermanifold.

Nilpotent BRST Symmetries: Anti-Chiral Superfield Approach

We generalize the **ordinary** fields of the Lagrangian density onto (D, 1)-dimensional anti-chiral super-submanifold (of the (D, 2)-dimensional supermanifold)) as follows:

$$
\begin{aligned}
A_\mu(x) &\longrightarrow B_\mu(x,\bar\theta) = A_\mu(x) + \bar\theta\, R_\mu(x),\\
C(x) &\longrightarrow F(x,\bar\theta) = C(x) + i\,\bar\theta\, B_1(x),\\
\bar C(x) &\longrightarrow \bar F(x,\bar\theta) \quad = \bar C_(x) + i\,\bar\theta\; B_2(x),\\
\psi(x) &\longrightarrow \Psi(x,\bar\theta) = \psi(x) + i\,\bar\theta\, b_1(x),\\
\bar\psi(x) &\longrightarrow \bar\Psi(x,\bar\theta) \quad = \bar\psi(x) + i\,\bar\theta\, b_2(x),\\
B(x) &\longrightarrow \tilde B(x,\bar\theta) = B(x) + i\,\bar\theta\, f(x),
\end{aligned}
$$

where the fields $(R_\mu,\ f)$ are the fermionic secondary fields and (B_1, B_2, b_1, b_2) are the bosonic secondary fields.

Nilpotent BRST Symmetries: Anti-Chiral Superfield Approach

According to the basic tenets of ACSA to BRST formalism, the BRST invariant quantities **must** remain independent of the Grassmannian variable $(\bar{\theta})$. Therefore, the appropriate BRST invariant quantities are:

$$s_b C = 0, \qquad s_b B = 0, \qquad s_b(\bar{\psi}\,\psi) = 0, \qquad s_b(\bar{\psi} D_\mu \psi) = 0,$$
$$s_b(A^\mu\,\partial_\mu C) = 0, \quad s_b(C\,\psi) = 0, \quad s_b(\bar{\psi}\,C) = 0,$$
$$s_b[A^\mu\,\partial_\mu B + i\,\partial_\mu \bar{C}\,\partial^\mu C] = 0,$$

Nilpotent BRST Symmetries: Anti-Chiral Superfield Approach

The above BRST invariant quantities are to be generalized onto (D, 1)-dimensional anti-chiral super-submanifold with the following restrictions:

$$F(x,\bar{\theta}) = C(x), \quad \bar{\Psi}(x,\bar{\theta})\, F(x,\bar{\theta}) = \bar{\psi}(x)\, C(x),$$
$$\bar{\Psi}(x,\bar{\theta})\, \Psi(x,\bar{\theta}) = \bar{\psi}(x)\, \psi(x), \quad \tilde{B}(x,\bar{\theta}) = B(x),$$
$$B^\mu(x,\bar{\theta})\, \partial_\mu F(x,\bar{\theta}) = A^\mu(x)\partial_\mu C(x),$$
$$F(x,\bar{\theta})\, \Psi(x,\bar{\theta}) = C(x)\, \psi(x),$$
$$\bar{\Psi}(x,\bar{\theta})\, \partial_\mu \Psi(x,\bar{\theta}) + i\, e\, \bar{\Psi}(x,\bar{\theta})\, B_\mu(x,\bar{\theta})\Psi(x,\bar{\theta}) =$$
$$\bar{\psi}(x)\partial_\mu \psi(x) + i\, e\, \bar{\psi}(x)\, A_\mu(x)\, \psi(x),$$
$$B^\mu(x,\bar{\theta})\, \partial_\mu \tilde{B}(x,\bar{\theta}) + i\, \partial_\mu\, \bar{F}(x,\bar{\theta})\, \partial^\mu F(x,\bar{\theta}) =$$
$$A^\mu(x)\, \partial_\mu B(x) + i\, \partial_\mu \bar{C}(x)\, \partial^\mu C(x).$$

Nilpotent BRST Symmetries: Anti-Chiral Superfield Approach

The above restrictions lead to the derivation of the expressions for the secondary fields of the super expansions in terms of the basic and auxiliary fields as:

$$R_\mu = \partial_\mu C, \qquad B_1(x) = 0, \qquad B_2(x) = B(x),$$
$$b_1 = -\, e\, C\, \psi, \qquad b_2 = -\, e\, \bar{\psi}\, C, \qquad f(x) = 0.$$

Nilpotent BRST Symmetries: Anti-Chiral Superfield Approach

Thus, finally, we have the following super expansions for **all** the superfields of our theory, namely;

$$\begin{aligned}
B_\mu^{(b)}(x,\bar\theta) &= A_\mu(x) + \bar\theta\,(\partial_\mu C) &&\equiv A_\mu(x) + \bar\theta\,(s_b\,A_\mu(x)),\\
F^{(b)}(x,\bar\theta) &= C(x) + \bar\theta\,(0) &&\equiv C(x) + \bar\theta\,(s_b\,C(x)),\\
\bar F^{(b)}(x,\bar\theta) &= \bar C(x) + \bar\theta\,(i\,B) &&\equiv \bar C(x) + \bar\theta\,(s_b\,\bar C(x)),\\
\Psi^{(b)}(x,\bar\theta) &= \psi(x) + \bar\theta\,(-\,i\,e\,C\,\psi) &&\equiv \psi(x) + \bar\theta\,(s_b\,\psi(x)),\\
\bar\Psi^{(b)}(x,\bar\theta) &= \bar\psi(x) + \bar\theta\,(-\,i\,e\,\bar\psi\,C) &&\equiv \bar\psi(x) + \bar\theta\,(s_b\,\bar\psi(x)),\\
\tilde B^{(b)}(x,\bar\theta) &= B(x) + \bar\theta\,(0) &&\equiv B(x) + \bar\theta\,(s_b\,B(x)),
\end{aligned}$$

where the superscript (b) on the anti-chiral superfields denotes that these superfields have been obtained after the use of BRST invariant quantities.

Nilpotent Anti-BRST Symmetries: Chiral Superfield Approach

We derive the nilpotent anti-BRST symmetry transformations by using the ACSA to BRST formalism. For this, we generalize the **ordinary** fields of Lagrangian density onto (D, 1)-dimensional chiral super submanifold as:

$$\begin{aligned}
A_\mu(x) &\longrightarrow B_\mu(x,\theta) = A_\mu(x) + \theta\,\bar{R}_\mu(x),\\
C(x) &\longrightarrow F(x,\theta) = C(x) + i\,\theta\,\bar{B}_1(x),\\
\bar{C}(x) &\longrightarrow \bar{F}(x,\theta) = \bar{C}_(x) + i\,\theta\,\bar{B}_2(x),\\
\psi(x) &\longrightarrow \Psi(x,\theta) = \psi(x) + i\,\theta\,\bar{b}_1(x),\\
\bar{\psi}(x) &\longrightarrow \bar{\Psi}(x,\theta) = \bar{\psi}(x) + i\,\theta\,\bar{b}_2(x),\\
B(x) &\longrightarrow \tilde{B}(x,\theta) = B(x) + i\,\theta\,\bar{f}(x),
\end{aligned}$$

where the secondary fields $(\bar{R}_\mu, \bar{f})$ are fermionic and secondary fields $(\bar{B}_1, \bar{B}_2, \bar{b}_1, \bar{b}_2)$ are bosonic in nature.

Nilpotent Anti-BRST Symmetries: Chiral Superfield Approach

According to the basic tenets of ACSA to BRST formalism, the anti-BRST invariant quantities **must** remain independent of the Grassmannian variable (θ). Therefore, we note the appropriate anti-BRST invariant quantities are:

$$
\begin{aligned}
& s_{ab}\,\bar{C} = 0, \quad s_{ab} B = 0, \quad s_{ab}\,(\bar{\psi}\,\psi) = 0, \quad s_{ab}\,(\bar{\psi}\,D_\mu \psi) = 0, \\
& s_{ab}\,(A^\mu \partial_\mu \bar{C}) = 0, \quad s_{ab}\,(\bar{C}\,\psi) = 0, \quad s_{ab}\,(\bar{\psi}\,\bar{C}) = 0, \\
& s_{ab}\,(A^\mu\,\partial_\mu B + i\,\partial_\mu \bar{C}\,\partial^\mu C) = 0.
\end{aligned}
$$

Nilpotent Anti-BRST Symmetries: Chiral Superfield Approach

The above anti-BRST invariant quantities are generalized onto the chiral super-submanifold. In other words, we have the following equalities:

$$
\begin{aligned}
&\bar{F}(x,\theta) = \bar{C}(x), \qquad \bar{\Psi}(x,\theta)\,\Psi(x,\theta) = \bar{\psi}(x)\,\psi(x),\\
&\bar{\Psi}(x,\bar{\theta})\,\bar{F}(x,\bar{\theta}) = \bar{\psi}(x)\,\bar{C}(x),\;\; \bar{F}(x,\theta)\,\Psi(x,\theta) = \bar{C}(x)\,\psi(x),\\
&B^\mu(x,\theta)\,\partial_\mu \bar{F}(x,\theta) = A^\mu(x)\partial_\mu \bar{C}(x), \quad \tilde{B}(x,\theta) = B(x),\\
&\bar{\Psi}(x,\theta)\,\partial_\mu \Psi(x,\theta) + i\,e\,\bar{\Psi}(x,\theta)\,B_\mu(x,\theta)\,\Psi(x,\theta) =\\
&\bar{\psi}(x)\,\partial_\mu \psi(x) + i\,e\,\bar{\psi}(x)\,A_\mu(x)\,\psi(x),\\
&B^\mu(x,\theta)\,\partial_\mu \tilde{B}(x,\theta) + i\,\partial_\mu \bar{F}(x,\theta)\,\partial^\mu F(x,\theta) =\\
&A^\mu(x)\,\partial_\mu B(x) + i\,\partial_\mu \bar{C}(x)\,\partial^\mu C(x).
\end{aligned}
$$

Nilpotent Anti-BRST Symmetries: Chiral Superfield Approach

The above restrictions lead to the derivation of the secondary fields in terms of the basic and auxiliary fields of the Lagrangian density as follows:

$$\bar{R}_\mu = \partial_\mu \bar{C}, \qquad \bar{b}_1 = -\, e\, \bar{C}\, \bar{\psi}, \qquad \bar{b}_2 = -\, e\, \bar{\psi}\, \bar{C},$$
$$\bar{f}(x) = 0, \qquad \bar{B}_1 = -\, B, \qquad \bar{B}_2 = 0.$$

Nilpotent Anti-BRST Symmetries: Chiral Superfield Approach

Thus, finally, we have the following super expansions for **all** the superfields of our theory, (on the chiral (D, 1)-dimensional super submanifold), namely;

$$
\begin{aligned}
B_\mu^{(ab)}(x,\theta) &= A_\mu(x) + \theta\,(\partial_\mu \bar{C}) &&\equiv A_\mu(x) + \theta\,(s_{ab}\,A_\mu(x)),\\
F^{(ab)}(x,\theta) &= C(x) + \theta\,(-\,i\,B) &&\equiv C(x) + \theta\,(s_{ab}\,C(x)),\\
\bar{F}^{(ab)}(x,\theta) &= \bar{C}(x) + \theta\,(0) &&\equiv \bar{C}(x) + \theta\,(s_{ab}\,\bar{C}(x)),\\
\Psi^{(ab)}(x,\theta) &= \psi(x) + \theta\,(-\,i\,e\,\bar{C}\,\psi) &&\equiv \psi(x) + \theta\,(s_{ab}\,\psi(x)),\\
\bar{\Psi}^{(ab)}(x,\theta) &= \bar{\psi}(x) + \theta\,(-\,i\,e\,\bar{\psi}\,\bar{C}) &&\equiv \bar{\psi}(x) + \theta\,(s_{ab}\,\bar{\psi}(x)),\\
\tilde{B}^{(ab)}(x,\theta) &= B(x) + \theta\,(0) &&\equiv B(x) + \theta\,(s_{ab}\,B(x)),
\end{aligned}
$$

where the superscript (ab) denotes the super expansions of the chiral superfields after the application of anti-BRST invariant restrictions.

We capture the nilpotency and absolute anticommutativity property of the BRST charges in ACSA to BRST formalism.

$$
\begin{aligned}
Q_b &= \frac{\partial}{\partial\bar\theta}\int d^{D-1}x \left[i\,\dot{\bar F}^{(b)}(x,\bar\theta)F^{(b)}(x,\bar\theta) - i\,\bar F^{(b)}(x,\bar\theta)\dot F^{(b)}(x,\bar\theta)\right] \\
&\equiv \int d\bar\theta \int d^{D-1}x\left[i\dot{\bar F}^{(b)}(x,\bar\theta)F^{(b)}(x,\bar\theta) - i\bar F^{(b)}(x,\bar\theta)\dot F^{(b)}(x,\bar\theta)\right] \\
Q_b &= \frac{\partial}{\partial\theta}\int d^{D-1}x \left[i\,F^{(ab)}(x,\theta)\dot F^{(ab)}(x,\theta)\right] \\
&\equiv \int d\theta \int d^{D-1}x \left[i\,F^{(ab)}(x,\theta)\dot F^{(ab)}(x,\theta)\right],
\end{aligned}
$$

where the superscripts $(a)b$ stand for the (anti-)chiral superfields that have been obtained after the application of (anti-)BRST invariant restrictions.

Thus, the nilpotency and absolute anticommutativity properties of the translational generators $(\partial_\theta, \partial_{\bar\theta})$ imply that

$$\partial_{\bar\theta}\, Q_b = 0 \iff \partial_{\bar\theta}^2 = 0 \iff s_b\, Q_b = -\, i\, \{Q_b, Q_b\} = 0,$$
$$\partial_\theta\, Q_b = 0 \iff \partial_\theta^2 = 0 \iff s_{ab}\, Q_b = -\, i\, \{Q_b, Q_{ab}\} = 0,$$

These show that there is a deep connection between the nilpotency $(\partial_{\bar\theta}^2 = 0)$ of the translational generator $(\partial_{\bar\theta})$ and the nilpotency (i.e. $Q_b^2 = 0$) of the BRST charge (Q_b). It is also interesting to point out that the above anticommutativity is connected with the nilpotency of the translational generators (∂_θ).

We capture the nilpotency and absolute anticommutativity of the anti-BRST charge within the framework of ACSA.

$$
\begin{aligned}
Q_{ab} &= \frac{\partial}{\partial \theta} \int d^{D-1}x \Big[i\, \bar{F}^{(ab)}(x,\theta)\, \dot{F}^{(ab)}(x,\theta) - i\, \dot{\bar{F}}^{(ab)}(x,\theta)\, F^{(ab)}(x,\theta)\Big] \\
&\equiv \int d\theta \int d^{D-1}x \Big[i\, \bar{F}^{(ab)}(x,\theta)\, \dot{F}^{(ab)}(x,\theta) - i\, \dot{\bar{F}}^{(ab)}(x,\theta)\, F^{(ab)}(x,\theta)\Big], \\
Q_{ab} &= \frac{\partial}{\partial \bar{\theta}} \int d^{D-1}x\, \Big[-\, \bar{F}^{(b)}(x,\bar{\theta})\, \dot{\bar{F}}^{(b)}(x,\bar{\theta})\Big] \\
&\equiv \int d\bar{\theta} \int d^{D-1}x\, \Big[-\, \bar{F}^{(b)}(x,\bar{\theta})\, \dot{\bar{F}}^{(b)}(x,\bar{\theta})\Big],
\end{aligned}
$$

where the superscript $(a)b$ stands for the (anti-)chiral superfield that have been obtained after the application of the (anti-)BRST invariant restrictions.

Conserved Anti-BRST Charge: (Anti-)Chiral Superfield Approach

Nilpotency and Absolute Anticommutativity Properties

Thus, the nilpotency and absolute anticommutativity property of the translational generators $(\partial_\theta, \partial_{\bar\theta})$ imply that

$$\begin{aligned} \partial_\theta\, Q_{ab} &= 0 \iff \partial_\theta^2 = 0 \iff s_{ab}\, Q_{ab} = -i\,\{Q_{ab}, Q_{ab}\} = 0. \\ \partial_{\bar\theta}\, Q_{ab} &= 0 \iff \partial_{\bar\theta}^2 = 0 \iff s_b\, Q_{ab} = -i\,\{Q_{ab}, Q_b\} = 0, \end{aligned}$$

The above observations show that there is a deep connection between the nilpotency $(\partial_\theta^2 = 0)$ of the translational generator (∂_θ) and the nilpotency (i.e. $Q_{ab}^2 = 0$) of the anti-BRST charge (Q_{ab}). It is **also** interesting to point out that the above anticommutativity is connected with the nilpotency of the translational generators $(\partial_{\bar\theta})$ along the $\bar\theta$-direction of the anti-chiral super-submanifold.

In the **Ordinary Space**, nilpotency and absolute anticommutativity of the conserved charges can be easily proven due to the following (anti-)BRST **exact** forms of them:

$$Q_b = s_b \int d^{D-1}x \, [i\,\dot{\bar{C}}\,C - i\,\bar{C}\,\dot{C}],$$

$$Q_b = s_{ab} \int d^{D-1}x \, (i\,C\,\dot{C}),$$

$$Q_{ab} = s_{ab} \int d^{D-1}x \, [i\,\bar{C}\,\dot{C} - i\,\dot{\bar{C}}\,C],$$

$$Q_{ab} = s_b \int d^{D-1}x \, (-\,i\,\bar{C}\,\dot{\bar{C}}).$$

Applying the symmetry principle on the fermionic operators, we obtain:

$$
\begin{aligned}
s_b Q_b &= -i\,\{Q_b, Q_b\} = 0 \;\longleftrightarrow\; Q_b^2 = 0 \;\Longleftrightarrow\; {s_b}^2 = 0,\\
s_{ab}\,Q_{ab} &= -i\,\{Q_{ab}, Q_{ab}\} = 0 \;\longleftrightarrow\; Q_{ab}^2 = 0 \;\Longleftrightarrow\; s_{ab}^2 = 0,\\
s_b Q_{ab} &= -i\,\{Q_{ab}, Q_b\} = 0 \;\longleftrightarrow\; \{Q_{ab}, Q_b\} = 0 \;\Longleftrightarrow\; s_b^2 = 0,\\
s_{ab} Q_b &= -i\,\{Q_b, Q_{ab}\} = 0 \;\longleftrightarrow\; \{Q_b, Q_{ab}\} = 0 \;\Longleftrightarrow\; s_{ab}^2 = 0.
\end{aligned}
$$

Summary of the Talk

- We have discussed the nilpotency and absolute anticommutativity properties of the conserved (anti-)BRST charges of the **ordinary** D-dimensional interacting Abelian 1-form gauge theory in the language of the (anti-)chiral superfield approach to BRST formalism.

- The **novel** observation is the proof of the **absolute anticommutativity** of the (anti-)BRST charges **despite** the fact that we have taken into account **only** the (anti-)chiral super expansions of the superfields.

- It is interesting to note that the nilpotency of the BRST and anti-BRST charges is connected with the nilpotency of the translational generators $\partial_{\bar{\theta}}$ and ∂_{θ}, respectively.

Summary of the Talk

- We have established that the absolute anticommutativity of the BRST charge with anti-BRST charge is connected with the nilpotency ($\partial_\theta^2 = 0$) of the translational generator (∂_θ), along the θ-direction of **chiral** super-submanifold.

- On the contrary, the absolute anticommutativity of the anti-BRST charge with BRST charge is connected with the nilpotency ($\partial_{\bar\theta}^2 = 0$) of the translational generator along $\bar\theta$-direction of the **anti-chiral** super-submanifold.

Thank You!!