Superfield Approach to 2D Non-Abelian 1-Form Gauge Theory

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This Talk is Based on

This talk is based on **"Superfield Approach to Nilpotency and Absolute Anticommutativity of Conserved Charges: 2D Non-Abelian 1-Form Gauge Theory"**

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Objective and Result

- We have exploited the Superfield Approach to derive the (anti-)BRST and (anti-)co-BRST symmetries in the case of 2D non-Abelian 1-form gauge theory.

- The derivation of the **(anti-)co-BRST symmetries** is **novel** result within the framework of superfield approach to BRST formalism.

Outline of the Talk

- Introduction
- (Anti-)BRST Lagrangian Densities and their Symmetries
- Horizontality Condition
- Dual Horizontality Condition
- Derivation of (Anti-)BRST and (Anti-)co-BRST symmetries using the application of Superfield Formalism
- Conclusion

Notations and Convention

For any arbitrary vectors P^a and Q^a in the $SU(N)$ Lie algebraic space, we have

- $P \cdot Q = P^a\, Q^a \quad \Longrightarrow$ Dot Product
- $(P \times Q)^a = f^{abc}\, P^b\, Q^c \Longrightarrow$ Cross Product
- $f^{abc} \quad \Longrightarrow$ Structure Constant
- $A_\mu = A_\mu \cdot T, F_{\mu\nu} = F_{\mu\nu} \cdot T$, etc.
- T: Stand for generator for the Lie Algebraic space $SU(N)$
- For $SU(N)$ Lie Algebra, $[T^a,\, T^b] = f^{abc}\, T^c$, etc.
- $a, b, c = 1, 2, 3, ..., (N^2 - 1)$ and
  $\mu, \nu = 0, 1, 2, 3, ..., (D - 1)$

Notations and Convention

- $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\nu + i\,(A_\mu \times A_\nu)^a$, etc.
- $E = F_{01} = -\varepsilon^{\mu\nu}[\partial_\mu A_\nu + \frac{1}{2}\, i\,(A_\mu \times A_\nu)]$, etc.
- 2D, Levi-Civita tensor obey the following properties $\varepsilon_{\mu\nu}\varepsilon^{\mu\nu} = -2!, \varepsilon_{\mu\nu}\varepsilon^{\mu\lambda} = -\,\delta^\lambda_\nu$
- $B + \bar{B} + (\bar{C} \times C)) = 0$: Curci-Ferrari Condition (CF-condition)
- (anti-)BRST symmetries transformations by $s_{(a)b}$ and (anti-)co-BRST symmetries transformations by $s_{(a)d}$

Non-Abelian 1-Form Gauge Theory

Let us begin with the (anti-)BRST invariant (coupled but equivalent) 2D non-Abelian 1-Form Lagrangian densities

$$\begin{aligned}\mathcal{L}_B &= \mathcal{B}\cdot E - \frac{1}{2}\mathcal{B}\cdot\mathcal{B} + B\cdot(\partial_\mu A^\mu) + \frac{1}{2}(B\cdot B + \bar{B}\cdot\bar{B}) \\ &- i\,\partial_\mu\bar{C}\cdot D^\mu C,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\bar{B}} &= \mathcal{B}\cdot E - \frac{1}{2}\mathcal{B}\cdot\mathcal{B} - \bar{B}\cdot(\partial_\mu A^\mu) + \frac{1}{2}(B\cdot B + \bar{B}\cdot\bar{B}) \\ &- i\,D_\mu\bar{C}\cdot\partial^\mu C,\end{aligned}$$

- where $\mathcal{B}$, B and $\bar{B}$ are the Nakanishi-Lautrup type auxiliary fields that have been invoked for various purposes.

- For instance, $\mathcal{B}$ is introduced in the theory to linearize the kinetic term $(-\frac{1}{4}F_{\mu\nu} \cdot F^{\mu\nu} = \frac{1}{2}E \cdot E \equiv \mathcal{B} \cdot E$ - $\frac{1}{2}\mathcal{B} \cdot \mathcal{B})$ and auxiliary fields B and $\bar{B}$ satisfy the Curci-Ferrari restriction:

$$B + \bar{B} + (C \times \bar{C}) = 0$$

- where the (anti-)ghost fields $\bar{C}$ and C are fermionic (i.e. $(C^a)^2 = (\bar{C}^a)^2 = 0, \quad C^a\bar{C}^b + \bar{C}^b C^a = 0, \quad C^a C^b + C^b C^a = 0, \quad \bar{C}^a\bar{C}^b + \bar{C}^b\bar{C}^a = 0, \quad \bar{C}^a C^b + C^b\bar{C}^a = 0,$ etc.) in nature and they are required in the theory for the validity of **unitarity.**

- $D_\mu C = \partial_\mu C + i\,(A_\mu \times C)$ and $D_\mu \bar{C} = \partial_\mu \bar{C} + i\,(A_\mu \times \bar{C})$ are the covariant derivatives on the (anti-)ghost fields in the adjoint representation

(Anti-)BRST Symmetries

- The Lagrangian densities respect the following off-shell nilpotent $(s_{(a)b}^2 = 0)$ (anti-)BRST symmetry transformations $(s_{(a)b})$

$$s_b\, A_\mu = D_\mu C, \quad s_b\, C = -\frac{i}{2}\,(C \times C), \quad s_b\, \bar{C} = i\, B, \quad s_b\, \bar{B} = i\,(\bar{B} \times C),$$
$$s_b\, B = 0, \quad C, \quad s_b\, E = i\,(E \times C), \quad s_b\, \mathcal{B} = i\,(\mathcal{B} \times C), \quad s_b\,(\mathcal{B} \cdot E) = 0,$$
$$s_{ab}\, A_\mu = D_\mu \bar{C}, \quad s_{ab}\, \bar{C} = -\frac{i}{2}\,(\bar{C} \times \bar{C}), \quad s_{ab}\, C = i\, \bar{B}, \quad s_{ab}\, B = i\,(B \times \bar{C}),$$
$$s_{ab}\, \bar{B} = 0, \quad s_{ab}\, E = i\,(E \times \bar{C}), \quad s_{ab}\, \mathcal{B} = i\,(\mathcal{B} \times \bar{C}), \quad s_{ab}\,(\mathcal{B} \cdot E) = 0.$$

- Symmetries are TRUE in any arbitrary dimension of spacetime.
- Under the above (anti-)BRST symmetry transformations, the kinetic term of the theory remains invariant i.e. $s_{(a)b}\,[-\frac{1}{4}\, F_{\mu\nu} \cdot F^{\mu\nu}] = 0.$

Invariance of the Lagrangian Densities

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- The Lagrangian densities $\mathcal{L}_B$ and $\mathcal{L}_{\bar{B}}$ transform under $s_{(a)b}$ as:

$$s_b \mathcal{L}_B = \partial_\mu (B \cdot D^\mu C), \qquad s_{ab} \mathcal{L}_{\bar{B}} = -\partial_\mu (\bar{B} \cdot D^\mu \bar{C}),$$

$$\begin{aligned} s_{ab} \mathcal{L}_B &= -\partial_\mu \left[\{ \bar{B} + (C \times \bar{C}) \} \cdot \partial^\mu \bar{C} \, \right] \\ &+ \{ B + \bar{B} + (C \times \bar{C}) \} \cdot D_\mu \partial^\mu \bar{C}, \end{aligned}$$

$$\begin{aligned} s_b \mathcal{L}_{\bar{B}} &= \partial_\mu \left[\{ B + (C \times \bar{C}) \} \cdot \partial^\mu C \, \right] \\ &- \{ B + \bar{B} + (C \times \bar{C}) \} \cdot D_\mu \partial^\mu C. \end{aligned}$$

Key Highlights

- It should be noted that *both* the Lagrangian densities respect *both* (i.e. BRST and anti-BRST) symmetries on the constrained hypersurface where the CF-condition $(B + \bar{B} + (C \times \bar{C}) = 0)$ is satisfied.
- In other words, we note that $s_b \mathcal{L}_{\bar{B}} = -\partial_\mu [\bar{B} \cdot \partial^\mu C]$ and $s_{ab} \mathcal{L}_B = \partial_\mu [B \cdot \partial^\mu \bar{C}]$ because of the validity of CF-condition.
- As a consequence, the action integrals $S_1 = \int d^2x \, \mathcal{L}_B$ and $S_2 = \int d^2x \, \mathcal{L}_{\bar{B}}$ remain invariant under the (anti-)BRST symmetries on the above hypersurface located in the 2D Minkowskian spacetime manifold.
- It is interesting to point out that the absolute anticommutativity $\{s_b, s_{ab}\} = 0$ is *also* satisfied on the above hypersurface which is defined by the field equation: $B + \bar{B} + (C \times \bar{C}) = 0$.

(Anti-)co-BRST Symmetries

- The Lagrangian densities *also* respect the following off-shell nilpotent ($s^2_{(a)d} = 0$) and absolutely anticommuting $(s_d s_{ad} + s_{ad} s_d = 0)$ (anti-)co-BRST [i.e. (anti-)dual BRST] symmetry transformations $(s_{(a)d})$

$$
\begin{aligned}
& s_{ad} A_\mu = -\varepsilon_{\mu\nu}\partial^\nu C, \; s_{ad} C = 0, \; s_{ad}\bar{C} = i\,\mathcal{B}, \; s_{ad} B = 0, \\
& s_{ad}\bar{B} = 0, \; s_{ad} E = D_\mu \partial^\mu C, \; s_{ad}(\partial_\mu A^\mu) = 0, \quad s_{ad}\mathcal{B} = 0, \\
& s_d A_\mu = -\varepsilon_{\mu\nu}\partial^\nu \bar{C}, \; s_d \bar{C} = 0, \; s_d C = -i\,\mathcal{B}, \; s_d B = 0, \\
& s_d \bar{B} = 0, \; s_d E = D_\mu \partial^\mu \bar{C}, \; s_d(\partial_\mu A^\mu) = 0, \; s_d \mathcal{B} = 0,
\end{aligned}
$$

- Under the (anti-)co-BRST symmetry transformations gauge-fixing $(\partial \cdot A)$ terms remain invariant.
- Symmetries (i.e. $s_{(a)d}$) are TRUE only in $2D$

- Under the above (anti-)co-BRST symmetries $(s_{(a)d})$ transformations, the Lagrangian densities transform as follows

$$s_{ad}\,\mathcal{L}_{\bar{B}} = \partial_\mu[\mathcal{B}\cdot\partial^\mu C], \quad s_d\mathcal{L}_B = \partial_\mu[\mathcal{B}\cdot\partial^\mu \bar{C}],$$

$$s_{ad}\mathcal{L}_B = \partial_\mu[\mathcal{B}\cdot D^\mu C + \varepsilon^{\mu\nu}\bar{C}\cdot(\partial_\nu C\times C)] \\ +i\,(\partial_\mu A^\mu)\cdot(\mathcal{B}\times C),$$

$$s_d\mathcal{L}_{\bar{B}} = \partial_\mu[\mathcal{B}\cdot D^\mu \bar{C} - \varepsilon^{\mu\nu}C\cdot(\partial_\nu \bar{C}\times\bar{C})] \\ +i\,(\partial_\mu A^\mu)\cdot(\mathcal{B}\times\bar{C}).$$

Key Highlights

- It is clear that *both* the Lagrangian densities respect *both* (i.e. co-BRST and anti-co-BRST) fermionic symmetry transformations on a hypersurface where the CF-type restrictions $\mathcal{B} \times C = 0$, $\mathcal{B} \times \bar{C} = 0$ are satisfied.
- We lay emphasis on the observation that absolute anticommutativity $\{s_d, s_{ad}\} = 0$ is satisfied **without** any use of CF-type restrictions $\mathcal{B} \times C = 0$, $\mathcal{B} \times \bar{C} = 0$
- It is interesting to note here that, while proving absolute anticommutativity of (anti-)BRST symmetries we have to invoke CF-type condition.

Motivation

Is there any way to derive these symmetries?

Yes: Superfield Formalism !!

Basic Features of the Superfield Formalism

- The ordinary spacetime manifold is generalized to an appropriate supermanifold.
- The supermanifold is parametrized by the ordinary spacetime coordinates x^μ and the Grassmannian variables $(\theta, \bar{\theta})$.
- The basic fields defined on the ordinary spacetime are generalized onto the corresponding superfields defined the supermanifold.
- We use the (anti-)BRST invariant quantities by using superfield formalism to derive the (anti-)BRST symmetries.

Superfield Formalism

- $\Phi(x) \longrightarrow \tilde{\Phi}(x,\theta,\bar{\theta})$

  Field → Superfield

  e.g. (D): Minkowski Spacetime → (D, 2)-dimensional: Supermanifold

- Algebra of Grassmannian variables and their derivatives

$$\theta\,\bar{\theta} + \bar{\theta}\,\theta = 0, \qquad \theta^2 = 0, \qquad \bar{\theta}^2 = 0,$$
$$\partial_\theta\,\partial_{\bar{\theta}} + \partial_{\bar{\theta}}\,\partial_\theta = 0, \qquad {\partial_\theta}^2 = 0, \qquad {\partial_{\bar{\theta}}}^2 = 0. \tag{1}$$

- Algebra of BRST (s_b) and anti-BRST (s_{ab}) Symmetries

$$s_b\,s_{ab} + s_{ab}\,s_b = 0, \qquad {s_b}^2 = 0, \qquad {s_{ab}}^2 = 0. \tag{2}$$

- Connection between (1) and (2) is established by superfield approach

$$s_b \longrightarrow \partial_{\bar{\theta}}\,|_{\theta=0}, \qquad s_{ab} \longrightarrow \partial_\theta\,|_{\bar{\theta}=0}\,.$$

Bonora and Tonin Phys. Lett. B. (1981)

Superfield Formalism

- Connection and Geometrical Interpretation

$$\tilde{\Phi}(x,\theta,\bar{\theta}) = \Phi(x) + \theta\left(\frac{\partial}{\partial\theta}\,\tilde{\Phi}(x,\theta,\bar{\theta})\right)\Big|_{\bar{\theta}=0} + \bar{\theta}\left(\frac{\partial}{\partial\bar{\theta}}\,\tilde{\Phi}(x,\theta,\bar{\theta})\right)\Big|_{\theta=0} + \theta\,\bar{\theta}\left(\frac{\partial^2}{\partial\theta\partial\bar{\theta}}\,\tilde{\Phi}(x,\theta,\bar{\theta})\right)$$

$$\equiv \Phi(x) + \theta\,(s_{ab}\,\Phi(x)) + \bar{\theta}\,(s_b\,\Phi(x)) + i\,\theta\bar{\theta}\,(s_b s_{ab}\,\Phi(x)),$$

$$s_b \longleftrightarrow \frac{\partial}{\partial\bar{\theta}}\Big|_{\theta=0}, \qquad s_{ab} \longleftrightarrow \frac{\partial}{\partial\theta}\Big|_{\bar{\theta}=0}.$$

Bonora and Tonin Phys. Lett. B. (1981)

Horizontality Condition (HC)

Horizontality Condition (HC): The physical quantities are independent of any mathematical artifact ("soul-flatness" condition: independent of the Grassmannian coordinates $(\theta, \bar{\theta})$).

- The super curvature 2-form is $\tilde{F}^{(2)} = \frac{1}{2}(dZ^M \wedge dZ^N)\tilde{F}_{MN}$
- $Z^M = (x^\mu, \theta, \bar{\theta})$: superspace coordinate
- $\tilde{F}_{MN} = (\tilde{F}_{\mu\nu}, \tilde{F}_{\mu\theta}, \tilde{F}_{\mu\bar{\theta}}, \tilde{F}_{\theta\bar{\theta}}, \tilde{F}_{\theta\theta}, \tilde{F}_{\bar{\theta}\bar{\theta}})(x, \theta, \bar{\theta})$

Horizontality Condition (HC)

- The celebrated horizontality condition (HC) requires that the Grassmannian components of $\tilde{F}_{MN}\,(x,\theta,\bar{\theta}) = (\tilde{F}_{\mu\nu}, \tilde{F}_{\mu\theta}, \tilde{F}_{\mu\bar{\theta}}, \tilde{F}_{\theta\theta}, \tilde{F}_{\theta\bar{\theta}}, \tilde{F}_{\bar{\theta}\bar{\theta}})$ should be set equal to zero so that, ultimately, we should have the following equality, namely;

$$-\frac{1}{4}\,\tilde{F}_{\mu\nu}(x,\theta,\bar{\theta})\cdot\tilde{F}^{\mu\nu}(x,\theta,\bar{\theta}) = -\frac{1}{4}\,F_{\mu\nu}(x)\cdot F^{\mu\nu}(x).$$

- Finally, one observe that the, Gauge invariant quantity (i.e. kinetic term) remains independent of Grassmannian variables.

Superfield Expansions

- The requirement of HC leads to the following

$$R_\mu = D_\mu C, \qquad \bar{R}_\mu = D_\mu \bar{C}, \qquad B_1 = -\frac{1}{2}(C \times C),$$
$$S_\mu = D_\mu B_2 + D_\mu C \times \bar{C} \equiv -D_\mu \bar{B}_1 - D_\mu \bar{C} \times C,$$
$$\bar{B}_2 = -\frac{1}{2}(\bar{C} \times \bar{C}), \quad s = i\,(\bar{B}_1 \times C), \quad \bar{s} = -i\,(B_2 \times \bar{C}),$$
$$\bar{B}_1 + B_2 = -(C \times \bar{C}) \to B + \bar{B} = -(C \times \bar{C}).$$

- The last entry is nothing but the celebrated CF-condition $(B + \bar{B} + (C \times \bar{C}) = 0)$ if we identify $\bar{B}_1 = \bar{B}$ and $B_2 = B$.
- It is crystal clear that the HC leads to the derivation of the secondary fields in terms of the auxiliary and basic fields of the starting Lagrangian densities.

Superfield Expansions

Substituting the values of the secondary fields, we finally obtain the following

$$\begin{aligned}
B_\mu^{(h)}(x,\theta,\bar\theta) &= A_\mu(x) + \theta\,(D_\mu \bar C) + \bar\theta\,(D_\mu C) + \theta\bar\theta\,\left[i\,\left(D_\mu B + \left(D_\mu C \times \bar C\right)\right)\right] \\
&\equiv A_\mu(x) + \theta\,(s_{ab}\,A_\mu) + \bar\theta\,(s_b\,A_\mu) + \theta\bar\theta\,(s_b s_{ab}\,A_\mu), \\
F^{(h)}(x,\theta,\bar\theta) &= C(x) + \theta\,(i\,\bar B) + \bar\theta\,(-\frac{i}{2}\,C \times C) + \theta\bar\theta\,(-\,\bar B \times C) \\
&\equiv C(x) + \theta\,(s_{ab}\,C) + \bar\theta\,(s_b\,C) + \theta\bar\theta\,(s_b s_{ab}\,C), \\
\bar F^{(h)}(x,\theta,\bar\theta) &= \bar C(x) + \theta\,(-\frac{i}{2}\,\bar C \times \bar C) + \bar\theta\,(i\,B) + \theta\bar\theta\,(B \times \bar C) \\
&\equiv \bar C(x) + \theta\,(s_{ab}\,\bar C) + \bar\theta\,(s_b\,\bar C) + \theta\bar\theta\,(s_b s_{ab}\,\bar C).
\end{aligned}$$

- Superscript (h) denotes the expansions of the superfields after application of HC.

Highlights

- **Thus, we see that we have derived all the (anti-)BRST symmetries transformation for the theory using the superfield formalism.**

- **HC (Horizontality Condition) has played a decisive role in the above.**

Dual Horizontality Condition

Q: How to derive the (anti-)co-BRST symmetry transformations ?

Ans. Using dual-HC.

Dual Horizontality Condition (DHC)

- We exploit here the dual-HC (DHC) to derive the (anti-)co-BRST symmetry transformations for the (anti-)ghost fields and gauge field $(A_\mu = A_\mu \cdot T)$ of our 2D non-Abelian theory.
- The gauge-fixing term $(\partial_\mu A^\mu)$ has its origin in the co-exterior derivative $(\delta = - * \; d \; *)$ of the differential geometry in the following sense

$$\delta A^{(1)} = - * \; d \; * (dx^\mu A_\mu) = \partial_\mu A^\mu, \qquad \delta^2 = 0,$$

Dual Horizontality Condition (DHC)

where,

- $\delta = - * \ d \, *$ is co-exterior derivative, $*$ is Hodge duality operator on any ordinary manifold.

- Lorentz gauge-fixing term $(\partial_\mu A^\mu)$ is a 0-form which emerges out from the 1-form $(A^{(1)} = dx^\mu A_\mu)$ due to application of the co-exterior derivative $(\delta = - * \ d \, *)$ which reduces the degree of a form by one.

Dual Horizontality Condition (DHC)

- We know that the gauge-fixing term $(\partial_\mu A^\mu)$ remains invariant under the (anti-)co-BRST symmetry transformations . We generalize this observation onto our chosen (2, 2)-dimensional supermanifold as follows

$$\tilde{\delta}\tilde{A}^{(1)} = \delta A^{(1)}, \qquad \tilde{\delta} = - \star \ \tilde{d} \star, \tilde{\delta}^2 = 0 \qquad \tilde{d}^2 = 0,$$

- where $\tilde{\delta}$ is the super co-exterior derivative defined on the (2, 2)-dimensional supermanifold and $\star$ is the Hodge duality operator on the (2, 2)-dimensional supermanifold
- R. P. Malik, *Int. J. Mod. Phys.* A **21**, 3307 (2006) "**Hodge Duality Operation And Its Physical Applications On Supermanifolds**"

Dual Horizontality Condition (DHC)

- The l.h.s. of above equation has already been computed in our previous work . We quote here the result of operation of $\tilde{\delta}$ on $\tilde{A}^{(1)}$ as 0-form, namely;

$$\tilde{\delta}\tilde{A}^{(1)} = \partial_\mu B^\mu + \partial_\theta \bar{F} + \partial_{\bar{\theta}} F + s^{\bar{\theta}\bar{\theta}}(\partial_{\bar{\theta}} \bar{F}) + s^{\theta\theta}(\partial_\theta F) = \partial_\mu A^\mu,$$

- where $s^{\theta\theta}$ and $s^{\bar{\theta}\bar{\theta}}$ appear in the following Hodge duality $\star$ operation:

$$\star\,(dx_\mu \wedge dx_\nu \wedge d\bar{\theta} \wedge d\bar{\theta}) = \varepsilon_{\mu\nu}\; s^{\bar{\theta}\bar{\theta}},$$
$$\star\,(dx_\mu \wedge dx_\nu \wedge d\theta \wedge d\theta) = \varepsilon_{\mu\nu}\; s^{\theta\theta},$$
$$\star\,(dx_\mu \wedge dx_\nu \wedge d\bar{\theta} \wedge d\bar{\theta}) = \varepsilon_{\mu\nu}\; s^{\bar{\theta}\bar{\theta}},$$
$$\star\,(dx_\mu \wedge dx_\nu \wedge d\theta \wedge d\theta) = \varepsilon_{\mu\nu}\; s^{\theta\theta}.$$

Dual Horizontality Condition (DHC)

- These factors (i.e. $s^{\theta\theta}$, $s^{\bar{\theta}\bar{\theta}}$) are essential to get back the 4-forms $(dx_\mu \wedge dx_\nu \wedge d\theta \wedge d\theta)$ and $(dx_\mu \wedge dx_\nu \wedge d\bar{\theta} \wedge d\bar{\theta})$ if we apply *another* $\star$ on the above.
- The equality in above equation ultimately, leads to

$$\partial_\theta F = 0, \qquad \partial_{\bar{\theta}} \bar{F} = 0, \qquad \partial_\mu B^\mu + \partial_\theta \bar{F} + \partial_{\bar{\theta}} F = \partial_\mu A^\mu,$$

- At this stage, we substitute the expressions of $B_\mu(x, \theta, \bar{\theta})$, $F(x, \theta, \bar{\theta})$ and $\bar{F}(x, \theta, \bar{\theta})$ to derive the following important relationships

Dual Horizontality Condition (DHC)

$$\partial_\mu R^\mu = 0, \quad \partial_\mu \bar{R}^\mu = 0, \quad \partial_\mu S^\mu = 0, \quad s = 0,$$
$$\bar{B}_1 = 0, \quad B_2 = 0, \quad \bar{s} = 0, \quad B_1 + \bar{B}_2 = 0.$$

- The last entry, in the above, is just like the CF-type restriction which is *trivial*. With the choices $B_1 = -\mathcal{B}$ and $\bar{B}_2 = \mathcal{B}$, we obtain the following expansions

$$F^{(dh)}(x, \theta, \bar{\theta}) = C(x) + \bar{\theta}\,(-\,i\mathcal{B}) \equiv C(x) + \bar{\theta}\,(s_d C),$$
$$\bar{F}^{(dh)}(x, \theta, \bar{\theta}) = \bar{C}(x) + \theta\,(\,i\mathcal{B}) \equiv \bar{C}(x) + \theta\,(s_{ad} \bar{C}),$$

Dual Horizontality Condition (DHC)

- For gauge Field A_μ, we have use the following restrictions

$$s_{(a)d}\,[\varepsilon^{\mu\nu} A_\nu \cdot \partial_\mu \mathcal{B} - i\,\partial_\mu \bar{C} \cdot \partial^\mu C] = 0,$$

- Thus, we have the following equality due to Augmented Version of Superfield Approach to BRST formalism:

$$\begin{aligned} & \varepsilon^{\mu\nu} B_\nu(x,\theta,\bar{\theta}) \cdot \partial_\mu \mathcal{B}(x) \\ - \;& i\,\partial_\mu \bar{F}^{(dh)}(x,\theta,\bar{\theta}) \cdot \partial^\mu F^{(dh)}(x,\theta,\bar{\theta}) \\ & \equiv \varepsilon^{\mu\nu} A_\nu(x) \cdot \partial_\mu \mathcal{B}(x) - i\,\partial_\mu \bar{C}(x) \cdot \partial^\mu C(x). \end{aligned}$$

- The substitution of the expansions $F^{(dh)}(x,\theta,\bar{\theta})$ and $\bar{F}^{(dh)}(x,\theta,\bar{\theta})$ yields the following

$$\varepsilon^{\mu\nu} \bar{R}_\nu + \partial^\mu C = 0, \quad \varepsilon^{\mu\nu} R_\nu + \partial^\mu \bar{C} = 0, \; \varepsilon^{\mu\nu} S_\nu - \partial^\mu \mathcal{B} = 0.$$

Dual Horizontality Condition (DHC)

- We have *not* taken any super expansion of $\mathcal{B}(x)$ on the l.h.s. in above equations because of the fact that $s_{(a)d}\mathcal{B}(x) = 0$
- Finally, we get the following results from the above relations:

$$R_\mu = -\,\varepsilon_{\mu\nu}\partial^\nu \bar{C}, \qquad \bar{R}_\mu = -\,\varepsilon_{\mu\nu}\partial^\nu C, \qquad S_\mu = \varepsilon_{\mu\nu}\partial^\nu \mathcal{B}.$$

- Finally, we get the expansions of $B_\mu^{(dg)}(x,\theta,\bar{\theta})$

$$\begin{aligned} B_\mu^{(dg)}(x,\theta,\bar{\theta}) &= A_\mu(x) + \theta\,(\varepsilon_{\mu\nu}\partial^\nu C) + \bar{\theta}\,(-\,\varepsilon_{\mu\nu}\partial^\nu \bar{C}) \\ &+ \theta\,\bar{\theta}\,(\varepsilon_{\mu\nu}\partial^\nu \mathcal{B}) \\ &\equiv A_\mu(x) + \theta\,(s_{ad}A_\mu) + \bar{\theta}\,(s_d A_\mu) \\ &+ \theta\,\bar{\theta}\,(s_d s_{ad} A_\mu). \end{aligned}$$

Dual Horizontality Condition (DHC)

- Here the superscript (dg) on $B_\mu^{(dg)}(x, \theta, \bar{\theta})$ denotes the expansion that has been obtained after the application of (anti-)co-BRST (i.e. dual gauge) invariant restriction.
- **Thus, we see that we have derived all the (anti-)co-BRST symmetry transformations for the theory using the Dual Horizontality Condition.**
- In our work, we have also captured the nilpotency and absolute anticommutativity of (anti-)co-BRST charges.

Conclusions

- The idea of dual-Horizontality is a novel concept where the Hodge duality operator on a supermanifold play a key role.
- The derivation of the proper (anti-)co-BRST symmetry transformations have been **one** of key results.
- We have *also* proven the nilpotency and absolute anticommutativity of the (anti-)BRST and (anti-)co-BRST charges within the framework of superfield formalism.
- **These results, which are connected with the (anti-)co-BRST symmetries transformation, have been accomplished for the first time.**

Thank You