Aspects Of Two Higgs Doublet Models

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Abstract

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Beyond Standard Model physics aims to answer the unanswered questions of the Standard Model and two Higgs doublet model (2HDM) is one of the simplest example of it. Since the LHC Higgs data seem to match the Standard Model predictions, its implication on the 2HDM scenario needs to be considered. This indeed puts constraints on various non-standard couplings. We first dealt with bounds on scalar masses resulting from a criterion of naturalness, in a broad class of two Higgs doublet models. Specifically, we assumed the cancellation of quadratic divergences in what are called the type I, type II, lepton-specific, and flipped two Higgs doublet models, with an additional U(1) symmetry. This resulted in a set of relations among masses of the physical scalars and coupling constants, a generalization of the Veltman conditions of the Standard Model. Playing with the values of α and β we arrived at various limits of 2HDMs namely alignment limit and the reverse alignment limit where the lighter and the heavier CP-even Higgs bosons correspond to the SM Higgs particle respectively and also the limit in which some of the Yukawa couplings of this particle are of opposite sign with respect to the vector boson couplings (wrong sign). For these limits the allowed masses of the remaining physical scalars based on naturalness, stability, perturbative unitarity, and constraints coming from the ρ parameter were determined. We also calculated the $h \to \gamma\gamma$ decay width in the wrong sign limit.

We further investigated the possibility of a Higgs-Higgs bound state in the two Higgs doublet model. We constructed an effective field theory formalism to examine the effect of dimension six operators, generated by new physics at a scale of a few TeV, on the self-couplings of the heavy CP-even scalar field in the model. The magnitudes of the attractive and repulsive coupling strengths were compared to estimate the possibility of the formation of the $H-H$ bound state. Another way to check if a bound state is formed or not is from the formation and decay times of the bound state. The possibilities in various types of two Higgs doublet models have been discussed elaborately.

An investment in knowledge pays the best interest. - Benjamin Franklin

In loving memory of my two grandmothers...

Dedicated to the three relevant women of my life -
my mother, my mother-in-law and my daughter.

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'Guru Brahma Gurur Vishnu Guru Devo Maheshwarah Guru Saakshat Para Brahma Tasmai Sree Gurave Namaha' - I thank my supervisor Prof. Amitabha Lahiri for his guidance and motivation during my PhD days. I got to learn many things from him apart from physics and it has been very pleasant working with him. Next I would like to thank my post graduation professors from University of Calcutta for such exuberant exposure to the dynamic field of High Energy Physics. At this point I would like to share a small story. After my MSc. finals I had almost made up my mind to go for civil service examinations. One fine day Prof. Anirban Kundu summoned me and literally scolded me for taking up such drastic decision. It was he who literally guided me to take up research. Now that I am jotting down my five years of research work in this thesis, I feel very satisfied with my career and thank AK for the rebuke.

A family is a constant source of support for any individual to succeed. I am indebted to my parents for their love, care and guidance throughout my life. My mother and father dedicated their entire lives to make me a successful person in the run of life (though still now I have many miles to go). My little brother always cheered me up with his subtle jokes when I went through many downs in life. During my PhD tenure I met a very wonderful and jovial person who has been very supportive in pursuing my career. He eventually happens to be my husband. He guided me how to cope up with stress and focus on work. The biggest boon is my daughter who just waves her magic wand with her little fingers and impregnates me with all the positivity. I would like to thank my in-laws and my relatives for their blessings.

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List of Publications

This thesis is based on the following:

- **JOURNAL PUBLICATIONS**

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  2. "Alignment, reverse alignment, and wrong sign Yukawa couplings in two Higgs doublet models", **Ambalika Biswas**, Amitabha Lahiri, Phys.Rev. D93 (2016) no.11, 115017 [arXiv:1511.07159 [hep-ph] — PDF].
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- **CONFERENCE PROCEEDINGS**

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  2. "Various perspectives of Two Higgs Doublet models and Naturalness criteria", **Ambalika Biswas**, Amitabha Lahiri, published in PoS ICHEP2016 (2016) 710.

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Abbreviations

SM	**Standard Model**
BSM	**Beyond Standard Model**
2HDM	**Two Higgs Doublet Model**
3HDM	**Three Higgs Doublet Model**
MSSM	**Minimal Supersymmetric Standard Model**
SUSY	**Supersymmetry**
HTM	**Higgs Triplet Model**
GUT	**Grand Unified Theory**
QFT	**Quantum Field Theory**
VEV	**Vacuum Expectation Value**
LHC	**Large Hadron Collider**
FCNC	**Flavour Changing Neutral Current**
NFC	**Natural Flavour Conservation**
MFV	**Minimal Flavour Violation**
GWP	**Glashow-Weinberg-Paschos**
CP	**Charge conjugation and Parity**
GCP	**General Charge conjugation and Parity**
HF	**Higgs Family**
BS	**Bound State**
RGE	**Renormalization Group Equations**
RH	**Right-handed**
LH	**Left-handed**
IR	**Infra-Red**
UV	**Ultra-Violet**

Chapter 1

Is Nature hinting at a wider horizon?

> Natural selection is a mechanism for generating an exceedingly high degree of improbability.
>
> *Ronald Aylmer Fisher*

The theories and discoveries of thousands of physicists since the 1930s have resulted in a remarkable insight into the fundamental structure of matter: everything in the universe is found to be made from a few basic building blocks called fundamental particles, governed by four fundamental forces. Our best understanding of how these particles and three of the forces are related to each other is encapsulated in the Standard Model of particle physics as shown in figure 1.1. Developed in the early 1970s, it has successfully explained almost all experimental results and precisely predicted a wide variety of phenomena. Over time and through many experiments, the Standard Model has become established as a well-tested physics theory. Discovery of all the fundamental particles as proposed by Standard Model was complete except the *Higgs Boson*. On 4 July 2012, the ATLAS [1] and CMS [2] experiments at CERN's Large Hadron Collider announced they had each observed a new particle in the mass region around 125 GeV. This particle is consistent with the Higgs boson predicted by the Standard Model.

Being a well celebrated model let us first have a look into the main principles involved in the foundation of the Standard Model. The three gauge groups $SU(3) \times SU(2) \times U(1)$ form the plinth of the Standard Model. Consequently there are three families of quarks and leptons in the representation 3×2, 3×1, 1×2 and 1×1. Mass is generated through the spontaneous electroweak symmetry breaking. This was formulated by Brout, Englert and Higgs and named Brout-Englert-Higgs mechanism [3–5]. This resulted in three Goldstone bosons and the Higgs boson. In the process the fermions and the gauge bosons gain their masses. Mixing of flavours occur with the help of the Cabibbo-Kobayashi-Maskawa (CKM) [6] and the Pontecorvo-Maki-Nakagava-Sakato (PMNS) [7] matrices and there is CP violation via the phase factors

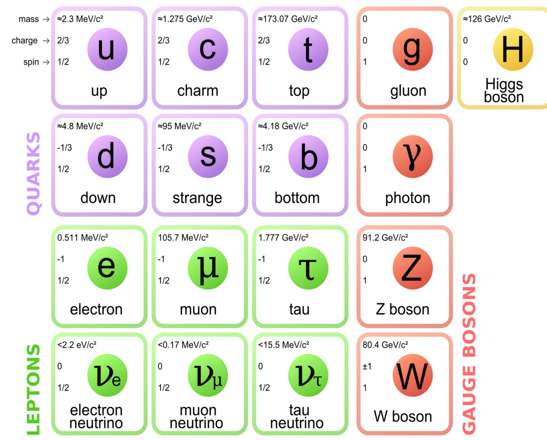


Figure 1.1: The Standard Model.(Src: Google)

in the flavour mixing matrices. Standard Model obeys the conservation of baryon and lepton numbers. Quarks and gluons are confined within the hadrons. CPT invariance accounts for the existence of antimatter.

Based on local quantum field theory the Standard Model is described by a Lagrangian which is built in accordance with the Lorentz invariance and invariance under three gauge groups. Also it is renormalizable which means that it contains only the operators of dimensions 2, 3 and 4 [8]. The Lagrangian density is written below.

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Higgs}}, \tag{1.1}$$

where

$$\begin{aligned}\mathcal{L}_{\text{gauge}} = & - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} - \frac{1}{4}A^i_{\mu\nu}A^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ & + i\overline{L}_\alpha\gamma^\mu D_\mu L_\alpha + i\overline{Q}_\alpha\gamma^\mu D_\mu Q_\alpha + i\overline{l}_\alpha\gamma^\mu D_\mu l_\alpha \\ & + i\overline{U}_\alpha\gamma^\mu D_\mu U_\alpha + i\overline{D}_\alpha\gamma^\mu D_\mu D_\alpha + \left(D_\mu H\right)^\dagger\left(D_\mu H\right) ,\end{aligned} \tag{1.2}$$

$$\mathcal{L}_{\text{Yukawa}} = y^l_{\alpha\beta}\overline{L}_\alpha l_\beta H + y^d_{\alpha\beta}\overline{Q}_\alpha D_\beta H + y^u_{\alpha\beta}\overline{Q}_\alpha U_\beta \tilde{H} + h.c. , \tag{1.3}$$

and

$$\mathcal{L}_{\text{Higgs}} = -V = m^2 H^\dagger H - \frac{\lambda}{2}\left(H^\dagger H\right)^2 . \tag{1.4}$$

Here $\tilde{H} = i\tau_2 H^\dagger$, y's are the Yukawa coupling constant matrices and λ is the Higgs coupling constant. L_α, Q_α, l_β, D_β and U_β are the left-handed lepton, left-handed quark, right-handed charged lepton, right-handed down-type quark and right-handed up-type quark matrices respectively. y and λ are both dimensionless and m which is the mass of the Higgs boson is the only dimensional mass parameter.

The symmetries of the Standard Model fixes all the interactions of the Standard Model fermions namely the quarks and the leptons. These interactions are performed by the exchange of the force carriers. Gluons are the force carriers in the case of strong interactions, W and Z bosons mediate the weak interactions, photons are exchanged in case of electromagnetic interactions and Yukawa interactions are mediated by the Higgs bosons. The only flexibility lies in the choice of the three gauge couplings g_i, the three Yukawa matrices $y^k_{\alpha\beta}$, the Higgs coupling λ, and the mass parameter m. All of them are not predicted by the SM but are measured experimentally. The existence of the right-handed neutrino leads to two additional terms in the Lagrangian viz, a kinetic term and the interaction of the neutrino with the Higgs boson. If the neutrino is a Majorana particle, then the Majorana mass term is added.

Though the Standard Model can precisely predict the masses and the interactions of the elementary particles but it has some drawbacks which we will discuss next. Landau pole problem is one of them. The running couplings of the SM tend to infinity at finite energies (the Landau pole [9]). Landau pole has been observed for the $U(1)$ and the Higgs couplings as discussed in [9]. Since the Landau pole has a wrong sign residue thus non-physical ghost fields appear. This leads to the violation of causality since Landau pole is an intrinsic problem of the theory. It takes place at energies much higher than the Planck mass where we assume quantum gravity might change everything but a theory with the Landau pole is not self consistent.

Stability of the electroweak vacuum is disturbed by radiative corrections rendering it either metastable or even unstable. This also affects the behaviour of the Higgs coupling which crosses zero and then becomes negative at the energies close to 10^{11} GeV as discussed in [10]. The accuracy of the measurement of the top quark and the Higgs boson masses and the order of perturbation theory affects this situation strongly. For higher orders the instability point moves toward higher energies and possibly might reach the Planck scale with increasing accuracy [11]. Beyond SM physics might attempt to change this situation.

The mass of the Higgs boson is not protected by any symmetry unlike the quarks, leptons and the intermediate weak gauge bosons. Thus new physics at the high energy scale might destroy the electroweak scale of the Standard Model due to radiative corrections. The radiative corrections to the mass of the Higgs boson being quadratically dependent on the same, destroy the electroweak scale. Thus there is no clear explanation of why the mass of the Higgs particle is not as large as the scale of any new physics, which may be $\sim 10^{16}$GeV (GUT scale) or even $\sim 10^{19}$GeV (Planck scale). This is known as the *hierarchy* problem. It is to be mentioned here that this is not a problem of the Standard Model itself since the quadratic divergences are absorbed into the redefinition of the bare mass which is not observable. But this instead leads to a quadratic dependence of low energy physics on unknown high energy one and that is not acceptable. The way out of this situation might be a new physics at intermediate energies.

Also it is not clear how some mechanisms inside the SM work. In particular, it is not clear how confinement actually works or how the neutrinos gain mass how small it might be. CP violation of the universe, the quark-hadron phase transition are among other mechanisms that need further explanation. In case all experimental data are not explained by the SM then there is an indication of physics beyond the SM. If a new scale physics is introduced to solve many of the above stated short comings of the SM, then we face the problem of protection of the SM from the new scale physics. There are some more unanswered questions that bother us like : is there another scale except for the electro-weak and the Planck scale? Is SM a self consistent quantum field theory? Moreover the dark matter issue questions the compatibility of the SM with Cosmology.

Introduction of a new physics seems inevitable to resolve these issues. Let us first look at the high energy physics panorama from the point of view of the energy scale. The electroweak scale lies at $\sim 10^2$ GeV and the Planck scale at $\sim 10^{19}$ GeV. The whole spectra of quark, lepton, intermediate vector boson and the Higgs boson masses lie in the electroweak scale. Apart from these two scales there is a scale of quantum chromodynamics at $\Lambda \sim 200$ MeV, the Grand unification scale at $\sim 10^{16}$ GeV and the vacuum stability scale at $\sim 10^{11}$ GeV.

Theory can suggest various ways of development but only experiment can show the right road. So far there has been no experimental evidence that all these high energy scales and new physics related to them exist. Today high energy physics is still hazy and the horizon of knowledge is yet to be expanded. But we hope that sooner or later the persistent efforts of the high energy physicists will clear the fog and the unanswered questions will be resolved. Below we encapsulate the various ways that can be adopted to go beyond the Standard Model.

1. **Extension of the symmetry group of the Standard Model** : If we expand the symmetry group of the SM then we have theories like Supersymmetry (SUSY), Grand Unified Theories (GUT), new $U(1)$ factors, etc. This may solve the problem of the Landau pole, the problem of stability, the hierarchy problem, and also the dark matter problem.

2. **Addition of new particles** : Models with extra generations of matter, extra gauge bosons, extra Higgs bosons, extra neutrinos, etc. have been proposed to this end. This way one may solve the problem of stability and the dark matter problem.

3. **Introduction of extra dimensions of space** : Introducing compact or flat extra dimensions have also been adopted by many physicists. This approach opens a whole new world of possibilities and one may solve the problem of stability and the hierarchy problem and even get a new insight into gravity.

4. **Transition beyond the local Quantum Field Theory** : Theories like string theory, brane world, etc. surfaced to answer questions that Standard Model failed to answer. The main aim of this approach is the unification of gravity with other interactions and the construction of quantum gravity.

We are in a paradoxical situation. Generally a new theory emerges to explain the observables which are not explained by an old existing theory but now we try to construct a new theory and persistently look for experimental data which go beyond the Standard Model. We have not been able to find substantial amount of data so far. The existing small deviations from the Standard Model at the level of a few sigma such as in the forward-backward asymmetries in electron-positron scattering or in the anomalous magnetic moment of muon might be due to uncertainty of the experiment or data processing. The Standard Model gets slightly modified to incorporate the neutrino masses as indicated by the neutrino oscillation experiments. Heavy Majorana neutrinos may account for dark matter, though many models which incorporate extra particles do suggest probable dark matter candidates. Nevertheless, there is a vast field of new theoretical models which persistently tries to solve the mysteries that SM has failed to. The question is which of these models will persist and be adequate to Nature? The driving idea behind the attempts to go beyond the Standard Model is unification. Maxwell unified electricity and magnetism in his theory, electromagnetic and weak forces were unified in electroweak theory, the three forces were unified in the Grand unified theory and subsequent attempts are being made to unify the remaining force that is the gravitational force.

1.1 Beyond Standard Model Physics

Extending our horizon of knowledge beyond the Standard Model seems inevitable from the above discussion. The last brick in the Standard Model was cemented with the discovery of the Higgs boson. The Higgs boson, as proposed within the Standard Model, is the simplest manifestation of the Brout-Englert-Higgs mechanism. The discovery of the Higgs boson was celebrated as a huge success since it seemed to complete the menagerie of fundamental particles in the Standard Model.

But human beings barely know the fact that Nature is enigmatic and unfolds itself with new myths as mankind solves one. The origin of neutrino mass, dark matter and baryon asymmetry of the universe are some of the unanswered problems that surfaced and could not be satisfactorily answered by the SM alone. The gauge boson and fermion sectors of the Standard Model of the electroweak interactions have been extremely well probed phenomenologically but its scalar sector could be probed further. Hence the question: *The Higgs or A Higgs*? This intrigued the grey cells to think about physics beyond the standard model (BSM physics). People have proposed many theories with extended families of fermions and family mixing but these seem to be very complicated theories. On the other hand if the scalar sector of the standard model is extended things are much easier to handle. In the standard model the simplest possible scalar structure is just one SU(2) doublet [3–5, 12, 13]. When an extra SU(2) doublet was added to the standard model it was put to the 'ρ - parameter' test since ρ parameter carries substantial information about the scalar sector. In the SU(2)×U(1) gauge theory, if there are n scalar multiplets Φ_i, with weak isospin I_i, weak hypercharge Y_i, and vacuum expectation value (vev) of the neutral components v_i, then the parameter ρ at tree level is given by [14],

$$\rho = \frac{\sum_{i=1}^{n} \left[I_i(I_i+1) - \frac{1}{4}Y_i^2 \right] v_i}{\sum_{i=1}^{n} \frac{1}{2} Y_i^2 v_i} . \tag{1.5}$$

Experimentally ρ is very close to unity [15]. SU(2) doublets with $Y = \pm 1$ give $\rho = 1$ according to Eq. (1.5) since SU(2) doublets have $I(I+1) = \frac{3}{4}Y^2$. Thus adding a second scalar doublet seemed feasible. Since the standard model was extended by adding a second scalar doublet hence the nomenclature two Higgs doublet model(2HDM) [16,17].

The two Higgs doublet model got its motivation from supersymmetry [18]. In supersymmetric theories the scalars belong to chiral multiplets and their complex conjugates belong to multiplets of the opposite chirality. A single Higgs doublet is unable to give mass simultaneously to the up-type and bottom-type quarks since multiplets of different chiralities cannot couple together in the Lagrangian. The cancellation of anomalies also requires that an additional doublet be added since scalars and chiral spin- 1/2 fields occur together in the chiral multiplets. Thus, addition of a second Higgs doublet seemed to be justified. Since the Minimal Supersymmetric Standard Model (MSSM) contains two Higgs doublets, it can be said that 2HDM is contained in MSSM.

Peccei and Quinn showed in their work [19] that if a global $U(1)$ symmetry is imposed then a possible CP-violating term in the QCD Lagrangian can be done away with. Though this CP violating term is phenomenologically known to be very small but its existence is not desirable. With single Higgs doublet this symmetry cannot be imposed. Rotation requires minimum two Higgs doublets. This provided yet another motivation for two Higgs doublet models. Though experiment has ruled out the simplest versions of the Peccei–Quinn model (in which all the New Physics was at the TeV scale), there are variations with singlets at a higher scale that are acceptable, and the effective low-energy theory for those models still requires two Higgs doublets [20].

Now since we all know that the SM is unable to generate a baryon asymmetry of the Universe of sufficient size [21] introduction of beyond standard model physics seemed necessary as discussed in the previous section. Two-Higgs-doublet models have a rich parameter space and its CP violating version provides additional sources of CP violation thereby contributing to baryogenesis [22–29]. New possibilities for explicit or spontaneous CP violation have allured physicists to work with 2HDMs.

Another motivation, one that is important to us, is their use in models of dark matter [30–32]. These models are the inert doublet models, so called because one of the Higgs doublets does not couple to the fermions. Of the 2HDMs we will consider, the Yukawa couplings of one model (type I) approach the inert doublet model for large values of the ratio of the vacuum expectation values (VEVs) of the two Higgs fields. The other models also have small couplings to one or more types of fermions in that limit.

1.2 The two Higgs doublet model

Two Higgs doublet model has been used as a benchmark model by both ATLAS and CMS in its CP-conserving, softly broken Z_2 symmetric version to search for new physics beyond the standard model. First proposed by T. D. Lee [16], 2HDMs have been used as benchmark models not only for the LHC searches but also theoretically. Theoretical motivations stem from the rich scalar structure of the model that allow for instance the introduction of CP violation in the scalar sector, controlled flavour changing neutral currents or dark matter candidates. Simultaneous breaking of both charge and CP renders this model a very different vacuum structure than the SM one.

In general, the vacuum structure of 2HDMs is very rich and the most general scalar potential can have CP-conserving, CP-violating, and charge-violating minima. However, most phenomenological studies of 2HDMs make several simplifying assumptions. We have assumed that CP is conserved in the Higgs sector (then the distinction between scalars and pseudoscalars become clear), that CP is not spontaneously broken, and that discrete symmetries eliminate from the potential all quartic terms odd in either of the doublets. We will work with the scalar potential [16, 33]

$$
\begin{aligned}
V = & \lambda_1 \left(|\Phi_1|^2 - \frac{v_1^2}{2} \right)^2 + \lambda_2 \left(|\Phi_2|^2 - \frac{v_2^2}{2} \right)^2 \\
& + \lambda_3 \left(|\Phi_1|^2 + |\Phi_2|^2 - \frac{v_1^2 + v_2^2}{2} \right)^2 \\
& + \lambda_4 \left(|\Phi_1|^2 |\Phi_2|^2 - |\Phi_1^\dagger \Phi_2|^2 \right) \\
& + \lambda_5 \left(\frac{1}{2} \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 - v_1 v_2 \right) \right)^2 \\
& + \lambda_6 \left(\frac{1}{2i} \left(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1 \right) \right)^2 \,, \qquad (1.6)
\end{aligned}
$$

where the λ's are real because of the hermiticity of the Lagrangian.

For a region of parameter space, the minimization of this potential gives

$$
\langle \Phi_i \rangle_0 = \begin{pmatrix} 0 \\ \frac{v_i}{\sqrt{2}} \end{pmatrix} , \qquad i = 1, 2 \qquad (1.7)
$$

where the VEVs v_i of the neutral components of the two SU(2) scalar doublets may be taken to be real and positive without any loss of generality. v_1 and v_2 are related to the electroweak vacuum expectation value v by

$$
v = \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV} \,. \qquad (1.8)
$$

With two SU(2) scalar doublets there are eight real scalar fields. In the charge eigenstate these fields are given as

$$\Phi_i = \begin{pmatrix} w_i^+(x) \\ \dfrac{v_i + h_i(x) + iz_i(x)}{\sqrt{2}} \end{pmatrix}, \qquad i = 1, 2. \tag{1.9}$$

Now we can construct the mass matrices for the charged, CP-odd and CP-even scalars using the potential given by Eq. (1.6). Since we have assumed that the parameters of the potential are all real without any loss of generality the bilinear mixing terms like $h_k z_l$ will not be present. As a consequence the neutral mass eigenstates will also be the eigenstates of CP. The mass matrices for the charged, CP-odd and CP-even sectors are given below,

$$V_{\text{charged}} = \begin{pmatrix} w_1^+ & w_2^+ \end{pmatrix} M_C^2 \begin{pmatrix} w_1^- \\ w_2^- \end{pmatrix} \tag{1.10}$$

with

$$M_C^2 = \frac{\lambda_4}{2} \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix}. \tag{1.11}$$

$$V_{\text{CP-odd}} = \begin{pmatrix} z_1 & z_2 \end{pmatrix} \frac{1}{2} M_P^2 \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \tag{1.12}$$

with

$$M_P^2 = \frac{\lambda_6}{2} \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix}. \tag{1.13}$$

$$V_{\text{CP-even}} = \begin{pmatrix} h_1 & h_2 \end{pmatrix} \frac{1}{2} M_S^2 \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \tag{1.14}$$

with

$$M_S^2 = \begin{pmatrix} A_S & B_S \\ B_S & C_S \end{pmatrix} \tag{1.15}$$

where

$$A_S = 2(\lambda_1 + \lambda_3) v_1^2 + \frac{\lambda_5}{2} v_2^2 \tag{1.16}$$

$$B_S = 2(\lambda_3 + \frac{\lambda_5}{4}) v_1 v_2 \tag{1.17}$$

$$C_S = 2(\lambda_2 + \lambda_3) v_2^2 + \frac{\lambda_5}{2} v_1^2 . \tag{1.18}$$

Since the mass matrices, M_C^2, M_P^2 and M_S^2 are not diagonal in the charge basis, we rotate away to the mass basis where the mass matrices are diagonal and the mass eigenstates can be identified with the physical scalars. The rotation from the charge basis to the mass basis is[1]

$$\begin{pmatrix} \omega^\pm \\ \xi^\pm \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} w_1^\pm \\ w_2^\pm \end{pmatrix}. \tag{1.19}$$

[1] $c_\alpha \equiv \cos\alpha$, $s_\beta \equiv \sin\beta$ etc.

This yields a pair of physical charged Higgs bosons $\xi^{\pm}$ and a pair of charged Goldstone bosons $\omega^{\pm}$. The mass of the charged Higgs pair is found to be

$$m_{\xi}^2 = \frac{\lambda_4}{2}v^2 \tag{1.20}$$

and the charged Goldstone bosons get 'eaten away' by the $W^{\pm}$ bosons which in turn obtain their masses. For the pseudoscalar part the rotation angle remains the same as the charged scalar part and we obtain a physical pseudoscalar (A) and a neutral Goldstone (ζ),

$$\begin{pmatrix} \zeta \\ A \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} . \tag{1.21}$$

The mass of the pseudoscalar is given by,

$$m_A^2 = \frac{\lambda_6}{2}v^2 . \tag{1.22}$$

ζ gets 'eaten away' by the neutral gauge boson Z which then gains its mass. For the CP-even scalar part the rotation angle is different from the other two sections since it is not protected by the custodial symmetry unlike the other two sectors. Due to custodial symmetry, the linear combination that gives the charged Goldstone bosons also gives the neutral Goldstone boson. So the angle of rotation is the same in these two sectors. For the CP-even sectors the rotation to mass eigenstates yields a heavy CP-even scalar H and a light CP-even scalar h. The rotation is shown below,

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} , \tag{1.23}$$

where the masses

$$m_H^2 = \frac{1}{2}\left[(A_S + C_S) + \sqrt{(A_S - C_S)^2 + B_S^2}\right] , \tag{1.24}$$

$$m_h^2 = \frac{1}{2}\left[(A_S + C_S) - \sqrt{(A_S - C_S)^2 + B_S^2}\right] . \tag{1.25}$$

Two crucial angles have been introduced while rotating from the charge basis to the mass basis. $\angle\beta$ is the angle of rotation in the charged and CP-odd sectors and $\angle\alpha$ is the angle of rotation in the CP-even sector. These are so chosen that,

$$\tan\beta = \frac{v_2}{v_1} \tag{1.26}$$

and

$$\tan 2\alpha = \frac{2B_S}{(A_S - C_S)} \tag{1.27}$$

$$= \frac{2\left(\lambda_3 + \frac{1}{4}\lambda_5\right) v_1 v_2}{\lambda_1 v_1^2 - \lambda_2 v_2^2 + \left(\lambda_3 - \frac{1}{4}\lambda_5\right)\left(v_1^2 - v_2^2\right)} . \tag{1.28}$$

The ranges of β and α are, $\angle\beta \in \left[0, \frac{\pi}{2}\right]$ and $\angle\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

To start with there were eight parameters, viz: v_1, v_2 and the 6 λ's. v_1 and v_2 can be traded away for v and $\tan\beta$ using Eq. (1.8) and Eq. (1.26). Except λ_5 all other λ's can be expressed in terms of the masses of the physical Higgs bosons and the angle α. Amongst these new eight parameters two are known. One is the standard electroweak vev, $v = 246$ GeV and the other is the mass of one of the neutral CP-even physical Higgs bosons which is taken to be 125 GeV. This can be m_h or m_H depending on the limit in which we are working. The remaining parameters need to be constrained both theoretically and experimentally. The relations among the λ's and the masses of the physical Higgs bosons are given below [34],

$$\lambda_1 = \frac{1}{2v^2c_\beta^2}\left[c_\alpha^2 m_H^2 + s_\alpha^2 m_h^2 - \frac{s_\alpha c_\alpha}{\tan\beta}(m_H^2 - m_h^2)\right] - \frac{\lambda_5}{4}(\tan^2\beta - 1)\,, \tag{1.29}$$

$$\lambda_2 = \frac{1}{2v^2s_\beta^2}\left[s_\alpha^2 m_H^2 + c_\alpha^2 m_h^2 - s_\alpha c_\alpha \tan\beta(m_H^2 - m_h^2)\right] - \frac{\lambda_5}{4}\left(\frac{1}{\tan^2\beta} - 1\right)\,, \tag{1.30}$$

$$\lambda_3 = \frac{1}{2v^2}\frac{s_\alpha c_\alpha}{s_\beta c_\beta}(m_H^2 - m_h^2) - \frac{\lambda_5}{4}\,, \tag{1.31}$$

$$\lambda_4 = \frac{2}{v^2}m_\xi^2\,, \tag{1.32}$$

$$\lambda_6 = \frac{2}{v^2}m_A^2\,. \tag{1.33}$$

We take a note that λ_5 appears on the right hand side of the first three of the above set of equations.

1.2.1 The Higgs basis

Unlike the mass eigenstates since the two doublets Φ_1 and Φ_2 are not physical fields, therefore any linear combination of the doublets which preserves the form of the kinetic terms of the theory is equally acceptable. This freedom of reparameterization implies that different bases of the doublet fields can be chosen, without changing the physical predictions of the model and potentially simplifying the theory. It is sometimes useful to work in the so-called Higgs basis, wherein one performs a $U(2)$ transformation on Φ_1 and Φ_2 in such a manner that only the first of the transformed fields H_1 and H_2, acquires a vacuum expectation value. We can define the Higgs basis up to an arbitrary complex phase multiplying the second doublet. Performing a $U(2)$ transformation we obtain,

$$\begin{aligned}\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} &= R_H \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \\ &\equiv \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}.\end{aligned} \tag{1.34}$$

Thereby the potential turns out to be [35]

$$\begin{aligned} V(H_1,H_2) &= -\frac{1}{2}\left[Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + \left(Y_3 H_1^\dagger H_2 + \text{h.c.}\right)\right] + \frac{1}{2}Z_1(H_1^\dagger H_1)^2 + \frac{1}{2}Z_2(H_2^\dagger H_2)^2 \\ &+ Z_3(H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4(H_1^\dagger H_2)(H_2^\dagger H_1) \\ &+ \left[\frac{1}{2}Z_5(H_1^\dagger H_2)^2 + \left(Z_6 H_1^\dagger H_1 + Z_7 H_2^\dagger H_2\right) H_1^\dagger H_2 + \text{h.c.}\right]. \end{aligned} \tag{1.35}$$

The minimization of the scalar potential yields

$$Y_1 = Z_1 v^2 \,, \tag{1.36}$$

$$Y_3 = Z_6 v^2 \,. \tag{1.37}$$

Higgs basis is particularly useful in understanding the fine difference between alignment limit and decoupling limit which has been discussed in Section 4.1.

1.2.2 CP violation in 2HDM

The most general two Higgs doublet model has three neutral physical Higgs bosons but these need not be eigenstates of CP. This can be illustrated by going to the Higgs basis where the two Higgs doublets are parametrized as

$$\Phi_1 = \begin{pmatrix} G^+(x) \\ \dfrac{v + \eta_1(x) + iG_0(x)}{\sqrt{2}} \end{pmatrix}, \tag{1.38}$$

$$\Phi_2 = \begin{pmatrix} H^+(x) \\ \dfrac{\eta_2(x) + i\chi_2(x)}{\sqrt{2}} \end{pmatrix}. \tag{1.39}$$

The potential is given by Eq. (1.35). The parameters Y_3 together with Z_5, Z_6 and Z_7 can be complex in general. The mass-squared matrix will depend on the parameters Y_2, Z_1, Z_3, Z_4, Z_5 and Z_6. It takes the form,

$$\mathcal{M}^2 = \begin{pmatrix} Z_1 v^2 & \text{Re}Z_6 v^2 & -\text{Im}Z_6 v^2 \\ \text{Re}Z_6 v^2 & \frac{1}{2}\left(-Y_2 + (Z_3 + Z_4 + \text{Re}Z_5)v^2\right) & -\frac{1}{2}\text{Im}Z_5 v^2 \\ -\text{Im}Z_6 v^2 & -\frac{1}{2}\text{Im}Z_5 v^2 & \frac{1}{2}\left(-Y_2 + (Z_3 + Z_4 - \text{Re}Z_5)v^2\right) \end{pmatrix}. \tag{1.40}$$

The mass matrix does not include Z_7. Mixing between CP-even and CP-odd fields can be avoided if Z_5 and Z_6 can be made simultaneously real by a redefinition of Φ_2. This is the case for instance if $Z_5 = 0$ or $Z_6 = 0$. In particular, for $Z_6 = 0$, Z_5 can be made real by rephasing Φ_2 and the mass matrix becomes automatically diagonal. However, in order to conclude that CP is conserved one must check whether or not Z_7 can also be made real with the same rephasing of Φ_2 that makes Z_5 and Z_6 real, otherwise there will be CP violation in the trilinear and quartic couplings.

These conditions will look different in a general (non-Higgs) basis, but the different possibilities of having CP conservation or violation can be sorted out by exploring the basis-transformation invariants mentioned above [36–39]. A different approach is to ask whether a basis exists in which the potential and the vacuum expectation values are simultaneously real [40].

It is worth mentioning that we have considered that there is no CP violation in the vacuum expectation values (vevs) of the scalar doublets $\Phi_{1,2}$. This means that $v_{1,2}$ are both real and non-negative without any loss of generality.

1.2.3 Flavour conservation

In the SM, things are pretty simple since there is a single $SU(2)$ doublet and thus a single Yukawa structure in each of the quark sectors - the up and down which is both responsible for:

1. The generation of mass upon spontaneous breaking of $SU(2)_L \bigotimes U(1)_Y$ into $U(1)_{EM}$ and
2. The Yukawa couplings of the quarks to the only fundamental scalar leftover, the Higgs boson, after associating the three would-be Goldstone bosons to the longitudinal polarizations of the massive Z and $W^{\pm}$ gauge bosons.

Thus, there are no tree level Flavour Changing Neutral Couplings(FCNC) of the Higgs to quarks.

But in two Higgs doublet models the situation is dramatically changed since there are two SU(2) scalar doublets and hence two independent Yukawa structures are available in each quark sector. Thus flavour changing neutral couplings of Higgs to quarks arise at tree level. To which extent they appear in the couplings of the different physical neutral scalars depends on the details of the scalar potential. If the 125 GeV scalar is a mixture of the true but unphysical Higgs and the additional neutral scalars, FCNC will "leak" into its couplings through that mixing [41].

These FCNCs can cause severe phenomenological difficulties. As for example, the $\bar{d}sh$ interaction will lead to K–$\bar{K}$ mixing at tree level. If the coupling is as large as the b-quark Yukawa coupling, the mass of the exchanged scalar would have to exceed 10 TeV [42, 43]. Thus the different ways to dispense away the FCNC couplings and the conditions for their appearance or absence, has drawn sustained attention over the years.

To make things explicit, let us write down the Yukawa part of a 2HDM Lagrangian as follows:

$$\mathcal{L}_Y = \sum_{i=1,2} \left[-\bar{l}_L \Phi_i G_e^i e_R - \bar{Q}_L \tilde{\Phi}_i G_u^i u_R - \bar{Q}_L \Phi_i G_d^i d_R + h.c. \right] \tag{1.41}$$

where $l_L\,, Q_L$ are left-handed 3-vectors of iso-doublets in the space of generations, $e_R\,, u_R\,, d_R$ are right-handed 3-vectors of singlets, G_e^i, G_u^i and G_d^i are complex 3×3 matrices in generation space containing the Yukawa coupling constants in the up, down and charged lepton sectors respectively. Here $\tilde{\Phi}_i = i\tau_2\Phi_i^*$, where τ_2 is the second Pauli matrix. Also we have suppressed the flavor indices in Eq. (1.41). We have also assumed that the neutrinos are massless. From Eq. (1.41) one can write the mass matrix for the down type quarks, for example, as follows:

$$M_d = G_d^1 \langle\Phi_1\rangle + G_d^2 \langle\Phi_2\rangle \ , \tag{1.42}$$

where $\langle\Phi_i\rangle = v_i/\sqrt{2}$ denotes the vacuum expectation value (VEV) of Φ_i. Since G_d^1 and G_d^2, in general, can be arbitrary, there is no reason for them to be simultaneously diagonal once M_d is diagonalized using a biunitary transformation. Therefore, there will be Higgs mediated FCNC at the tree level in the most general 2HDMs.

In the mass equation Eq. (1.42) if one of the Yukawa couplings G_d^1 or G_d^2 vanishes then the remaining Yukawa coupling gets simultaneously diagonalized when the mass matrix M_d is diagonalized and tree level FCNC can be avoided in the down sector. The same method can be applied to remove the tree-level FCNCs from the up quark and the charged lepton sectors too. Thus if all fermions with the same quantum numbers (which are thus capable of mixing) couple to the same Higgs multiplet, then FCNC will be absent. This was formalized by the Glashow-Weinberg-Paschos (GWP) theorem [44,45] which states that a necessary and sufficient condition for the absence of FCNC at tree level is that all fermions of a given charge and helicity transform according to the same irreducible representation of SU(2), correspond to the same eigenvalue of T_3 and that a basis exists in which they receive their contributions in the mass matrix from a single source. In the Standard Model with left-handed doublets and right-handed singlets, this theorem implies that all right-handed quarks of a given charge must couple to a single Higgs multiplet. In the 2HDM, this can only be ensured by the introduction of a discrete or continuous symmetry which classifies the two Higgs doublet models into various types as discussed below.

If we first deal with the quark sector of the 2HDM, there are only two possibilities within the purview of the GWP prescription. Let us look into the possibilities elaborately. In type I 2HDM, all quarks couple to just one of the Higgs doublets (conventionally chosen to be Φ_2). In type II 2HDM, the Q = 2/3 right-handed (RH) quarks couple to one Higgs doublet (conventionally chosen to be Φ_2) and the Q = $-1/3$ RH quarks couple to the other Higgs doublet (Φ_1). The type I 2HDM can be enforced with a simple $\Phi_1 \to -\Phi_1$ discrete Z_2 symmetry, whereas the type II 2HDM is enforced with a $\Phi_1 \to -\Phi_1$, $d_R^i \to -d_R^i$ discrete Z_2 symmetry. This Z_2 symmetry, when extended to the full Lagrangian, also prevents the corresponding Yukawa couplings from getting generated via quantum corrections.

Now coming to the leptons, it is conventionally assumed, in discussions of type I and type II 2HDMs, that the right-handed charged leptons satisfy the same discrete symmetry as the d_R^i and thus the charged leptons couple to the same Higgs doublet as the Q = $-1/3$ quarks. However, the Glashow–Weinberg-Pascos theorem does not demand this and thus there are two other possibilities. In the third type of model, the "lepton-specific" model, the right-handed quarks all couple to Φ_2 and the right-handed charged leptons couple to Φ_1. There is yet another model the "flipped" model, where the Q = 2/3 RH quarks couple to Φ_2 and the Q = $-1/3$ RH quarks coupling to Φ_1 , as in the type II 2HDM, but now the RH charged leptons couple to Φ_2. The phenomenology of these models is quite different. Names type X and type Y were used for the lepton-specific and flipped models respectively in [46]. These four types of two Higgs doublet models subjected to the GWP theorem has been tabulated in table 1.1.

Model	u_R^i	d_R^i	e_R^i
Type I	Φ_2	Φ_2	Φ_2
Type II	Φ_2	Φ_1	Φ_1
Lepton-specific	Φ_2	Φ_2	Φ_1
Flipped	Φ_2	Φ_1	Φ_2

Table 1.1: Two Higgs doublet models that lead to natural Flavour conservation (NFC).

It has been suggested by Pich and Tuzon [47] that an alternative way to avoid tree-level FCNC is to make the two Yukawa matrices for fermions of a particular charge proportional to each other. This technique will also diagonalize the Yukawa matrices simultaneously as the mass matrix will be diagonalized. These types of 2HDMs are called aligned 2HDMs (A2HDMs). However, the Yukawa alignment imposed at a certain energy scale does not ensure, in general, that the alignment will be maintained at different energy scales too [48, 49]. But it has been shown that the FCNCs generated due to such misalignments are expected to be small [50] because the Yukawa aligned 2HDMs belong to a general category of models with minimal flavor violation (MFV) [51]. The mathematics of simultaneous diagonalisation has been done in Chapter A

At the end of the day, as with many New Physics avenues, the presence of FCNC is a double edged feature: since the competing SM gauge mediated contributions to FCNC processes are loop induced, those transitions pose severe constraints while, on the same grounds, provide immediate opportunities to discover deviations from the SM picture.

1.2.4 Symmetries and the scalar potential

With the introduction of a second scalar doublet, the parameter space of 2HDMs is largely increased and the theory becomes less predictive. Ways to constrain the parameter space is therefore largely welcome. One way of doing so is by the imposition of symmetries. Also, as we discussed in previous sections, the 2HDM is in general plagued by flavour-changing neutral currents, which however may be eliminated or strongly suppressed by imposing an internal symmetry on the 2HDM. These symmetries leave the kinetic terms unchanged. Symmetries leaving the kinetic terms unchanged may be of either one of two types:

1. Higgs Family (HF) symmetries
   Here one Higgs doublet is related to the other via some unitary transformation of the form,

$$\Phi_a \to \Phi_a^S = \sum_{b=1}^{2} S_{ab}\Phi_b\,, \tag{1.43}$$

   where S is a unitary matrix. We then require the potential to be invariant under this transformation. As a result of this invariance the quartic couplings are transformed as,

$$\lambda_{ab,cd} = \sum_{e,f,g,h=1}^{2} S_{ae}S_{cf}\lambda_{eg,fh}S^*_{bg}S^*_{dh}\,. \tag{1.44}$$

2. General Charge conjugation-Parity symmetry

   In this case Φ_a is related through some unitary transformation of Φ_b^* as shown below:

$$\Phi_a \to \Phi_a^{GCP} = \sum_{b=1}^{2} X_{ab}\Phi_b^*\,, \tag{1.45}$$

   where X is an arbitrary unitary matrix. We then require the potential to be invariant under this transformation. As a result of this invariance the quartic couplings are transformed as,

$$\lambda_{ab,cd} = \sum_{e,f,g,h=1}^{2} X_{ae}X_{cf}\lambda^*_{eg,fh}X^*_{bg}X^*_{dh}\,. \tag{1.46}$$

Under a global basis transformation from Φ_a to Φ'_a,

$$\Phi'_a = \sum_{b=1}^{2} U_{ab}\Phi_b\,, \tag{1.47}$$

the specific forms of the HF and GCP symmetries get altered respectively into,

$$S' = USU^\dagger \tag{1.48}$$
$$X' = UXU^T\,. \tag{1.49}$$

Thus, if the coefficients of the potential are written using a different basis for the Higgs doublet, the symmetry relations among the coefficients of the scalar potential will in general adopt a different form.

We have imposed a global U(1) symmetry on the scalar potential which is a generalization of the most common form of symmetry imposed on 2HDMs viz., the discrete Z_2 symmetry. This potential is invariant under the U(1) symmetry which when imposed on the Higgs doublets transforms them as,

$$\Phi_1 \to e^{i\theta}\Phi_1\,, \Phi_2 \to \Phi_2\,, \tag{1.50}$$

The symmetry is realized by putting $\lambda_5 = \lambda_6$ in the scalar 2HDM potential of Eq. (1.6) which then takes the form,

$$\begin{aligned} V = {} & \lambda_1 \left(|\Phi_1|^2 - \frac{v_1^2}{2} \right)^2 + \lambda_2 \left(|\Phi_2|^2 - \frac{v_2^2}{2} \right)^2 \\ & + \lambda_3 \left(|\Phi_1|^2 + |\Phi_2|^2 - \frac{v_1^2 + v_2^2}{2} \right)^2 \\ & + \lambda_4 \left(|\Phi_1|^2 |\Phi_2|^2 - |\Phi_1^\dagger \Phi_2|^2 \right) \\ & + \lambda_5 \left| \Phi_1^\dagger \Phi_2 - \frac{v_1 v_2}{2} \right|^2 . \end{aligned} \tag{1.51}$$

The potential contains a term $\lambda_5 v_1 v_2 \Re(\Phi_1^\dagger \Phi_2)$ which softly breaks the $U(1)$ symmetry. Additional dimension-4 terms, including one allowed by a softly broken Z_2 symmetry [52] are also set to zero by this U(1) symmetry.

The imposition of this global U(1) symmetry changes the last relation Eq. (1.33) between the quartic coupling and the physical Higgs boson masses.

$$\lambda_5 = \lambda_6 = \frac{2}{v^2} m_A^2\,. \tag{1.52}$$

Thus, now the theory has one parameter less on the imposition of U(1) symmetry.

1.2.5 Some lessons from the previous studies made on various 2HDMs

- Proposed by Barger and others in [53] the flipped two Higgs doublet model is the least studied model of all the four types of two Higgs doublet models. Even works that discuss all four models generally focus less on this structure than the others. The only paper dedicated entirely to the flipped model was that of Logan and Maclennan [54]. They studied the charged Higgs phenomenology in that model, including branching ratios and indirect constraints and analyse prospects at the LHC.

- The lepton-specific model was first discussed in two papers by Barnett et al. [55, 56] in the context of extremely light Higgs scalars. Later Su and Thomas analysed this model extensively in [57]. They studied theoretical and experimental constraints on the model and showed that there can be substantial enhancement of the couplings between the charged leptons and the neutral Higgs scalar. Logan and Maclennan in their paper [58] considered the constraints on the charged Higgs mass, with bounds arising from lepton flavour universality and direct searches and direct prospects at the LHC. Goh, Hall and Kumar [59] discussed the leptonic cosmic ray signals seen by PAMELA and ATIC with the help of the lepton specific model and studied the implications for the LHC. The work of Boucenna and Profumo [60] may be referred for the analyses of astrophysical results and direct dark matter detection with the lepton specific model. In another analysis Cao et al. [61] assumed that the 3σ discrepancy [62] between theory and experiment in the $g-2$ of the muon is primarily due to the lepton-specific model. This requirement substantially reduces the available parameter space, forcing the model to have a very light pseudoscalar and very large values of $\tan\beta$, and they analysed this parameter space. Finally Aoki et al. [63] looked at neutrino masses and dark matter in the lepton-specific model, but did add singlets.

- The type I 2HDM [64] is the second most studied model[1]. In the quark sector, it is identical to the lepton-specific model, thus many results from the studies of the type I 2HDM apply to the lepton-specific as well. A special limit of the type I 2HDM is $\alpha = \frac{\pi}{2}$, in which case the fermions all completely decouple from the lightest Higgs; this limit is referred to as the fermiophobic limit. We note that even in this limit, the coupling does reappear at the one-loop level, but it will in any event be very small. Moving away from the fermiophobic limit, there are many papers looking at the type I 2HDM. A very early discussion of the fermiophobic, gauge-phobic and fermiophilic limits was given by Pois, Weiler and Yuan [65], who studied top production below the $t\bar{t}$ threshold in Higgs decays. In a series of papers, Akeroyd and collaborators [66–70] considered Higgs decays into lighter Higgs bosons, charged Higgs decays into a W and a pseudoscalar, double-Higgs production, Higgs decays to $\gamma\gamma$, $\tau^+\tau^-$ at the LHC, and the possibility of a very light Higgs, respectively all in context of the type I 2HDM. Type I 2HDM have been used to study its contribution to the anomalous magnetic moment of the muon in [71, 72].

- The most popular among all four is the type II 2HDM which is by far the most studied since it is the structure present in supersymmetric models. A voluminous Physics Reports review article in 2008 by Djouadi [73] analyses the Higgs bosons of the Minimal Supersymmetric Standard Model in great detail. Although 2HDM is embedded in the minimal supersymmetric standard model yet both the models have evolved with their distinct characteristics. The most crucial difference between 2HDM and MSSM is that the general type II 2HDM does not have a strict upper bound on the mass of the lightest Higgs boson, which is an important characteristic of the MSSM. In addition the scalar self-couplings are now arbitrary. Another important difference is that the mixing parameter α, which in MSSM is given in terms of $\tan\beta$ and the scalar and pseudoscalar masses, is now arbitrary. Finally in the MSSM, the charged scalar and the pseudoscalar masses are so close that the decay of the charged Higgs into a pseudoscalar and a

[1] Developments in string phenomenology suggest that a type I 2HDM is generic among the vacua of the heterotic string, providing new motivation for the study of this model.

real W is kinematically forbidden, while it is generally allowed in type II 2HDM (refer [74, 75] for possible exceptions).

- In addition there is another model which has natural flavour conservation, in which the quarks and charged leptons all couple to Φ_2, but the right-handed neutrino couples to Φ_1. In this model there are Dirac neutrino masses, and the vacuum expectation value of Φ_1 must be zero. The only way to have such a small vacuum expectation value is through an appropriate symmetry. The model originally used a Z_2 symmetry which was softly [76] or spontaneously [77] broken, but this allows for right-handed neutrino masses. Extending the symmetry to $U(1)$ symmetry and breaking it softly (to avoid a Goldstone boson) gives the model of Davidson and Logan [78].

References

[1] G. Aad *et al.* [ATLAS Collaboration], *"Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,"* Phys. Lett. B **716**, 1 (2012).

[2] S. Chatrchyan *et al.* [CMS Collaboration], *"Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC,"* Phys. Lett. B **716**, 30 (2012).

[3] F. Englert and R. Brout, *"Broken Symmetry and the Mass of Gauge Vector Mesons",* Phys. Rev. Lett. 13, 321.

[4] Peter W. Higgs, *"Broken Symmetries and the Masses of Gauge Bosons",* Phys. Rev. Lett. 13, 508.

[5] Peter W. Higgs, *"Spontaneous Symmetry Breakdown without Massless Bosons",* Phys. Rev. 145, 1156.

[6] M. Kobayashi, T. Maskawa, *"CP-Violation in the Renormalizable Theory of Weak Interaction",* Progress of Theoretical Physics, Volume 49, Issue 2, 1 February 1973, Pages 652–657, doi.org/10.1143/PTP.49.652.

[7] Z. Maki, M. Nakagawa, S. Sakata, *"Remarks on the Unified Model of Elementary Particles",* Progress of Theoretical Physics, Volume 28, Issue 5, 1 November 1962, Pages 870–880, doi.org/10.1143/PTP.28.870.

[8] C. Patrignani et al., *"The Review of Particle Physics (2017)",* Particle Data Group, Chin. Phys. C40 (2016) 100001 and 2017 update.

[9] L. D. Landau, *"On quantum field theory in 'Niels Bohr and the Development of Physics",* London: Pergamon Press, 1955.

[10] G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, and A. Strumia, *"Higgs mass and vacuum stability in the Standard Model at NNLO",* JHEP 1208 (2012) 098, arXiv:1205.6497.

[11] J. R. Espinosa, *"Vacuum stability and the Higgs boson",* Conference: C13-07-29.1 Proceedings, arXiv:1311.1970 (2013).

[12] G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Lett. 13 (1964) 585.

[13] T. W. B. Kibble, Phys. Rev. 155 (1967) 1554.

[14] P. Langacker, Phys. Rep. 72 (1981) 185.

[15] K. Nakamura et al. [Particle Data Group], J. Phys. G 37 (2010) 075021.

[16] T. D. Lee, *"A Theory of Spontaneous T Violation,"* Phys. Rev. D **8**, 1226 (1973).

[17] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, *"Theory and phenomenology of two-Higgs-doublet models,"* Phys. Rept. **516**, 1 (2012)

[18] H. E. Haber and G. L. Kane, Phys. Rep. 117 (1985) 75.

[19] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38 (1977) 1440.

[20] J. E. Kim, Phys. Rep. 150 (1987) 1.

[21] M. Trodden, hep-ph/9805252.

[22] N. Turok and J. Zadrozny, Nucl. Phys. B 358 (1991) 471.

[23] M. Joyce, T. Prokopec, and N. Turok, Phys. Rev. D 53 (1996) 2958 [hep-ph/9410282].

[24] K. Funakubo, A. Kakuto, and K. Takenaga, Prog. Theor. Phys. 91 (1994) 341[hep-ph/9310267].

[25] A. T. Davies, C. D. Froggatt, G. Jenkins, and R. G. Moorhouse, Phys. Lett. B 336 (1994) 464.

[26] J. M. Cline, K. Kainulainen, and A. P. Vischer, Phys. Rev. D 54 (1996) 2451 [hep-ph/9506284].

[27] J. M. Cline and P. A. Lemieux, Phys. Rev. D 55 (1997) 3873 [hep-ph/9609240].

[28] M. Laine and K. Rummukainen, Nucl. Phys. B 597 (2001) 23 [hep-lat/0009025].

[29] L. Fromme, S. J. Huber, and M. Seniuch, JHEP 0611 (2006) 038 [hep-ph/0605242].

[30] E. Ma, *"Verifiable radiative seesaw mechanism of neutrino mass and dark matter,"* Phys. Rev. D **73**, 077301 (2006).

[31] E. Ma, *"Utility of a Special Second Scalar Doublet,"* Mod. Phys. Lett. A **23**, 647 (2008).

[32] R. Barbieri, L. J. Hall and V. S. Rychkov, *"Improved naturalness with a heavy Higgs: An Alternative road to LHC physics,"* Phys. Rev. D **74**, 015007 (2006) [hep-ph/0603188].

[33] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *"The Higgs Hunter's Guide,"* Front. Phys. **80**, 1 (2000).

[34] A. G. Akeroyd, A. Arhrib and E. M. Naimi, *"Note on tree level unitarity in the general two Higgs doublet model,"* Phys. Lett. B **490**, 119 (2000)

[35] S. Davidson and H. E. Haber, Phys. Rev. D72, 035004 (2005), hep-ph/0504050, [Erratum: Phys. Rev. D72, 099902 (2005)].

[36] G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, *"CP-odd invariants in models with several Higgs doublets,"* Phys. Lett. B 614 (2005) 187 doi:10.1016/j.physletb.2005.03.075 [hep-ph/0502118].

[37] J. F. Gunion and H. E. Haber, *"Conditions for CP-violation in the general two-Higgs-doublet model,"* Phys. Rev. D 72 (2005) 095002 doi:10.1103/PhysRevD.72.095002 [hep-ph/0506227].

[38] L. Lavoura and J. P. Silva, *"Fundamental CP violating quantities in a SU(2)× U(1) model with many Higgs doublets,"* Phys. Rev. D 50 (1994) 4619 doi:10.1103/PhysRevD.50.4619 [hep-ph/9404276].

[39] F. J. Botella and J. P. Silva, *"Jarlskog - like invariants for theories with scalars and fermions,"* Phys.Rev. D 51 (1995) 3870 doi:10.1103/PhysRevD.51.3870 [hep-ph/9411288].

[40] J. F. Gunion and H. E. Haber, *"The CP conserving two Higgs doublet model: The Approach to the decoupling limit,"* Phys. Rev. D 67 (2003) 075019 doi:10.1103/PhysRevD.67.075019 [hep-ph/0207010].

[41] H. Georgi and D. V. Nanopoulos, *"Suppression of Flavor Changing Effects From Neutral Spinless Meson Exchange in Gauge Theories",* Phys. Lett. B82 (1979) 95.

[42] B. McWilliams and L. F. Li, Nucl. Phys. B 179 (1981) 62.

[43] O. U. Shanker, Nucl. Phys. B 206 (1982) 253.

[44] S. L. Glashow and S. Weinberg, Phys. Rev. D 15 (1977) 1958.

[45] E. A. Paschos, Phys. Rev. D 15 (1977) 1966.

[46] M. Aoki, S. Kanemura, K. Tsumura, and K. Yagyu, Phys. Rev. D 80 (2009) 015017 [arXiv:0902.4665 [hep-ph]].

[47] A. Pich and P. Tuzon, *"Yukawa Alignment in the Two-Higgs-Doublet Model",* Phys. Rev. D80 (2009) 091702, [arXiv:0908.1554].

[48] P. M. Ferreira, L. Lavoura, and J. P. Silva, *"Renormalization-group constraints on Yukawa alignment in multi-Higgs-doublet models",* Phys. Lett. B688 (2010) 341–344, [arXiv:1001.2561].

[49] F. J. Botella, G. C. Branco, A. M. Coutinho, M. N. Rebelo, and J. I. Silva-Marcos, *"Natural Quasi-Alignment with two Higgs Doublets and RGE Stability",* Eur. Phys. J. C75 (2015) 286, [arXiv:1501.07435].

[50] C. B. Braeuninger, A. Ibarra, and C. Simonetto, *"Radiatively induced flavour violation in the general two-Higgs doublet model with Yukawa alignment",* Phys. Lett. B692 (2010) 189–195, [arXiv:1005.5706].

[51] A. J. Buras, M. V. Carlucci, S. Gori, and G. Isidori, *"Higgs-mediated FCNCs: Natural Flavour Conservation vs. Minimal Flavour Violation",* JHEP 10 (2010) 009,[arXiv:1005.5310].

[52] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *"Errata for the Higgs Hunter's guide,"* arXiv:hep-ph/9302272.

[53] V. Barger, H. E. Logan and G. Shaughnessy, Phys.Rev.D79(2009)115018 [arXiv:09020170[hep-ph]].

[54] H. E. Logan and D. Maclennan, Phys.Rev.D81(2010)075016 [arXiv:1002.4916[hep-ph]].

[55] R. M. Barnett, G. Senjanović, L. Wolfenstein and D. Wyler, Phys.LettB 136(1984)191.

[56] R. M. Barnett, G. Senjanović, D. Wyler, Phys.Rev.D30(1984)1529.

[57] S. Su and B. Thomas, Phys.Rev.D79(2009)095014 [arXiv:0903.0667[hep-ph]].

[58] H. E. Logan and D. Maclennan, Phys.Rev.D79(2009)115022 [arXiv:0903.2246[hep-ph]].

[59] H. S. Goh, L. J. Hall and P. Kumar, JHEP 0905(2009)097 [arXiv:0902.0814[hep-ph]].

[60] M. S. Boucenna and S. Profumo, Phys. Rev.D84(2011)055011 [arXiv:1106.3368[hep-ph]] .

[61] J. Cao, P. Wan, L. Wu and J. M. Yang, Phys.Rev.D80(2009)071701 [arXiv:0909.5148][hep-ph].

[62] F. Jegerlehner and A. Nyffeler, Phys.Rep.477(2009) [arXiv:0902.3360[hep-ph]].

[63] M. Aoki, S. Kanemura and O. Seto, Phys.Rev.Lett.102(2009)051805 [arXiv:0807.0361[hep-ph]].

[64] H. E. Haber, G. L. Kane and T. Sterling, Nucl.Phys.B 161(1979)493.

[65] H. Pois, T. J. Weiler and T. C. Yuan, Phys.Rev.D47(1993)3886 [hep-ph/9303277].

[66] A. G. Akeroyd, Nucl.Phys.B544(1999)557 [hep-ph/9806337].

[67] A. G. Akeroyd, A. Arhrib and E. M. Naimi, Eur.Phys.J.C12(2000)451 [Erratum ibid.14(2000)371][hep-ph/9811431].

[68] A. G. Akeroyd, M. A. Diáz and F. J. Pacheco, Phys.Rev.D70(2004)075002 [hep-ph/0312231].

[69] A. G. Akeroyd and M. A. Diáz, Phys.Rev.D67(2003)095007 [hep-ph/0301203].

[70] A. G. Akeroyd, J.Phys.G24(1998)1983 [hep-ph/9803324].

[71] A. Dedes and H. E. Haber, JHEP0105(2001)006 [hep-ph/0102297].

[72] M. Krawczyk and J. Zochowski, Phys.Rev.D55(1997)6968 [hep-ph/9608321].

[73] A. Djouadi, Phys.Rep.459(2008) [hep-ph/0503173].

[74] A. G. Akeroyd and S. Baek, Phys.Lett.B525(2002)315 [hep-ph/0105228].

[75] S. Litsey and M. Sher, Phys.Rev.D80 (2009)057701 [arXiv:0908.0502[hep-ph]].

[76] E. Ma, Phys.Rev.Lett.86 (2001)2502 [hep-ph/0011121].

[77] S. Gabriel and S. Nandi, Phys.Lett.B655(2007)141 [hep-ph/0610253].

[78] S. M. Davidson and H. E. Logan, Phys.Rev.D80(2009)095008 [arXiv:0906.3335[hep-ph]].

Chapter 2

Nature is a strict mother with all its constraints

A kite needs to be tied down in order to fly. I learned how important restrictions can sometimes be in order to experience freedom.

Damien Rice

Even in a fairy tale if *Cinderella* gets her time restricted by her fairy God-mother, then why not two Higgs doublet model by Nature? Nature has imposed many restrictions on the two Higgs doublet models from experimental evidences. These experimental evidences in turn put theoretical constraints on the model and its parameter space gets narrowed down. Until and unless these restrictions are put the unknown parameter space will be very tough to solve.

The primary restriction came when ATLAS and CMS both observed a peak at around 125 GeV and it was mandatory that one of the CP even scalars of the two Higgs doublet model need to have a mass of 125 GeV. Among many other constraints, the one that is very obvious is that the potential has to be bounded from below. With the mass of the Higgs boson determined with a good precision, the discussion about the stability of the SM Higgs potential has gained attention. This involves studying the evolution of the SM quartic coupling λ with the renormalization group equations (RGE). Let us first discuss it for the standard model post Higgs discovery. We can then easily argue the same for the simplest extension of the standard model i.e., the two Higgs doublet model. Two effects come into play:

1. The quartic coupling itself has a positive contribution to its own RGE evolution, and therefore tends to increase its value as one goes to higher energy scales;

2. The top quark Yukawa coupling has a negative contribution to the RGE of the quartic coupling λ, and tends to reduce its value as one goes up in energy scale.

As a result of these two effects, if the value of the quartic coupling at the weak scale is too small to start with, the contribution of the top quark Yukawa coupling to the RGE evolution will be more than the positive contribution of the quartic coupling itself to its renormalization group equation evolution and cause λ to turn negative at some point, and therefore the potential becomes unstable. If, however, the starting value of the quartic coupling is too large, its renormalization group equation evolution will drive it to even higher values so that the theory eventually ceases to be perturbative and λ develops a Landau pole. Both these situations are undesirable. These arguments were used to constrain the mass of the SM Higgs boson [1–10] prior to its discovery. Now that we know its mass, we can verify whether the potential remains stable, and the theory perturbative, all the way up to the Planck scale. If that were not the case, that would most likely be a sign of the existence of new physics, hitherto undiscovered, which would stabilize the renormalization group equation evolution of the couplings. It has been shown in [11–13], in fact, that the SM vacuum is metastable if the theory is to be valid up to the Planck scale. The only way to have a stable electroweak vacuum, according to these results, would therefore be for new physics to exist at a scale well below the Planck scale. The stability of the electroweak vacuum can be ensured with the addition of extra scalar degrees of freedom. With all the parameters of the SM determined, the addition of a scalar singlet is enough to cure the problem [14–17]. As shown in [17], the addition of a complex singlet not only provides a vacuum stable up to the Planck scale but in the broken phase of the model one of the new scalars can have a mass below 125 GeV. For this particular model only one scalar with a mass above 125 GeV is needed to stabilize the vacuum. It has been shown, however, in the context of the SM, that the presence of new physics very close to the Planck scale can alter considerably such conditions of stability of the potential [18–20], and likewise eventual gravity contributions near the Planck scale can have a sizeable impact [21].

Let us now move on to the two Higgs doublet model where an extra scalar doublet enlarges the SM scalar sector, but the remaining fields (gauge and fermion) remain the same, as do the gauge symmetries of the model. A larger scalar sector implies a more involved scalar potential. And as in the SM, one can ask whether the potential remains stable and perturbative, as one considers progressively larger energy scales. As such, the evolution of the renormalization group equations of the quartic couplings of the 2HDM were studied by several authors [22–25] to ascertain the validity of the model up to higher energy scales and, prior to the Higgs discovery, to attempt to impose constraints on the unknown scalar masses of the model. After the Higgs boson was discovered the stability of the several versions of the 2HDM was revisited in a number of papers [26–34]. In all these works, the lightest CP-even scalar was considered to be the discovered Higgs boson, and there was a common conclusion that, with all relevant theoretical and experimental constraints taken into account, there always exists a region of the parameter space where the 2HDM is valid up to the Planck scale. Notice, however, that these studies assume a softly broken Z_2 symmetry, the most popular version of the 2HDM, and the region of parameter space found always included the soft Z_2 breaking term. On the other hand, in reference [26] a type II version with an exact Z_2 symmetric model was analysed, concluding that the exact Z_2 conserving potential cannot be valid beyond 10 TeV without the intervention of new physics, a conclusion that was then confirmed in later works. However, this conclusion is heavily dependent on the value of the charged Higgs mass, m_ξ. Chapter B contains the RGEs depicting the evolution of the gauge couplings, the Yukawa couplings, the quartic couplings and the quadratic couplings of the two Higgs doublet potential with energy. In this chapter we will be discussing the constraints arising from the stability of the two Higgs doublet model potential, the perturbative unitarity condition and from new physics corrections.

2.1 Stability of the 2HDM potential

The two Higgs doublet model scalar potential can be written in two notations both of which are invariant under the global $U(1)$ symmetry. The two notations are inter-convertible. They are:

$$\begin{aligned} V\left(\Phi_1,\Phi_2\right) &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right) + \frac{\Lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\Lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 \\ &\quad + \Lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \Lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \left(\frac{\Lambda_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right) \end{aligned} \tag{2.1}$$

$$\begin{aligned} V\left(\Phi_1,\Phi_2\right) = &\ \lambda_1\left(|\Phi_1|^2 - \frac{v_1^2}{2}\right)^2 + \lambda_2\left(|\Phi_2|^2 - \frac{v_2^2}{2}\right)^2 \\ &+ \lambda_3\left(|\Phi_1|^2 + |\Phi_2|^2 - \frac{v_1^2+v_2^2}{2}\right)^2 \\ &+ \lambda_4\left(|\Phi_1|^2|\Phi_2|^2 - |\Phi_1^\dagger\Phi_2|^2\right) \\ &+ \lambda_5\left(\text{Re}\left(\Phi_1^\dagger\Phi_2\right) - \frac{v_1v_2}{2}\right)^2 \\ &+ \lambda_6\left(\text{Im}\left(\Phi_1^\dagger\Phi_2\right)\right)^2 \end{aligned} \tag{2.2}$$

All the parameters in the 2HDM potential are considered to be real in order to make the model CP conserving. As we note that the $U(1)$ symmetry is softly broken by the terms m_{12}^2 and λ_5 in Eq. (2.1) and Eq. (2.2) respectively. Potential in Eq. (2.1) can be cast into the potential in Eq. (2.2) by minimizing Eq. (2.1) and using the two minimization conditions to trade m_{11}^2 and m_{22}^2 for v_1 and v_2. The relations between the parameters are written below.

$$\begin{aligned} m_{11}^2 &= -\left(\lambda_1 v_1^2 + \lambda_3 v^2\right)\ ,\ m_{22}^2 = -\left(\lambda_2 v_2^2 + \lambda_3 v^2\right)\ ,\ m_{12}^2 = \frac{\lambda_5}{2}v_1v_2\ ,\ \Lambda_1 = 2\left(\lambda_1+\lambda_3\right)\ , \\ \Lambda_2 &= 2\left(\lambda_2+\lambda_3\right)\ ,\ \Lambda_3 = 2\lambda_3+\lambda_4\ ,\ \Lambda_4 = \frac{\lambda_5+\lambda_6}{2} - \lambda_4\ ,\ \Lambda_5 = \frac{\lambda_5-\lambda_6}{2}\,. \end{aligned} \tag{2.3}$$

The above two notations will be required to discuss this and the successive sections. For this section it is convenient to use the potential of the first notation in Eq. (2.1). In order to bound the potential from below it is sufficient to examine the quartic terms since only the quartic terms will be dominant for larger field values of Φ_1 and Φ_2. The stability conditions are constraints on the parameters Λ_i's. We introduce a few more terms like, $a = \Phi_1^\dagger\Phi_1$, $b = \Phi_2^\dagger\Phi_2$, $c = \text{Re }\Phi_1^\dagger\Phi_2$ and $d = \text{Im }\Phi_1^\dagger\Phi_2$ for the sake of convenience of calculation. We note that,

$$ab \geq c^2 + d^2 \,. \tag{2.4}$$

Using these notations we can write the quartic part of the scalar potential which we denote by V_{IV} as follows [35],

$$\begin{aligned} V_{IV} &= \frac{1}{2}\left(\sqrt{\Lambda_1}a - \sqrt{\Lambda_2}b\right)^2 + \left(\Lambda_3 + \sqrt{\Lambda_1\Lambda_2}\right)\left(ab - c^2 - d^2\right) + 2\left(\Lambda_3 + \Lambda_4 + \sqrt{\Lambda_1\Lambda_2}\right)c^2 \\ &\quad + \left(\text{Re }\Lambda_5 - \Lambda_3 - \Lambda_4 - \sqrt{\Lambda_1\Lambda_2}\right)\left(c^2 - d^2\right) - 2cd\,\text{Im }\Lambda_5 \,. \end{aligned} \tag{2.5}$$

In order to guarantee the stability of the potential, we have to ensure that the quartic part of the potential i.e., V_{IV} never becomes infinitely negative along any of the field directions that is for any value of the eight field parameters, four for Φ_1 and four for Φ_2. Also as we will see, this condition is independent of the CP conserving nature of the potential, that is to say that the stability conditions are independent of the reality of the Λ_i's. Since Φ_1 and Φ_2 are two component column matrices it is possible to choose two arbitrary non-zero values for a and b even when c and d are both zero. But when either of a and b or both are zero, then it forces c and d to be zero. Keeping these facts in mind we now consider various possibilities that bound the potential from below.

- When $b = 0$ and $a \to \infty$ thereby forcing $c = d = 0$, then $V_{IV} = \frac{\Lambda_1 a^2}{2}$. Thus for V_{IV} not to be largely negative requires,

$$\Lambda_1 \geq 0 \,. \tag{2.6}$$

- When $a = 0$ and $b \to \infty$ thereby forcing $c = d = 0$, then $V_{IV} = \frac{\Lambda_2 b^2}{2}$. Thus for V_{IV} not to be largely negative requires,

$$\Lambda_2 \geq 0 \,. \tag{2.7}$$

- Next if we consider the field direction along which $a = \sqrt{\Lambda_2/\Lambda_1}b$ and $c = d = 0$, then the first term in Eq. (2.5) vanishes. Now if we go to large field values in this direction i.e., $a, b \to \infty$ then $V_{IV} = \left(\Lambda_3 + \sqrt{\Lambda_1\Lambda_2}\right)ab$. Since $a, b > 0$ by definition, the condition for the potential to be bounded from below turns out to be,

$$\Lambda_3 + \sqrt{\Lambda_1\Lambda_2} \geq 0 \,. \tag{2.8}$$

- Now if we again choose the field direction to be $a = \sqrt{\Lambda_2/\Lambda_1}b$ and further impose $ab = c^2 + d^2$ then the quartic potential takes the below form,

$$V_{IV} = Xc^2 + 2Ycd + Zd^2\,, \tag{2.9}$$

$$\begin{aligned}
\text{where,}\quad X &= \text{Re}\,\Lambda_5 + \beta\,, &(2.10)\\
Y &= -\text{Im}\,\Lambda_5\,, &(2.11)\\
Z &= -\text{Re}\,\Lambda_5 + \beta\,, &(2.12)\\
\text{with}\quad \beta &= \Lambda_3 + \Lambda_4 + \sqrt{\Lambda_1\Lambda_2}\,. &(2.13)
\end{aligned}$$

Still now c and d are arbitrary and thereby choosing $d = 0$, $c \to \infty$ and $c = 0$, $d \to \infty$ successively we land up with two conditions,

$$X = \text{Re}\,\Lambda_5 + \beta \geq 0\,, \tag{2.14}$$

$$Z = -\text{Re}\,\Lambda_5 + \beta \geq 0\,. \tag{2.15}$$

This enforces $\beta \geq 0$.

Let us recast Eq. (2.9) as,

$$V_{IV} = X\left(c + \frac{Y}{X}d\right)^2 + \left(Z - \frac{Y^2}{X}\right)d^2\,. \tag{2.16}$$

If we now choose $c = -(Y/X)\,d$ with $d \to \infty$, then we have,

$$\begin{aligned}
Z - \frac{Y^2}{X} &> 0\\
\Rightarrow XZ &> Y^2\,, &(2.17)
\end{aligned}$$

where we have used the fact that $X > 0$ from Eq. (2.14). After substituting for X, Y, Z we get from Eq. (2.17),

$$\begin{aligned}
\beta^2 - (\text{Re}\,\Lambda_5)^2 &> (\text{Im}\,\Lambda_5)^2\\
\Rightarrow \beta^2 &> \mid\Lambda_5\mid^2\\
\Rightarrow \beta &> \mid\Lambda_5\mid\,, &(2.18)
\end{aligned}$$

where we have used the fact of non-negativity of β. Since $\mid\Lambda_5\mid > \pm\text{Re}\,\Lambda_5, 0$ therefore $\beta > \mid\Lambda_5\mid$ puts a stronger constraint on β than $\beta > 0$. Substituting for β, the above constraint on β takes the form,

$$\Lambda_3 + \Lambda_4 + \sqrt{\Lambda_1\Lambda_2} > \mid\Lambda_5\mid\,. \tag{2.19}$$

We use Eq. (2.3) to express the above obtained four stability conditions Eq. (2.6), Eq. (2.7), Eq. (2.8) and Eq. (2.19) in terms of the parameters of Eq. (2.2) i.e., λ's.

$$\lambda_1 + \lambda_3 > 0\,,$$
$$\lambda_2 + \lambda_3 > 0\,,$$
$$(2\lambda_3 + \lambda_4) + 2\sqrt{(\lambda_1 + \lambda_3)\,(\lambda_2 + \lambda_3)} > 0\,,$$
$$2\lambda_3 + \frac{\lambda_5 + \lambda_6}{2} - \frac{|\,\lambda_5 - \lambda_6\,|}{2} + 2\sqrt{(\lambda_1 + \lambda_3)\,(\lambda_2 + \lambda_3)} > 0\,. \quad (2.20)$$

2.2 Perturbative unitarity constraints

Every scattering amplitude can be expanded in terms of the partial waves,

$$\mathcal{M}\,(\theta) = 16\pi \sum_{l=0}^{\infty} (2l+1)\, a_l P_l\,(\cos\theta)\ , \quad (2.21)$$

where $\mathcal{M}$ is the scattering amplitude, θ is the angle of scattering, a_l is the partial wave amplitude and $P_l\,(x)$ is the *Legendre* polynomial of order l. *'Unitarity condition'* on the partial wave amplitude is,

$$|a_l| \leq 1 \quad (2.22)$$

and if we hold the hands of perturbative calculations then the above unitarity condition must be satisfied order by order [36, 37]. This means it must hold at tree level also. The fundamentals used to derive some restrictions on the Higgs masses from this knowledge of unitarity condition is explained now. The Feynman amplitude of a certain $2 \rightarrow 2$ scattering amplitude was calculated from which the partial wave amplitude a_l was obtained by utilizing the property of orthonormality of the *Legendre* polynomials. Lee, Quigg and Thacker (LQT) [38] had done pioneering work in the context of standard model. They had analysed several two body scatterings involving longitudinal gauge bosons and physical Higgs in the SM. Similar analysis was done for two Higgs doublet models [39–42] considering similar two body scattering processes where using the equivalence theorem [43, 44] unphysical Higgs masses were used instead of the longitudinal components of the gauge bosons in the high energy limit. For this analysis the potential was written in the second notation as in Eq. (2.2). It was shown that all such scattering amplitudes are proportional to the Higgs quartic couplings in the high energy limit. The trilinear vertices were suppressed by a factor of E^2 coming from the intermediate propagator therefore their contribution to the scattering amplitudes at high energies were neglected. Next the $l = 0$ partial wave amplitude denoted by a_0 was extracted from these amplitudes and a matrix was formed having different two-body states as rows and columns. This is the well known S-matrix. It was shown that the largest eigenvalue of the S-matrix is bounded by the unitarity constraint, $|a_0| \leq 1$. The largest eigenvalue can be written in terms of the quartic Higgs self couplings and therefore the quartic Higgs self couplings as well as the non-physical Higgs masses get restricted to a maximum value. Thus our purpose is to find a_0 for every possible $2 \rightarrow 2$ scattering process and then cast them in the form of an S-matrix which is constructed by taking the different two-body channels as rows and columns. Unitarity will restrict the magnitude of each of the eigenvalues of this S-matrix to lie below unity.

If we consider the below parametrization for the fields as given in Eq. (1.9) ,

$$\Phi_i = \begin{pmatrix} w_i^+(x) \\ \dfrac{v_i + h_i(x) + i z_i(x)}{\sqrt{2}} \end{pmatrix}, \qquad i = 1, 2\,, \tag{2.23}$$

the possible two particle states are made of fields $w_i^\pm$, h_i and z_i. For our purpose we will confine ourselves to the neutral two-particle states and singly charged two-particle states viz., $w_i^+ w_j^-$, $h_i h_j$, $z_i z_j$, $h_i z_j$, $w_i^+ h_j$, $w_i^+ z_j$ etc. Generalizing the result for n- doublets Φ_k where $k = 1, \cdots, n$ there will be $3n^2 + n$ neutral two-particle states and $2n^2$ charged two-particle states. Thus the S-matrix will be of dimension $\big(3n^2 + n\big) \times \big(3n^2 + n\big)$ for the neutral $2 \to 2$ scattering processes and of dimension $2n^2 \times 2n^2$ for charged $2 \to 2$ scattering processes.

In our case where there are two Higgs doublets i.e., $n = 2$, the neutral channel S-matrix will be a 14×14 matrix. The rows and columns will be the following two-particle states:

$$w_1^+ w_1^-\,,\; w_2^+ w_2^-\,,\; w_1^+ w_2^-\,,\; w_2^+ w_1^-\,,\; \frac{h_1 h_1}{\sqrt{2}}\,,\; \frac{z_1 z_1}{\sqrt{2}}\,,$$
$$\frac{h_2 h_2}{\sqrt{2}}\,,\; \frac{z_2 z_2}{\sqrt{2}}\,,\; h_1 z_2\,,\; h_2 z_1\,,\; z_1 z_2\,,\; h_1 h_2\,,\; h_1 z_1\,,\; h_2 z_2\,.$$

Bose symmetry gives rise to a factor of $1/\sqrt{2}$ when identical particle states arise. Without any symmetry, finding the eigenvalues of a 14×14 matrix will be a tedious task to accomplish. But with the form of the potential in Eq. (2.2), there are some obvious symmetries involved with the quartic terms. These symmetries make our life a bit easy since now we can decompose the 14×14 S-matrix into smaller blocks. It is to be noted that the quartic part of the potential always contain even number of indices, 1 or 2. Consequently a state $x_1 y_1$ or $x_2 y_2$ will always go into $x_1 y_1$ or $x_2 y_2$ but not into $x_1 y_2$ or $x_2 y_1$ and vice versa. Furthermore since CP symmetry is conserved there will be no mixing between the CP-even and the CP-odd states i.e., a neutral state with combination $h_i h_j$ or $z_i z_j$ will never end up going into $h_i z_j$. Keeping these facts in mind we now decompose the S-matrix in the neutral sector into smaller blocks.

$$\mathcal{M}_N = \begin{pmatrix} \big(\mathcal{M}_N^{11}\big)_{6\times 6} & 0 & 0 \\ 0 & \big(\mathcal{M}_N^{11}\big)_{2\times 2} & 0 \\ 0 & 0 & \big(\mathcal{M}_N^{12}\big)_{6\times 6} \end{pmatrix}. \tag{2.24}$$

The sub-matrices are given below,

$$\big(\mathcal{M}_N^{11}\big)_{6\times 6} = \begin{pmatrix} \big(\mathcal{A}_N^{11}\big)_{3\times 3} & \big(\mathcal{B}_N^{11}\big)_{3\times 3} \\ \big(\mathcal{B}_N^{11}\big)^\dagger_{3\times 3} & \big(\mathcal{C}_N^{11}\big)_{3\times 3} \end{pmatrix}, \tag{2.25}$$

where,

$$\left(\mathcal{A}_N^{11}\right)_{3\times3} = \begin{array}{c} \\ w_1^+w_1^- \\ w_2^+w_2^- \\ \frac{z_1z_1}{\sqrt{2}} \end{array} \begin{array}{c} \begin{array}{ccc} w_1^+w_1^- & w_2^+w_2^- & \frac{z_1z_1}{\sqrt{2}} \end{array} \\ \begin{pmatrix} 4\left(\lambda_1+\lambda_3\right) & 2\lambda_3+\frac{\lambda_5+\lambda_6}{2} & \sqrt{2}\left(\lambda_1+\lambda_3\right) \\ 2\lambda_3+\frac{\lambda_5+\lambda_6}{2} & 4\left(\lambda_2+\lambda_3\right) & \sqrt{2}\left(\lambda_3+\frac{\lambda_4}{2}\right) \\ \sqrt{2}\left(\lambda_1+\lambda_3\right) & \sqrt{2}\left(\lambda_3+\frac{\lambda_4}{2}\right) & 3\left(\lambda_1+\lambda_3\right) \end{pmatrix} \end{array}, \tag{2.26}$$

$$\left(\mathcal{B}_N^{11}\right)_{3\times3} = \begin{array}{c} \\ w_1^+w_1^- \\ w_2^+w_2^- \\ \frac{z_1z_1}{\sqrt{2}} \end{array} \begin{array}{c} \begin{array}{ccc} \frac{h_1h_1}{\sqrt{2}} & \frac{z_2z_2}{\sqrt{2}} & \frac{h_2h_2}{\sqrt{2}} \end{array} \\ \begin{pmatrix} \sqrt{2}\left(\lambda_1+\lambda_3\right) & \sqrt{2}\left(\lambda_3+\frac{\lambda_4}{2}\right) & \sqrt{2}\left(\lambda_3+\frac{\lambda_4}{2}\right) \\ \sqrt{2}\left(\lambda_3+\frac{\lambda_4}{2}\right) & \sqrt{2}\left(\lambda_2+\lambda_3\right) & \sqrt{2}\left(\lambda_2+\lambda_3\right) \\ \left(\lambda_1+\lambda_3\right) & \left(\lambda_3+\frac{\lambda_5}{2}\right) & \left(\lambda_3+\frac{\lambda_6}{2}\right) \end{pmatrix} \end{array}, \tag{2.27}$$

$$\left(\mathcal{C}_N^{11}\right)_{3\times3} = \begin{array}{c} \\ \frac{h_1h_1}{\sqrt{2}} \\ \frac{z_2z_2}{\sqrt{2}} \\ \frac{h_2h_2}{\sqrt{2}} \end{array} \begin{array}{c} \begin{array}{ccc} \frac{h_1h_1}{\sqrt{2}} & \frac{z_2z_2}{\sqrt{2}} & \frac{h_2h_2}{\sqrt{2}} \end{array} \\ \begin{pmatrix} 3\left(\lambda_1+\lambda_3\right) & \left(\lambda_3+\frac{\lambda_6}{2}\right) & \left(\lambda_3+\frac{\lambda_5}{2}\right) \\ \left(\lambda_3+\frac{\lambda_6}{2}\right) & 3\left(\lambda_2+\lambda_3\right) & \left(\lambda_2+\lambda_3\right) \\ \left(\lambda_3+\frac{\lambda_5}{2}\right) & \left(\lambda_2+\lambda_3\right) & 3\left(\lambda_2+\lambda_3\right) \end{pmatrix} \end{array}. \tag{2.28}$$

$$\left(\mathcal{M}_N^{11}\right)_{2\times2} = \begin{array}{c} \\ h_1z_1 \\ h_2z_2 \end{array} \begin{array}{c} \begin{array}{cc} h_1z_1 & h_2z_2 \end{array} \\ \begin{pmatrix} 2\left(\lambda_1+\lambda_3\right) & \left(\frac{\lambda_5-\lambda_6}{2}\right) \\ \left(\frac{\lambda_5-\lambda_6}{2}\right) & 2\left(\lambda_2+\lambda_3\right) \end{pmatrix} \end{array}, \tag{2.29}$$

$$\left(\mathcal{M}_N^{12}\right)_{6\times6} = \begin{pmatrix} \left(\mathcal{A}_N^{12}\right)_{3\times3} & \left(\mathcal{B}_N^{12}\right)_{3\times3} \\ \left(\mathcal{B}_N^{12}\right)^\dagger_{3\times3} & \left(\mathcal{C}_N^{12}\right)_{3\times3} \end{pmatrix}, \tag{2.30}$$

where,

$$\left(\mathcal{A}_N^{12}\right)_{3\times3} = \begin{array}{c} \\ w_1^+w_2^- \\ w_2^+w_1^- \\ h_1z_2 \end{array} \begin{array}{c} \begin{array}{ccc} w_1^+w_2^- & w_2^+w_1^- & h_1z_2 \end{array} \\ \begin{pmatrix} 2\lambda_3+\frac{\lambda_5+\lambda_6}{2} & \lambda_5-\lambda_6 & -\frac{i}{2}\left(\lambda_4-\lambda_6\right) \\ \lambda_5-\lambda_6 & 2\lambda_3+\frac{\lambda_5+\lambda_6}{2} & \frac{i}{2}\left(\lambda_4-\lambda_6\right) \\ \frac{i}{2}\left(\lambda_4-\lambda_6\right) & -\frac{i}{2}\left(\lambda_4-\lambda_6\right) & 2\lambda_3+\lambda_6 \end{pmatrix} \end{array}, \tag{2.31}$$

$$\left(\mathcal{B}_N^{12}\right)_{3\times3} = \begin{array}{c} \\ w_1^+w_2^- \\ w_2^+w_2^- \\ h_1z_2 \end{array} \begin{array}{c} \begin{array}{ccc} h_2z_1 & z_1z_2 & h_1h_2 \end{array} \\ \begin{pmatrix} \frac{i}{2}\left(\lambda_4-\lambda_6\right) & \frac{\lambda_5-\lambda_4}{2} & \frac{\lambda_5-\lambda_4}{2} \\ -\frac{i}{2}\left(\lambda_4-\lambda_6\right) & \frac{\lambda_5-\lambda_4}{2} & \frac{\lambda_5-\lambda_4}{2} \\ \frac{\lambda_5-\lambda_6}{2} & 0 & 0 \end{pmatrix} \end{array}, \tag{2.32}$$

$$\left(\mathcal{C}_N^{11}\right)_{3\times 3} = \begin{array}{c} \\ h_2z_1 \\ z_1z_2 \\ h_1h_2 \end{array} \!\!\begin{array}{c} \begin{array}{ccc} h_2z_1 & z_1z_2 & h_1h_2 \end{array} \\ \begin{pmatrix} 2\lambda_3+\lambda_6 & 0 & 0 \\ 0 & 2\lambda_3+\lambda_5 & \frac{\lambda_5-\lambda_6}{2} \\ 0 & \frac{\lambda_5-\lambda_6}{2} & 2\lambda_3+\lambda_5 \end{pmatrix} \end{array}. \tag{2.33}$$

The eigenvalues in the neutral sector are,

- $\left(\mathcal{M}_N^{11}\right)_{6\times 6} \,:\, a_1^{\pm}\,, a_2^{\pm}\,, a_3^{\pm}$
- $\left(\mathcal{M}_N^{11}\right)_{2\times 2} \,:\, a_3^{\pm}$
- $\left(\mathcal{M}_N^{12}\right)_{6\times 6} \,:\, b_1\,, b_2\,, b_3\,, b_4\,, b_5$

b_5 is two-fold degenerate. The explicit expressions for the eigenvalues will be listed later on.

When we repeat the same procedure for the singly charged two particle states we land up to a 8×8 matrix which can be similarly decomposed into block diagonal form. In the charged sector the corresponding matrices are written below.

$$\mathcal{M}_C = \begin{pmatrix} \left(\mathcal{M}_C^{11}\right)_{4\times 4} & 0 \\ 0 & \left(\mathcal{M}_C^{12}\right)_{4\times 4} \end{pmatrix}, \tag{2.34}$$

where the submatrices are,

$$\left(\mathcal{M}_C^{11}\right)_{4\times 4} = \begin{array}{c} \\ h_1w_1^+ \\ h_2w_2^+ \\ z_1w_1^+ \\ z_2w_2^+ \end{array} \!\!\begin{array}{c} \begin{array}{cccc} h_1w_1^+ & h_2w_2^+ & z_1w_1^+ & z_2w_2^+ \end{array} \\ \begin{pmatrix} 2\left(\lambda_1+\lambda_3\right) & \frac{\lambda_5-\lambda_4}{2} & 0 & -\frac{i}{2}\left(\lambda_4-\lambda_6\right) \\ \frac{\lambda_5-\lambda_4}{2} & 2\left(\lambda_2+\lambda_3\right) & -\frac{i}{2}\left(\lambda_4-\lambda_6\right) & 0 \\ 0 & \frac{i}{2}\left(\lambda_4-\lambda_6\right) & 2\left(\lambda_1+\lambda_3\right) & \frac{\lambda_5-\lambda_4}{2} \\ \frac{i}{2}\left(\lambda_4-\lambda_6\right) & 0 & \frac{\lambda_5-\lambda_4}{2} & 2\left(\lambda_2+\lambda_3\right) \end{pmatrix} \end{array}, \tag{2.35}$$

$$\left(\mathcal{M}_C^{12}\right)_{4\times 4} = \begin{array}{c} \\ h_1w_2^+ \\ h_2w_1^+ \\ z_1w_2^+ \\ z_2w_1^+ \end{array} \!\!\begin{array}{c} \begin{array}{cccc} h_1w_2^+ & h_2w_1^+ & z_1w_2^+ & z_2w_1^+ \end{array} \\ \begin{pmatrix} 2\lambda_3+\lambda_4 & \frac{\lambda_5-\lambda_4}{2} & 0 & \frac{i}{2}\left(\lambda_4-\lambda_6\right) \\ \frac{\lambda_5-\lambda_4}{2} & 2\lambda_3+\lambda_4 & \frac{i}{2}\left(\lambda_4-\lambda_6\right) & 0 \\ 0 & -\frac{i}{2}\left(\lambda_4-\lambda_6\right) & 2\lambda_3+\lambda_4 & \frac{\lambda_5-\lambda_4}{2} \\ -\frac{i}{2}\left(\lambda_4-\lambda_6\right) & 0 & \frac{\lambda_5-\lambda_4}{2} & 2\lambda_3+\lambda_4 \end{pmatrix} \end{array}. \tag{2.36}$$

The eigenvalues in the charged sector are,

- $\left(\mathcal{M}_C^{11}\right)_{4\times 4} \,:\, a_2^{\pm}\,, a_3^{\pm}$
- $\left(\mathcal{M}_C^{12}\right)_{4\times 4} \,:\, b_2\,, b_4\,, b_5\,, b_6\,.$

The explicit expressions for the eigenvalues in the neutral and charged sectors are listed below.

$$
\begin{aligned}
a_1^{\pm} &= 3\left(\lambda_1+\lambda_2+2\lambda_3\right) \pm \sqrt{9\left(\lambda_1-\lambda_2\right)^2+\left(4\lambda_3+\lambda_4+\frac{\lambda_5+\lambda_6}{2}\right)^2}\,, && (2.37)\\
a_2^{\pm} &= \left(\lambda_1+\lambda_2+2\lambda_3\right) \pm \sqrt{\left(\lambda_1-\lambda_2\right)^2+\frac{1}{4}\left(2\lambda_4-\lambda_5-\lambda_6\right)^2}\,, && (2.38)\\
a_3^{\pm} &= \left(\lambda_1+\lambda_2+2\lambda_3\right) \pm \sqrt{\left(\lambda_1-\lambda_2\right)^2+\frac{1}{4}\left(\lambda_5-\lambda_6\right)^2}\,, && (2.39)\\
b_1 &= 2\lambda_3-\lambda_4-\frac{1}{2}\lambda_5+\frac{5}{2}\lambda_6\,, && (2.40)\\
b_2 &= 2\lambda_3+\lambda_4-\frac{1}{2}\lambda_5+\frac{1}{2}\lambda_6\,, && (2.41)\\
b_3 &= 2\lambda_3-\lambda_4+\frac{5}{2}\lambda_5-\frac{1}{2}\lambda_6\,, && (2.42)\\
b_4 &= 2\lambda_3+\lambda_4+\frac{1}{2}\lambda_5-\frac{1}{2}\lambda_6\,, && (2.43)\\
b_5 &= 2\lambda_3+\frac{1}{2}\lambda_5+\frac{1}{2}\lambda_6\,, && (2.44)\\
b_6 &= 2\left(\lambda_3+\lambda_4\right)-\frac{1}{2}\lambda_5-\frac{1}{2}\lambda_6\,. && (2.45)
\end{aligned}
$$

Coming back to the theory explained initially, each of these eigenvalues get an upper and lower bound from the unitarity constraint as,

$$|a_i^{\pm}|\,,\ |b_i| \leq 16\pi\,. \tag{2.46}$$

Hence the unitarity constraints.

2.3 New Physics corrections

The information about the relative strength between the neutral current and the charged current interactions in four fermion processes at zero momentum transfer is carried by the ρ parameter [45]. It is defined as

$$\rho = \frac{m_W^2}{m_Z^2\cos^2\theta_W}\,, \tag{2.47}$$

where m_W and m_Z are the masses of the $W^{\pm}$ and Z^0 gauge bosons respectively and θ_W is the weak mixing angle. ρ is unity at the tree level for the Standard model. At one-loop level, the vacuum polarization effects due to fields that couple either to the $W^{\pm}$ or to Z^0 produce the vacuum polarization tensors,

$$\Pi_{VV}^{\mu\nu}(q) = g^{\mu\nu}A_{VV}\left(q^2\right) + q^{\mu}q^{\nu}B_{VV}\left(q^2\right)\,, \tag{2.48}$$

where V represents either W or Z bosons and q^μ is their four momentum. Thus at one-loop level the deviations from unity arise due to the difference in the self energies as given below [45, 46],

$$\rho - 1 = \frac{A_{WW}(0)}{m_W^2} - \frac{A_{ZZ}(0)}{m_Z^2}. \tag{2.49}$$

[47] can be referred for the three loop SM corrections to the above equation upto leading order terms. The precise measurement of the $W^\pm$ and Z^0 self energies at the LEP [48] are in striking agreement with the SM predictions [49] and thereby strongly constrains the extended electroweak models. Though as discussed in Section 1.1 the value of ρ remains close to unity if additional $SU(2)$ scalar doublets are added to the SM whose hypercharge is $\pm 1/2$ still they are constrained by the experimental data. Thus the parameter space for the non-standard variables of the extended models get restricted. We explore this to provide some mass bounds to the non-standard Higgs bosons of the two Higgs doublet models [50, 51].

We define $\Delta\rho$ as the deviation of the ρ parameter from the SM value. It is defined as the non-standard part of the difference in self energies Eq. (2.49) and is given as,

$$\Delta\rho = \left[\frac{A_{WW}(0)}{m_W^2} - \frac{A_{ZZ}(0)}{m_Z^2}\right]_{\text{non}-\text{SM}} - \left[\frac{A_{WW}(0)}{m_W^2} - \frac{A_{ZZ}(0)}{m_Z^2}\right]_{\text{SM}}. \tag{2.50}$$

It is to be noted that the function $A_{VV}(q^2)$ carries more information than $\Delta\rho$. Since the consistent SM subtraction to the above equation requires only results upto one loop therefore we can replace m_Z^2 by $m_W^2/c_{\theta_W}^2$ where $c_{\theta_W} \equiv \cos\theta_W$ and θ_W is the well known Weinberg angle. Making this replacement Eq. (2.50) looks like,

$$\Delta\rho = \left[\frac{A_{WW}(0) - c_{\theta_W}^2 A_{ZZ}(0)}{m_W^2}\right]_{\text{non}-\text{SM}} - \left[\frac{A_{WW}(0) - c_{\theta_W}^2 A_{ZZ}(0)}{m_W^2}\right]_{\text{SM}}. \tag{2.51}$$

When new physics is introduced the contributing Feynman diagrams are mostly divergent but it is observed through elaborate calculation that the divergent parts cancel out among different Feynman diagrams between $A_{WW}(0)$ and $c_{\theta_W}^2 A_{ZZ}(0)$. The leftover divergent contributions get cancelled with the SM subtraction as laid down in Eq. (2.51). After these cancellations we are left with either quadratic or logarithmic dependence of $\Delta\rho$ on the masses of the new physics particles. $\Delta\rho$ is finite if the new physics is a renormalizable model. If the masses of the non-standard particles are large then this leaves a pronounced effect on $\Delta\rho$ and this is used to probe physics beyond the Standard Model.

A detailed analysis of the "oblique corrections" for physics whose scale is much above the electroweak scale lead to the identification of three relevant observables in this respect. These are the S, T and U parameters as defined in [52] and ϵ_1, ϵ_2 and

ϵ_3 as designated in [53]. Without going into the precise definitions of these two set of observables, we choose to relate the quantities which interest us in our further work.

$$\Delta\rho = \alpha T = \epsilon_1 \,, \tag{2.52}$$

where $\alpha = e^2/4\pi = g^2 s^2_{\theta_W}/4\pi$ is the fine structure constant.

It is not straightforward to obtain a bound on $\Delta\rho$ from electroweak precision data. However to get an idea about the order of magnitude of $\Delta\rho$ we quote the number

$$T = -0.03 \pm 0.09 \; (+0.09) \;, \tag{2.53}$$

which was obtained in [49] by setting $U = 0$. Higgs boson mass, m_h was assumed to be 117 GeV for the mean value of T and the mean value in parenthesis is for $m_h = 300$ GeV. Eq. (2.53) translates to $\Delta\rho = -0.0002 \pm 0.0007 \; (+0.0007)$.

The expression for $\Delta\rho$ for a new physics model containing $n-$ $SU(2)$ scalar Higgs doublets with hypercharge 1/2, $p-$ number of complex $SU(2)$ singlets with hypercharge 1 and $q-$ number of real $SU(2)$ singlets with hypercharge 0 has been derived elaborately in [54]. We simplify it for 2HDMs. It is convenient to work in the Higgs basis for this calculation in which the vacuum expectation value is associated with the first Higgs doublet only. In this basis

$$\Phi_1 = \begin{pmatrix} G^+ \\ \left(v + H + iG^0\right)/\sqrt{2} \end{pmatrix} , \quad \Phi_2 = \begin{pmatrix} S_2^+ \\ \left(R + iI\right)/\sqrt{2} \end{pmatrix} . \tag{2.54}$$

Here G^+ and G^0 are the charged and neutral Goldstone bosons and S_2^+ is the physical charged scalar with mass m_2. Thus the matrix which relates the charged components of Φ_1 and Φ_2 to the corresponding mass eigenstates is the unit matrix in the Higgs basis. We denote this unit matrix by U. H, R and I are the real fields which are related to their corresponding mass eigenstates $S^0_{2,3,4}$ through an orthogonal matrix O as shown below,

$$\begin{pmatrix} H \\ R \\ I \end{pmatrix} = O \begin{pmatrix} S_2^0 \\ S_3^0 \\ S_4^0 \end{pmatrix} . \tag{2.55}$$

We can choose det$O = +1$ without any loss of generality. We now define a $2\times$ 4 matrix V as,

$$\begin{pmatrix} H + iG^0 \\ R + iI \end{pmatrix} = V \begin{pmatrix} G^0 \\ S_2^0 \\ S_3^0 \\ S_4^0 \end{pmatrix} , \tag{2.56}$$

where V has the form,

$$V = \begin{pmatrix} i & O_{11} & O_{12} & O_{13} \\ 0 & O_{21} + iO_{31} & O_{22} + iO_{32} & O_{23} + iO_{33} \end{pmatrix} . \tag{2.57}$$

Thus,

$$V^\dagger V = \begin{pmatrix} 1 & -iO_{11} & -iO_{12} & -iO_{13} \\ iO_{11} & 1 & iO_{13} & -iO_{12} \\ iO_{12} & -iO_{13} & 1 & iO_{11} \\ iO_{13} & iO_{12} & -iO_{11} & 1 \end{pmatrix} . \tag{2.58}$$

Grimus and others have derived the value of $\Delta\rho$ in their paper [54]. We quote the expression here,

$$\begin{aligned} \Delta\rho &= \frac{g^2}{64\pi^2 m_W^2}\Bigg(\sum_{a=2}^{4}\left(1-O_{1a-1}^2\right)F\left(m_2^2,\mu_a^2\right) \\ &\quad -O_{13}^2 F\left(\mu_2^2,\mu_3^2\right) - O_{12}^2 F\left(\mu_2^2,\mu_4^2\right) - O_{11}^2 F\left(\mu_3^2,\mu_4^2\right) \\ &\quad +3\sum_{a=2}^{4} O_{1b-1}^2\left[F\left(m_Z^2,\mu_a^2\right) - F\left(m_W^2,\mu_a^2\right) - F\left(m_Z^2,m_h^2\right) + F\left(m_W^2,m_h^2\right)\right]\Bigg) \end{aligned} \tag{2.59}$$

where $\mu_{2,3,4}$ denote the masses of $S^0_{2,3,4}$ respectively and m_h is the mass of the SM Higgs boson.

The function F of two non-negative arguments x and y is defined as,

$$F(x,y) \equiv \begin{cases} \frac{x+y}{2} - \frac{xy}{x-y}\ln\frac{x}{y} & \text{for} \quad x \neq y\,, \\ 0 & \text{for} \quad x = y\,. \end{cases} \tag{2.60}$$

$F(x,y)$ is a non-negative function and is symmetrical under the interchange of its arguments. It vanishes if and only if the two arguments are equal. $F(x,y)$ grows linearly with the maximum of the two arguments i.e. quadratically with the heaviest scalar mass when the scalar becomes obese. Only cancellations can prevent the divergence of $\Delta\rho$.

The scalar potential [55, 56] with a U(1) symmetry which forbids flavor-changing neutral currents (FCNCs) is written below,

$$\begin{aligned} V &= \lambda_1\left(|\Phi_1|^2 - \frac{v_1^2}{2}\right)^2 + \lambda_2\left(|\Phi_2|^2 - \frac{v_2^2}{2}\right)^2 \\ &\quad +\lambda_3\left(|\Phi_1|^2 + |\Phi_2|^2 - \frac{v_1^2+v_2^2}{2}\right)^2 \\ &\quad +\lambda_4\left(|\Phi_1|^2|\Phi_2|^2 - |\Phi_1^\dagger\Phi_2|^2\right) \\ &\quad +\lambda_5\left|\Phi_1^\dagger\Phi_2 - \frac{v_1 v_2}{2}\right|^2 , \end{aligned} \tag{2.61}$$

with real λ_i.

When the scalar doublets are parametrized as in Eq. (1.9)

$$\Phi_i = \begin{pmatrix} w_i^+(x) \\ \frac{v_i + h_i(x) + i z_i(x)}{\sqrt{2}} \end{pmatrix} , \qquad i = 1, 2 \tag{2.62}$$

where the VEVs v_i may be taken to be real and positive without any loss of generality. Three of these fields get "eaten" by the $W^\pm$ and Z^0 gauge bosons; the remaining five are physical scalar fields. There is a pair of charged scalars denoted by $\xi^\pm$, two neutral CP-even scalars H and h, and one CP-odd pseudoscalar denoted by A. The two CP-even scalars have distinct masses, and $m_h < m_H$. With

$$\tan\beta = \frac{v_2}{v_1}\,, \tag{2.63}$$

the scalar fields are given by the combinations

$$\begin{pmatrix} \omega^\pm \\ \xi^\pm \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} w_1^\pm \\ w_2^\pm \end{pmatrix}, \tag{2.64}$$

$$\begin{pmatrix} \zeta \\ A \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \tag{2.65}$$

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \tag{2.66}$$

where $c_\alpha \equiv \cos\alpha$, etc. These have already been discussed in Chapter 1 through Eq. (1.19), Eq. (1.21) and Eq. (1.23). We will assume, without loss of generality, that $0 \le \beta \le \frac{\pi}{2}$, and $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$.

With this parametrization, $\Delta\rho$ in Eq. (2.59) takes the below form,

$$\begin{aligned}\Delta\rho = \frac{g^2}{64\pi^2 m_w^2}\Big(& F(m_\xi^2, m_A^2) + \sin^2(\beta-\alpha) F(m_\xi^2, m_H^2) + \cos^2(\beta-\alpha) F(m_\xi^2, m_h^2) \\ & - \sin^2(\beta-\alpha) F(m_A^2, m_H^2) - \cos^2(\beta-\alpha) F(m_A^2, m_h^2) \\ & + 3\cos^2(\beta-\alpha)\left[F(m_Z^2, m_H^2) - F(m_W^2, m_H^2)\right] \\ & + 3\sin^2(\beta-\alpha)\left[F(m_Z^2, m_h^2) - F(m_W^2, m_h^2)\right] \\ & - 3\left[F(m_Z^2, m_{h_{SM}}^2) - F(m_W^2, m_{h_{SM}}^2)\right]\Big)\,, \end{aligned} \tag{2.67}$$

PDG quotes $\rho_0 = 1.00039 \pm 0.00019$ for the global fit [57] of precision electroweak observables. When the work on finding the mass ranges of the non-standard Higgs bosons was done the then current experimental bound on the total new physics contribution to ρ was given by $\delta\rho = -0.00011$ [58].

Higgs contributions to the oblique electroweak parameters are shown in Chapter C.

References

[1] P. Q. Hung, Phys. Rev. Lett. 42, 873 (1979).

[2] N. Cabibbo, L. Maiani, G. Parisi, and R. Petronzio, Nucl. Phys. B158, 295 (1979).

[3] R. A. Flores and M. Sher, Phys. Rev. D27, 1679 (1983).

[4] M. Lindner, Z. Phys. C31, 295 (1986).

[5] M. Sher, Phys. Rept. 179, 273 (1989).

[6] M. Lindner, M. Sher, and H. W. Zaglauer, Phys. Lett. B228, 139 (1989).

[7] C. Ford, D. R. T. Jones, P. W. Stephenson, and M. B. Einhorn, Nucl. Phys. B395, 17 (1993), hep-lat/9210033.

[8] M. Sher, Phys. Lett. B317, 159 (1993), hep-ph/9307342, [Addendum: Phys.Lett.B331,448(1994)].

[9] G. Isidori, G. Ridolfi, and A. Strumia, Nucl. Phys. B609, 387 (2001), hep-ph/0104016.

[10] M. B. Einhorn and D. R. T. Jones, JHEP 04, 051 (2007), hep-ph/0702295.

[11] F. Bezrukov, M. Yu. Kalmykov, B. A. Kniehl, and M. Shaposhnikov, JHEP 10, 140 (2012), 1205.2893, [,275(2012)].

[12] G. Degrassi et al., JHEP 08, 098 (2012), 1205.6497.

[13] D. Buttazzo et al., JHEP 12, 089 (2013), 1307.3536

[14] O. Lebedev, Eur. Phys. J. C72, 2058 (2012), 1203.0156.

[15] J. Elias-Miro, J. R. Espinosa, G. F. Giudice, H. M. Lee, and A. Strumia, JHEP 06, 031 (2012), 1203.0237.

[16] G. M. Pruna and T. Robens, Phys. Rev. D88, 115012 (2013), 1303.1150.

[17] R. Costa, A. P. Morais, M. O. P. Sampaio, and R. Santos, Phys. Rev. D92, 025024 (2015), 1411.4048.

[18] V. Branchina and E. Messina, Phys. Rev. Lett. 111, 241801 (2013), 1307.5193.

[19] V. Branchina, E. Messina, and A. Platania, JHEP 09, 182 (2014), 1407.4112.

[20] V. Branchina, E. Messina, and M. Sher, Phys. Rev. D91, 013003 (2015), 1408.5302.

[21] V. Branchina, E. Messina, and D. Zappala, EPL 116, 21001 (2016), 1601.06963.

[22] G. Kreyerhoff and R. Rodenberg, Phys. Lett. B226, 323 (1989).

[23] S. Nie and M. Sher, Phys. Lett. B449, 89 (1999), hep-ph/9811234.

[24] S. Kanemura, T. Kasai, and Y. Okada, Phys. Lett. B471, 182 (1999), hep-ph/9903289.

[25] P. M. Ferreira and D. R. T. Jones, JHEP 08, 069 (2009), 0903.2856.

[26] N. Chakrabarty, U. K. Dey, and B. Mukhopadhyaya, JHEP 12, 166 (2014), 1407.2145.

[27] P. S. Bhupal Dev and A. Pilaftsis, JHEP 12, 024 (2014), 1408.3405 [Erratum:JHEP11,147(2015)].

[28] D. Das and I. Saha, Phys. Rev. D91, 095024 (2015), 1503.02135.

[29] D. Chowdhury and O. Eberhardt, JHEP 11, 052 (2015), 1503.08216.

[30] P. Ferreira, H. E. Haber, and E. Santos, Phys. Rev. D92, 033003 (2015), 1505.04001, [Erratum: Phys. Rev. D94, no.5, 059903 (2016)].

[31] N. Chakrabarty and B. Mukhopadhyaya, Eur. Phys. J. C77, 153 (2017), 1603.05883.

[32] V. Cacchio, D. Chowdhury, O. Eberhardt, and C. W. Murphy, JHEP 11, 026 (2016), 1609.01290.

[33] N. Chakrabarty and B. Mukhopadhyaya, (2017), 1702.08268.

[34] S. Gori, H. E. Haber, and E. Santos, JHEP 06, 110 (2017), 1703.05873.

[35] J. F. Gunion and H. E. Haber, *"The CP conserving two Higgs doublet model: The Approach to the decoupling limit"*, Phys.Rev. D67 (2003) 075019, [hep-ph/0207010].

[36] G. Bhattacharyya, *"A Pedagogical Review of Electroweak Symmetry Breaking Scenarios",* Rept.Prog.Phys. 74 (2011) 026201, [arXiv:0910.5095].

[37] M. S. Chanowitz, M. Furman, and I. Hinchliffe, *"Weak Interactions of Ultra heavy Fermions"*. 2., Nucl.Phys. B153 (1979) 402.

[38] B. W. Lee, C. Quigg, and H. Thacker, *"Weak Interactions at Very High-Energies: The Role of the Higgs Boson Mass"*, Phys.Rev. D16 (1977) 1519.

[39] J. Maalampi, J. Sirkka, and I. Vilja, *"Tree level unitarity and triviality bounds for two Higgs models"*, Phys.Lett. B265 (1991) 371–376.

[40] S. Kanemura, T. Kubota, and E. Takasugi, *"Lee-Quigg-Thacker bounds for Higgs boson masses in a two doublet model"*, Phys.Lett. B313 (1993) 155–160, [hep-ph/9303263].

[41] A. G. Akeroyd, A. Arhrib, and E. M. Naimi, *"Note on tree level unitarity in the general two Higgs doublet model",* Phys.Lett. B490 (2000) 119–124, [hep-ph/0006035].

[42] J. Horejsi and M. Kladiva, *"Tree-unitarity bounds for THDM Higgs masses revisited",* Eur.Phys.J. C46 (2006) 81–91, [hep-ph/0510154].

[43] P. B. Pal, *"What is the equivalence theorem really?"*, hep-ph/9405362.

[44] J. Horejsi, *"Electroweak interactions and high-energy limit: An Introduction to equivalence theorem"*, Czech.J.Phys. 47 (1997) 951–977, [hep-ph/9603321].

[45] D. A. Ross and M. J. G. Veltman, *"Neutral currents in neutrino experiments",* Nucl. Phys. B 95 (1975) 135; M. J. G. Veltman, *"Second threshold in weak interactions",* Acta Phys. Polon. B 8 (1977) 475; M. J. G. Veltman, *"Limit on mass differences in the Weinberg model"*, Nucl. Phys. B 123 (1977) 89.

[46] M. B. Einhorn, D. R. T. Jones and M. J. G. Veltman, *"Heavy particles and the rho pa- rameter in the Standard Model",* Nucl. Phys. B 191 (1981) 146.

[47] R. Boughezal, J. B. Tausk and J. J. van der Bij, *"Three-loop electroweak correction to the rho parameter in the large Higgs mass limit",* Nucl. Phys. B 713 (2005) 278 [hep-ph/0410216], and references therein.

[48] W. M. Yao et al. (Particle Data Group), Review of particle physics, J. Phys. G (Par- ticles and Fields) 33 (2006) 1.

[49] J. Erler and P. Langacker in [48], p. 119.

[50] S. Bertolini, *"Quantum effects in a two-Higgs-doublet model of the electroweak interactions",* Nucl. Phys. B 272 (1986) 77.

[51] A. Denner, R. J. Guth and J. H. Kühn, *"Relaxation of top-mass limits in the two Higgs doublet model",* Phys. Lett. B 240 (1990) 438; C. D. Froggatt, R. G. Moorhouse and I. G. Knowles, *"Leading radiative corrections in two-scalar-doublet models",* Phys. Rev. D 45 (1992) 2471.

[52] M. E. Peskin and T. Takeuchi, *"A new constraint on a strongly interacting Higgs sector",* Phys. Rev. Lett. 65 (1990) 964; M. E. Peskin and T. Takeuchi, *"Estimation of oblique electroweak corrections",* Phys.Rev. D 46 (1992) 381.

[53] G. Altarelli and R. Barbieri, *"Vacuum-polarization effects of new physics on electroweak processes",* Phys. Lett. B 253 (1991) 161; G. Altarelli, R. Barbieri and S. Jadach, *"Toward a model-independent analysis of electroweak data",* Nucl. Phys. B 369 (1992) 3 [erratum ibid. B 376 (1992) 444].

[54] W. Grimus, L. Lavoura, O. M. Ogreid and P. Osland, *"A precision constraint on multi-Higgs doublet models",* J.Phys.G35:075001, 2008.

[55] T. D. Lee, *"A Theory of Spontaneous T Violation,"* Phys. Rev. D **8**, 1226 (1973).

[56] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *"The Higgs Hunter's Guide,"* Front. Phys. **80**, 1 (2000).

[57] The Review of Particle Physics (2018), M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).

[58] K. A. Olive *et al.* [Particle Data Group Collaboration], *"Review of Particle Physics,"* Chin. Phys. C **38**, 090001 (2014). doi:10.1088/1674-1137/38/9/090001

Chapter 3

On Naturalness

> But man has still another powerful resource: natural science with its strictly objective methods.
>
> *Ivan Pavlov*

3.1 A brief background

Physics below 300 GeV is termed infrared and physics above 1 TeV is called ultraviolet. Infrared physics is usually set by the Fermi mass which is the mass given by Fermi coupling constant of weak interactions, being about 350 GeV. High energy is anything above 1 TeV that is large with respect to the Fermi mass. The most tantalizing question in particle physics is - what is the structure, the spectrum and what are the forces at very high energies? Below the Fermi mass we do have a reasonably satisfactory theory, the Standard Model. It contains electromagnetic, weak and strong interactions all of them described by a gauge theory with the U(1), SU(2) and SU(3) symmetries.

On the other end of the scale we have the Planck mass, related to the gravitational coupling constant and of the order of 10^{19} GeV. In this mass scale the physics is much less satisfactory and all we can say is that perhaps supergravity may be a promising theory.

There lies a great deal of ignorance in between these two extreme ends. SU(5) Grand Unified Theory (GUT) proposed by Georgi et al. in their paper [1] bears the idea that there is nothing all the way from the Fermi mass up to the unification point at 10^{17} GeV. Many physicists find it hard to accept and the most obvious reason for this unwillingness is the unnaturalness of the SU(5) scheme.

Unnaturalness of the cosmological constant in relation to spontaneous symmetry breaking was earlier pointed out in [2]. It seemed reasonable enough to investigate the possibility that the Higgs be removed i.e. made very heavy [3]. Even the Yale group [4] addressed the problem and obtained a classification of the effects arising in this limit. The Higgs sector in this limit corresponds to the non-linear σ model [5]. This model is non-renormalizable and cut-off dependent effects can be observed. However at low energies there is a screening effect and the cut-off dependent terms become unobservable. Whether or not the non-linear σ model is a viable theory in four dimensions is not known; but it is very well conceivable that at least in some domain (1 to 10 TeV) the theory behaves effectively like this model.

In the early days of the renormalizable gauge theories ideas that Higgs is a composite bound state of two fermions surfaced [6]. Strong interactions similar to quantum chromodynamics were suggested to this end [7]. The crucial remark however was made by Susskind, reviving an old remark by Wilson [8] who observed that scalar particles, in particular Higgs, are unnatural. Following up on this argument the Stanford group and also Eichten and Lane and others have investigated the possibility of composite Higgs with new strong interactions called technicolor [9].

The problem can be approached from the ultraviolet limit. Ellis et al. [10] have started from a supersymmetric theory and those parts that would survive in the infrared limit were identified. Even though the method was questionable nevertheless it was interesting since something came out that resembled the known structure. In the process the discussion on anomalies became important.

A refreshingly new approach that at high energies the world is sort of random but in the infrared only the renormalizable gauge symmetric theories as observed would survive was proposed by Foerster, Nielsen and Ninomiya [11], Maiani et al. and Iliopoulos et al. [12].

't Hooft in his paper [13] put forward the idea that neither the infrared nor the ultraviolet theory should contain anomalies. The compositness of the known fermions was not compatible with Hooft's idea. Yet the situation was not really so simple. If we consider that there are anomalies below 1 TeV (which there are not, as far as we know), then both top and bottom quarks will be heavier than 1 TeV while τ lepton and its neutrino will be where they are now. Now the question is would such a theory display bad effects? One would not think so because the kind of the non-renormalizable effects associated with anomalies are rather hidden and quite buried in perturbation theory. The anomalies themselves are not even sensitive to the actual mass values. As a matter of fact, cut-off dependent effects due to anomalies appear only at the 3-loop level. Yet it is not possible to let the mass of fermions become large since in this case there is no decoupling, not even at the one-loop level [14, 15]. However, this problem can be cured by the introduction of further heavy particles (scalars) and it may be possible that under certain conditions anomalies are relatively harmless.

3.2 Reviving the question of naturalness of the standard model

The standard model of the electroweak interactions has been very successful in describing the known subatomic world in terms of SU(3)$\bigotimes$SU(2)$\bigotimes$U(1) gauge dynamics. In the standard model an elementary Higgs field is introduced with a negative mass term which induces an instability and causes the Higgs field to condense generating a spontaneous symmetry breaking. Thus masses of the electroweak gauge bosons and fermions are generated through the gauge and Yukawa couplings of the Higgs field. The scale of the negative Higgs mass term determines the scale of this electroweak symmetry breaking and the magnitude of the resulting gauge boson and fermion masses. This Higgs mass term can be strongly affected by quantum corrections and thereby the scale of the electroweak symmetry breaking too gets shifted in the energy scale. If the standard model were to represent the correct physics up to a high scale, it is usually assumed that the quantum corrections shift the Higgs mass term by large amount due to quadratic divergences of the loop amplitudes. The fine-tuning required to keep the effective Higgs mass term at the electroweak scale and not at the high energy scale represents a naturalness problem for the standard model [3, 8].

The standard model Higgs Lagrangian is usually written as,

$$\mathcal{L}_h = \left(D^\mu \bar{h}\right)\left(D_\mu h\right) - \bar{Q}_L G_U U \cdot \tilde{h} - \bar{Q}_L G_D D \cdot h - m_h^2 \bar{h} h \tag{3.1}$$

where the quark Yukawa couplings have been displayed explicitly.

The quantum corrections to the Higgs mass terms from gauge bosons and fermion loops are usually thought to generate quadratic divergences representing large shifts in the effective Higgs mass. If $\Lambda_f(\Lambda_b)$ is the momentum cut-off used for fermions (bosons) loops and v is the electroweak vacuum expectation value then at one loop order the Higgs mass counter term may be written as

$$\Delta m_h^2 = -4\sum_f m_f^2\left(\Lambda_f^2/v^2\right) + \left(2m_W^2 + m_Z^2 + m_h^2\right)\left(\Lambda_b^2/v^2\right) . \tag{3.2}$$

If the standard model is to be valid up to a high energy domain then the cut-offs are at a high scale and the mass shift of Higgs boson as stated in Eq. (3.2) is expected to be large. This is an alarming situation since the mass of the Higgs boson needs to be in the electroweak scale and not just blow up as we go high up on the energy scale. Thus these quantum corrections need to get cancelled. This is a central puzzle of the standard model which has gained attention over the years.

Let us investigate into the various possibilities that could solve this naturalness problem. One approach, usually rejected, is that the "bare" Higgs mass term is fine-tuned in each order in perturbation theory to precisely cancel the large corrections of Eq. (3.2). Another possibility to cancel the quadratic divergences of the Higgs mass term is to establish a relationship between the Yukawa couplings and the gauge coupling constants [16]. Such a relationship implicitly bears the assumption that the fermion and bosons have a common cut-off. This common cut-off need not be a property of the quantum theory. It is also somewhat problematic that a consistent coupling constant relation could be maintained in higher orders of the perturbation theory or in a precise non-perturbative formulation of the full theory. The usual statement of the naturalness theorem is that the absence of large corrections can only be maintained through symmetries which protect the Higgs mass term [13, 17].

Within the framework of dimensional regularization, Veltman in his paper [16] suggested that a suitable criterion to address the issue of quadratic divergences is the occurrence of poles in the complex D - dimensional plane for D less than four. In particular at the n-loop level, a quadratic divergence corresponds to a pole at $D = 4-2/n$. Naive quadratic divergences at the one-loop level thus correspond to poles for D = 2.

It was realized by Veltman that poles existed in vector boson and Higgs self energy diagrams for $D = 2$ in the SM[1]. For the Higgs mass they correspond to the shift, $m_h^2 \to m_h^2 + \Delta m_h^2$ as already stated above. If a common cut-off scale is considered, the divergence in Eq. (3.2) has the form[2],

$$\Delta m_h^2 = \frac{\Lambda^2}{16\pi^2} C_V \,, \qquad C_V = \sum_{n\geqslant 1} C_{V_n} \,, \tag{3.3}$$

where the contribution C_{V_n} is associated to n loops. In particular for one-loop the standard model result is,

$$C_{V_1} = \frac{3}{v^2}\left(m_h^2 + m_Z^2 + 2m_W^2 - 4m_t^2\right) \tag{3.4}$$

which stems from Eq. (3.2) when $\Lambda_f = \Lambda_b = \Lambda$ and only the top quark contribution is considered while the contribution from other quarks are too small with respect to the top quark contribution. The condition for the absence of the quadratic divergences at one loop arising from the cancellation between the fermion and boson masses, i.e., $C_{V_1} = 0$, is known as the Veltman condition at 1-loop [16] and was dubbed "semi-natural" by Veltman himself.

A very simple way to understand the Veltman condition in the Standard Model and generalizations thereof is starting from the one-loop effective potential in the presence of the (constant) background Higgs field configuration Φ. The one-loop effective potential is given by,

$$V^{(1)}(\Phi) = \frac{1}{64\pi^2}\int d^4k \, \mathrm{STr}\left[\log\left(k^2 + \mathcal{M}^2(\Phi)\right)\right] \tag{3.5}$$

[1] And also in tadpole diagrams and in connection with the cosmological constant
[2] NB: A similar structure holds for vector bosons

where,

$$\mathrm{STr}\left[\log\left(k^2+\mathcal{M}^2(\Phi)\right)\right]=\sum_{J=0,\frac{1}{2},1}(-1)^{2J}(2J+1)\,\mathrm{Tr}\left[\log\left(k^2+\mathcal{M}_J^2(\Phi)\right)\right]\,. \qquad (3.6)$$

Here $\mathcal{M}_J^2(\Phi)$ is the matrix of the second derivatives of the Lagrangian at zero momentum k for spin J fields[3]. The mass matrix is thus obtained from $\mathcal{M}_J^2(\Phi)$ by inserting the vacuum expectation value $<\Phi>=v$, where v is the location of the minimum of the effective potential.

The UV divergences of the one loop effective potential can be displayed by expanding the integrand in powers of large k. Writing

$$\log\left(k^2+\mathcal{M}_J^2\right)=\log k^2+\frac{\mathcal{M}_J^2}{k^2}-\frac{1}{2}\frac{\mathcal{M}_J^4}{k^4}+\cdots \qquad (3.7)$$

leads to,

$$V^{(1)}(\Phi)=\frac{1}{64\pi^2}\left[\mathrm{STr}\,\mathbb{I}\int\frac{d^4k}{(2\pi)^4}\log k^2+\mathrm{STr}\mathcal{M}^2(\Phi)\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}+\cdots\right]. \qquad (3.8)$$

If a UV cut-off Λ is introduced then the first term is a pure cosmological constant term with coefficient proportional to $\mathrm{STr}\,\mathbb{I}=n_B-n_F$ which vanishes in theories like supersymmetry where there are equal number of bosonic (n_B) and fermionic (n_F) degrees of freedom. The second term is of order Λ^2 and determines the presence of quadratic divergences at one-loop level. Therefore quadratic divergences are absent provided that $\mathrm{STr}\mathcal{M}^2(\Phi)=0$. Even $\mathrm{STr}\mathcal{M}^2(\Phi)=$ constant is permissible since this would correspond to a shift of the zero point energy which remains undetermined in the absence of coupling to gravity.

Now, for the SM potential,

$$V(\Phi)=-\frac{m^2}{2}\Phi^2+\frac{\lambda}{4}\Phi^4 \qquad (3.9)$$

$\mathrm{STr}\mathcal{M}^2(\Phi)$ will be a function of the renormalization scale $\mu\sim\langle\Phi\rangle$ and will be given by,

$$\mathrm{STr}\mathcal{M}^2(\Phi)=H(\mu)+3G(\mu)+6W(\mu)+3Z(\mu)-12T(\mu)\,. \qquad (3.10)$$

The numerical coefficients in Eq. (3.10) come from the number of degrees of freedom of the physical Higgs boson H (one), the Goldstone bosons G (three), the massive gauge bosons Z (three) and W (six) and the top, a Dirac fermion T (twelve). The terms in Eq. (3.10) can be explicitly written as,

[3]For spin 1/2 fields one should replace $\mathcal{M}_{1/2}^2(\Phi)\to\mathcal{M}_{1/2}^\dagger(\Phi)\mathcal{M}_{1/2}(\Phi)$

$$\begin{aligned} H(\mu) &= -m^2(\mu) + 3\lambda(\mu)\Phi^2 \\ G(\mu) &= -m^2(\mu) + \lambda(\mu)\Phi^2 \\ W(\mu) &= \frac{1}{4}g^2(\mu)\Phi^2 \\ Z(\mu) &= \frac{1}{4}\left(g^2(\mu) + g'^2(\mu)\right)\Phi^2 \\ T(\mu) &= \frac{1}{2}y_t^2(\mu)\Phi^2\,, \end{aligned} \qquad (3.11)$$

where y_t is the top Yukawa coupling and g, g' are the electroweak gauge couplings. When $\langle\Phi\rangle = v = 246$ GeV, the functions H, W, Z, T become the physical masses m_h^2, m_W^2, m_Z^2 and m_t^2 while G=0 since Goldstones are massless. Terms linear in Φ are absent in Eq. (3.10) because the SM does not have a cubic scalar invariant term in the Lagrangian. Clearly in the SM it is not possible to have $\mathrm{STr}\mathcal{M}^2 = 0$ for general Φ, since the mass squared terms in Eq. (3.10) do not cancel. The vanishing of $\mathrm{STr}\mathcal{M}^2$ will happen only at some specific value of Φ. Since in the RGE we are identifying Φ with the renormalization scale μ, therefore for large field values the terms proportional to Φ^2 in Eq. (3.10) will neatly dominate; in other words the absence of quadratic divergences is provided by the condition [18],

$$\frac{\partial \mathrm{STr}\mathcal{M}^2(\Phi)}{\partial\Phi^2} = 6\lambda(\mu) + \frac{9}{4}g^2(\mu) + \frac{3}{4}g'^2(\mu) - 6y_t^2(\mu) = 0\,, \qquad (3.12)$$

which is precisely the Veltman condition at one-loop since the right hand side of Eq. (3.4) can be written in terms of the running couplings as,

$$\begin{aligned} C_{V_1} &= \frac{3}{v^2}\left(m_h^2 + m_Z^2 + 2m_W^2 - 4m_t^2\right) \\ &= 6\lambda(\mu) + \frac{9}{4}g^2(\mu) + \frac{3}{4}g'^2(\mu) - 6y_t^2(\mu)\,. \end{aligned} \qquad (3.13)$$

If we include two-loop (or higher-loop) corrections then the Veltman condition for one loop will get modified by a loop suppressing factor $\mathcal{O}\left(1/(4\pi)^2\right)$ which will translate into a tiny modification of the one-loop Veltman scale μ_{V_1} [19, 20].

3.3 Veltman conditions in the framework of 2HDMs

In two Higgs doublet models the two Higgs doublets Φ_1 and Φ_2 couple to fermions and to each other in any way consistent with the $SU(2)\times U(1)$ gauge symmetry. Initially we work with the most general 2HDM Lagrangian density where no additional symmetries are imposed. In particular the special case that one doublet gives mass only to the up quark while the other doublet gives mass only to the down quarks, as in supersymmetry, is not insisted here. Later on we will impose a global $U(1)$ symmetry to avoid FCNCs which will further simplify our equations. The Yukawa potential for the most general

2HDM is given as follows:

$$\begin{aligned}\mathcal{L}_Y = &- \left[\left(\bar{\Psi}_{lL}\Phi_1\right) G_e^1 \Psi_{eR} + \bar{\Psi}_{eR} G_e^{1\dagger} \left(\Phi_1^\dagger \Psi_{lL}\right) \right] - \left[\left(\bar{\Psi}_{lL}\Phi_2\right) G_e^2 \Psi_{eR} + \bar{\Psi}_{eR} G_e^{2\dagger} \left(\Phi_2^\dagger \Psi_{lL}\right) \right] \\ &- \left[\left(\bar{\Psi}_{qL}\tilde{\Phi}_1\right) G_u^1 \Psi_{uR} + \bar{\Psi}_{uR} G_u^{1\dagger} \left(\tilde{\Phi}_1^\dagger \Psi_{qL}\right) \right] - \left[\left(\bar{\Psi}_{qL}\tilde{\Phi}_2\right) G_u^2 \Psi_{uR} + \bar{\Psi}_{uR} G_u^{2\dagger} \left(\tilde{\Phi}_2^\dagger \Psi_{qL}\right) \right] \\ &- \left[\left(\bar{\Psi}_{qL}\Phi_1\right) G_d^1 \Psi_{dR} + \bar{\Psi}_{dR} G_d^{1\dagger} \left(\Phi_1^\dagger \Psi_{qL}\right) \right] \\ &- \left[\left(\bar{\Psi}_{qL}\Phi_2\right) G_d^2 \Psi_{dR} + \bar{\Psi}_{dR} G_d^{2\dagger} \left(\Phi_2^\dagger \Psi_{qL}\right) \right]. \end{aligned} \tag{3.14}$$

Here Ψ_{eR}, Ψ_{uR} and Ψ_{dR} are three $SU(2)\times U(1)$ right handed singlets for charged leptons, up-type quarks and down-type quarks respectively in the space of generations and are given below.

$$\Psi_{eR} = \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \quad , \quad \Psi_{uR} = \begin{pmatrix} u'_R \\ c'_R \\ t'_R \end{pmatrix} \quad \text{and} \quad \Psi_{dR} = \begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix} . \tag{3.15}$$

Ψ_{lL} and Ψ_{qL} are three vectors of doublets as given below:

$$\Psi_{lL} = \begin{pmatrix} \begin{pmatrix} \nu'_{eL} \\ e'_L \end{pmatrix} \\ \begin{pmatrix} \nu'_{\mu L} \\ \mu'_L \end{pmatrix} \\ \begin{pmatrix} \nu'_{\tau L} \\ \tau'_L \end{pmatrix} \end{pmatrix} \quad ; \quad \Psi_{qL} = \begin{pmatrix} \begin{pmatrix} u'_L \\ d'_L \end{pmatrix} \\ \begin{pmatrix} c'_L \\ s'_L \end{pmatrix} \\ \begin{pmatrix} t'_L \\ b'_L \end{pmatrix} \end{pmatrix} . \tag{3.16}$$

The notation $\tilde{\Phi}_i$ is defined by $\tilde{\Phi}_i = i\tau_2\Phi_i^*$. The complex 3×3 matrices G_e^1 , G_e^2 , G_u^1 , G_u^2 , G_d^1 and G_d^2 contain the Yukawa coupling constants. The primed fields are not the physical fields since in the primed basis the mass matrices are not diagonal.

Let us start our discussion for this section with the most general two Higgs doublet model. But before we do that let me make a general comment on models with N Higgs doublets. In the N-Higgs doublet models, there are N real coupling constants μ_i associated with the quadratic terms and $\frac{1}{2}N^2(N^2+1)$ real coupling constants λ_j associated with the quartic terms in the Higgs potential. Thus for a two Higgs doublet model there will be two quadratic coupling constants and ten quartic coupling constants.

We will work with the scalar potential [21, 22] where the quadratic couplings are embedded within the quartic terms and since the potential obeys a $U(1)$ symmetry therefore few of the quartic terms have been set to zero by this symmetry as discussed in Section 1.2.4.

$$
\begin{aligned}
V = {} & \lambda_1 \left(|\Phi_1|^2 - \frac{v_1^2}{2} \right)^2 + \lambda_2 \left(|\Phi_2|^2 - \frac{v_2^2}{2} \right)^2 \\
& + \lambda_3 \left(|\Phi_1|^2 + |\Phi_2|^2 - \frac{v_1^2 + v_2^2}{2} \right)^2 \\
& + \lambda_4 \left(|\Phi_1|^2 |\Phi_2|^2 - |\Phi_1^\dagger \Phi_2|^2 \right) \\
& + \lambda_5 \left| \Phi_1^\dagger \Phi_2 - \frac{v_1 v_2}{2} \right|^2 ,
\end{aligned} \tag{3.17}
$$

with real λ_i. This potential is invariant under the symmetry $\Phi_1 \to e^{i\theta}\Phi_1 \, , \Phi_2 \to \Phi_2 \, ,$ except for a soft breaking term $\lambda_5 v_1 v_2 \Re(\Phi_1^\dagger \Phi_2)$. Additional dimension-4 terms, including one allowed by a softly broken Z_2 symmetry [23] are also set to zero by this U(1) symmetry.

Cancelling of the quadratic divergences of the 2HDM gives rise to the below mentioned four mass relations also known as the Veltman conditions.

$$2\,\mathrm{Tr}\, G_e^1 G_e^{1\dagger} + 6\,\mathrm{Tr}\, G_u^{1\dagger} G_u^1 + 6\,\mathrm{Tr}\, G_d^1 G_d^{1\dagger} = \frac{9}{4} g^2 + \frac{3}{4} g'^2 + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5 \tag{3.18}$$

$$2\,\mathrm{Tr}\, G_e^2 G_e^{2\dagger} + 6\,\mathrm{Tr}\, G_u^{2\dagger} G_u^2 + 6\,\mathrm{Tr}\, G_d^2 G_d^{2\dagger} = \frac{9}{4} g^2 + \frac{3}{4} g'^2 + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5 \tag{3.19}$$

$$2\,\mathrm{Tr}\, G_e^1 G_e^{2\dagger} + 6\,\mathrm{Tr}\, G_u^{1\dagger} G_u^2 + 6\,\mathrm{Tr}\, G_d^1 G_d^{2\dagger} = 0 \tag{3.20}$$

$$2\,\mathrm{Tr}\, G_e^2 G_e^{1\dagger} + 6\,\mathrm{Tr}\, G_u^{2\dagger} G_u^1 + 6\,\mathrm{Tr}\, G_d^2 G_d^{1\dagger} = 0 \tag{3.21}$$

It is noted that the first two of the above equations are real equations, while the last two equations are complex conjugates of each other. If the quadratic divergences of the Higgs self-energy are calculated in terms of the physical Higgs fields then the same mass relations are obtained but the algebra is much more lengthy.

The vacuum expectation values of $\Phi_i, i = 1\, ,\, 2$ are,

$$\langle \Phi_1 \rangle_0 = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix} \;,\; \langle \Phi_2 \rangle_0 = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix} . \tag{3.22}$$

To exhibit the decoupling of the physical Higgs fields from the unphysical Goldstone bosons and to simplify the equations that follow, it is convenient to define new fields ϕ_1 and ϕ_2 according to

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} , \tag{3.23}$$

where β is a crucial parameter of the theory and bears a direct relation with the vacuum expectation values of the doublets through the relation $\tan\beta = \frac{v_2}{v_1}$.

The immediate advantage of working with the fields ϕ_1 and ϕ_2 rather than Φ_1 and Φ_2 is that the Goldstone bosons G^+ , G^- and G_0 decouple from the five physical Higgs fields H^+ , H^- , H_1 , H_2 and H_3:

$$\phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + a + iG_0) \end{pmatrix} \;,\; \phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(b + ic) \end{pmatrix} . \tag{3.24}$$

In the above equations v is $\sqrt{v_1^2 + v_2^2}$ and carries the value of the electroweak vacuum expectation value of 246 GeV. The fields a, b and c are related to the three neutral Higgs fields H_1 , H_2 and H_3 by an orthogonal transformation $\mathbf{R}$ which diagonalizes the mass matrix:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} \;;\; \mathbf{R^T R} = \mathbf{I}\,. \tag{3.25}$$

The physical fermion fields are obtained by diagonalizing the fermion mass matrix. The physical leptons e, μ and τ are defined by a pair of unitary transformations U_e and V_e that act independently on the left-handed and right-handed fields as shown below,

$$\begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} = U_e \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \;;\; \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} = V_e \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} . \tag{3.26}$$

U_e and V_e are so chosen that they diagonalise the lepton mass matrix.

$$U_e^\dagger (G_e^1 \cos\beta + G_e^2 \sin\beta) V_e \;=\; \frac{\sqrt{2}}{v} M_e \,, \tag{3.27}$$

where M_e is diagonal:

$$M_e = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} . \tag{3.28}$$

Eq. (3.27) gives the coupling of ϕ_1 to the physical lepton fields. The coupling of ϕ_2 to the physical lepton fields is given by,

$$U_e^\dagger \left(-G_e^1 \sin\beta + G_e^2 \cos\beta\right) V_e = \frac{\sqrt{2}}{v} \left(-F_e^1 \tan\beta + F_e^2 \cot\beta\right) \,, \tag{3.29}$$

where F_e^1 and F_e^2 are 3×3 complex matrices satisfying $F_e^1 + F_e^2 = M_e$.

The physical quarks are obtained by an essentially identical procedure. The u, c and t quarks are obtained by a pair of unitary transformations as shown below,

$$\begin{pmatrix} u'_L \\ c'_L \\ t'_L \end{pmatrix} = U_u \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} \;;\; \begin{pmatrix} u'_R \\ c'_R \\ t'_R \end{pmatrix} = V_u \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} , \tag{3.30}$$

where U_u and V_u are the unitary matrices acting on the left handed and right handed up type quarks respectively. They satisfy the below written equality which is nothing but the coupling of the physical u, c and t quark fields to $\tilde{\phi}_1 = i\tau_2\phi_1^*$.

$$U_u^\dagger \left(G_u^1 \cos\beta + G_u^2 \sin\beta\right) V_u = \frac{\sqrt{2}}{v} M_u \,, \tag{3.31}$$

where M_u is diagonal:

$$M_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} . \tag{3.32}$$

Similarly the coupling of $\tilde{\phi}_2 = i\tau_2\phi_2^*$ to the physical u, c and t quarks are given by:

$$U_u^\dagger \left(-G_u^1 \sin\beta + G_u^2 \cos\beta\right) V_u = \frac{\sqrt{2}}{v} \left(-F_u^1 \tan\beta + F_u^2 \cot\beta\right) \,, \tag{3.33}$$

where F_u^1 and F_u^2 are 3×3 complex matrices satisfying $F_u^1 + F_u^2 = M_u$.

Moving forward with similar equations for the down type quarks we obtain,

$$\begin{pmatrix} d_L' \\ s_L' \\ b_L' \end{pmatrix} = U_d \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \;;\; \begin{pmatrix} d_R' \\ s_R' \\ b_R' \end{pmatrix} = V_d \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} , \tag{3.34}$$

which are chosen so that

$$U_d^\dagger \left(G_d^1 \cos\beta + G_d^2 \sin\beta\right) V_d = \frac{\sqrt{2}}{v} M_d \,. \tag{3.35}$$

The diagonal mass matrix M_d is given by,

$$M_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} . \tag{3.36}$$

The coupling of ϕ_1 to the d, s and b quarks is given by Eq. (3.35). The coupling of ϕ_2 to the down type quarks is as follows:

$$U_d^\dagger \left(-G_d^1 \sin\beta + G_d^2 \cos\beta\right) V_d \;=\; \frac{\sqrt{2}}{v} \left(-F_d^1 \tan\beta + F_d^2 \cot\beta\right) \,, \tag{3.37}$$

where again F_d^1 and F_d^2 are 3×3 complex matrices satisfying $F_d^1 + F_d^2 = M_d$.

We now write down a few general results using Eq. (3.27), Eq. (3.31) and Eq. (3.35),

$$\mathrm{Tr}\left[G_{1e}^{\dagger}G_{1e}\right] = \left(\frac{\sqrt{2}}{v\cos\beta}\right)^2 \left[m_e^2 + m_\mu^2 + m_\tau^2\right] \;, \tag{3.38}$$

$$\mathrm{Tr}\left[G_{2e}^{\dagger}G_{2e}\right] = \left(\frac{\sqrt{2}}{v\sin\beta}\right)^2 \left[m_e^2 + m_\mu^2 + m_\tau^2\right] \;, \tag{3.39}$$

$$\mathrm{Tr}\left[G_{1u}^{\dagger}G_{1u}\right] = \left(\frac{\sqrt{2}}{v\cos\beta}\right)^2 \left[m_u^2 + m_c^2 + m_t^2\right] \;, \tag{3.40}$$

$$\mathrm{Tr}\left[G_{2u}^{\dagger}G_{2u}\right] = \left(\frac{\sqrt{2}}{v\sin\beta}\right)^2 \left[m_u^2 + m_c^2 + m_t^2\right] \;, \tag{3.41}$$

$$\mathrm{Tr}\left[G_{1d}^{\dagger}G_{1d}\right] = \left(\frac{\sqrt{2}}{v\cos\beta}\right)^2 \left[m_d^2 + m_s^2 + m_b^2\right] \;, \tag{3.42}$$

$$\mathrm{Tr}\left[G_{2d}^{\dagger}G_{2d}\right] = \left(\frac{\sqrt{2}}{v\sin\beta}\right)^2 \left[m_d^2 + m_s^2 + m_b^2\right] \;. \tag{3.43}$$

Natural Flavour conservation restricts the couplings of Φ_1 and Φ_2 to the fermions for the four types of two Higgs doublet models by the imposition of a symmetry. Thus Eq. (3.38) to Eq. (3.43) have different values for different 2HDMs and hence the Veltman conditions are different for the various types of two Higgs doublet models. We now work to find out the Veltman conditions for the four types of two Higgs doublet models.

- **Type-I 2HDM**

Φ_2 couples to up-type quarks, down-type quarks and charged leptons whereas Φ_1 couples to none. So,

$$G_{1e} = 0 \quad , \quad G_{1d} = 0 \quad , \quad G_{1u} = 0\,. \tag{3.44}$$

Thus Eq. (3.20) and Eq. (3.21) are invariably satisfied for type I 2HDM when the Yukawa couplings have the values as in Eq. (3.44).

If we look at Eq. (3.18) keeping Eq. (3.44) in mind we obtain,

$$\begin{aligned}
-2\,\mathrm{Tr}\,G_e^{1\dagger}G_e^1 - 6\,\mathrm{Tr}\,G_u^{1\dagger}G_u^1 - 6\,\mathrm{Tr}\,G_d^{1\dagger}G_d^1 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\
\Rightarrow \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\
\Rightarrow 6\frac{M_W^2}{v^2} + 3\frac{M_Z^2}{v^2} + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0
\end{aligned}$$

Multiplying both sides of the above equation by $\frac{v^2}{4}$ we get ;

$$\begin{aligned}
\frac{6}{4}M_W^2 + \frac{3}{4}M_Z^2 + \left(\frac{v}{2}\right)^2 (6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5) &= 0 \\
\Rightarrow \frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{v^2}{4} (6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5) &= 0\,. \qquad (3.45)
\end{aligned}$$

Similarly using Eq. (3.39), Eq. (3.41) and Eq. (3.43) in Eq. (3.19) for type I we get,

$$\begin{aligned}
&-2\,\mathrm{Tr}\,G_e^{2\dagger}G_e^2 - 6\,\mathrm{Tr}\,G_u^{2\dagger}G_u^2 - 6\,\mathrm{Tr}\,G_d^{2\dagger}G_d^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5 = 0 \\
\Rightarrow\quad & 6\frac{M_W^2}{v^2} + 3\frac{M_Z^2}{v^2} + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5 = 2\left(\frac{\sqrt{2}}{v}\right)^2 \left[m_e^2 + m_\mu^2 + m_\tau^2\right] \csc^2\beta \\
&+6\left(\frac{\sqrt{2}}{v}\right)^2 \left[m_u^2 + m_c^2 + m_t^2\right] \csc^2\beta + 6\left(\frac{\sqrt{2}}{v}\right)^2 \left[m_d^2 + m_s^2 + m_b^2\right] \csc^2\beta \\
\Rightarrow\quad & 6\frac{M_W^2}{v^2} + 3\frac{M_Z^2}{v^2} + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5 = \frac{4}{v^2}\left[m_e^2 + m_\mu^2 + m_\tau^2\right] \csc^2\beta \\
&+\frac{12}{v^2}\left[m_u^2 + m_c^2 + m_t^2\right] \csc^2\beta + \frac{12}{v^2}\left[m_d^2 + m_s^2 + m_b^2\right] \csc^2\beta
\end{aligned}$$

Multiplying both sides of the above equation by $\frac{v^2}{4}$ we get ;

$$\begin{aligned}
\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{v^2}{4}(6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5) &= \Big[\left(m_e^2 + m_\mu^2 + m_\tau^2\right) + 3\left(m_u^2 + m_c^2 + m_t^2\right) \\
&\quad +3\left(m_d^2 + m_s^2 + m_b^2\right)\Big] \csc^2\beta\,. \qquad (3.46)
\end{aligned}$$

In getting the above two equations we have used

$$M_W = \frac{vg}{2} \quad , \quad M_Z = \frac{v}{2}\sqrt{g^2 + g'^2} \qquad (3.47)$$

Thus,

$$g = \frac{2M_W}{v} \qquad (3.48)$$

and

$$\begin{aligned}
g'^2 &= \frac{4M_Z^2}{v^2} - g^2 \\
\Rightarrow\quad g'^2 &= \frac{4M_Z^2}{v^2} - \frac{4M_W^2}{v^2} \qquad (3.49)
\end{aligned}$$

$$\begin{aligned}
\therefore\quad & \frac{9}{4}g^2 + \frac{3}{4}g'^2 \\
=\quad & \frac{9}{4}\left(\frac{2M_W}{v}\right)^2 + \frac{3}{4}\left(\frac{4M_Z^2}{v^2} - \frac{4M_W^2}{v^2}\right) \\
=\quad & 6\frac{M_W^2}{v^2} + 3\frac{M_Z^2}{v^2} \qquad (3.50)
\end{aligned}$$

- **Type-II 2HDM**

Here Φ_2 couples to up-type quarks and Φ_1 couples to down-type quarks and charged leptons. Thus ;

$$G_{2e} = 0 \quad , \quad G_{2d} = 0 \quad , \quad G_{1u} = 0\,. \tag{3.51}$$

Here again Eq. (3.20) and Eq. (3.21) are invariably satisfied for type II 2HDM when the Yukawa couplings have the values as in Eq. (3.51).

Now using Eq. (3.38) to Eq. (3.43) and Eq. (3.51) in Eq. (3.18) we get,

$$\begin{aligned}
-2\,\mathrm{Tr}\,G_e^{1\dagger}G_e^1 - 6\,\mathrm{Tr}\,G_u^{1\dagger}G_u^1 - 6\,\mathrm{Tr}\,G_d^{1\dagger}G_d^1 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\
\Rightarrow -2\,\mathrm{Tr}\,G_e^{1\dagger}G_e^1 - 6\,\mathrm{Tr}\,G_d^{1\dagger}G_d^1 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\
\Rightarrow 6\frac{M_{\rm w}^2}{v^2} + 3\frac{M_{\rm z}^2}{v^2} + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5 - \frac{4}{v^2}\left[m_e^2 + m_\mu^2 + m_\tau^2\right]\sec^2\beta - \frac{12}{v^2}\left[m_d^2 + m_s^2 + m_b^2\right]\sec^2\beta &= 0
\end{aligned}$$

Multiplying both sides of the above equation by $\frac{v^2}{4}$ we get ;

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{v^2}{4}\left(6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5\right) = \left[\,\left(m_e^2 + m_\mu^2 + m_\tau^2\right) + 3\left(m_d^2 + m_s^2 + m_b^2\right)\,\right]\sec^2\beta\,. \tag{3.52}$$

Similarly for Eq. (3.19),

$$\begin{aligned}
-2\,\mathrm{Tr}\,G_e^{2\dagger}G_e^2 - 6\,\mathrm{Tr}\,G_u^{2\dagger}G_u^2 - 6\,\mathrm{Tr}\,G_d^{2\dagger}G_d^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\
\Rightarrow -6\;TrG_u^{2\dagger}G_u^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\
\Rightarrow 6\frac{M_{\rm w}^2}{v^2} + 3\frac{M_{\rm z}^2}{v^2} + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5 - \frac{12}{v^2}\left[m_u^2 + m_c^2 + m_t^2\right]\csc^2\beta &= 0
\end{aligned}$$

Multiplying both sides of the above equation by $\frac{v^2}{4}$ we get ;

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{v^2}{4}\left(6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5\right) = 3\left(m_u^2 + m_c^2 + m_t^2\right)\csc^2\beta\,. \tag{3.53}$$

- **Lepton Specific 2HDM**

In this type of 2HDM, Φ_2 couples to up-type and down-type quarks, Φ_1 couples to charged leptons. Thus ;

$$G_{2e} = 0 \quad , \quad G_{1d} = 0 \quad , \quad G_{1u} = 0\,. \tag{3.54}$$

Eq. (3.20) and Eq. (3.21) are invariably satisfied for lepton specific models when the Yukawa couplings have the values as in Eq. (3.54).

Simplifying Eq. (3.18) we have,

$$
\begin{aligned}
-2\,\mathrm{Tr}\,G_e^{1\dagger}G_e^1 - 6\,\mathrm{Tr}\,G_u^{1\dagger}G_u^1 - 6\,\mathrm{Tr}\,G_d^{1\dagger}G_d^1 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\
\Rightarrow -2\,\mathrm{Tr}\,G_e^{1\dagger}G_e^1 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\
\Rightarrow 6\frac{M_{\mathrm{w}}^2}{v^2} + 3\frac{M_{\mathrm{z}}^2}{v^2} + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5 - \frac{4}{v^2}[m_e^2 + m_\mu^2 + m_\tau^2]\sec^2\beta &= 0
\end{aligned}
$$

Multiplying both sides of the above equation by $\frac{v^2}{4}$ we get ;

$$
\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{v^2}{4}\left(6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5\right) = \left(m_e^2 + m_\mu^2 + m_\tau^2\right)\sec^2\beta\,. \tag{3.55}
$$

For Eq. (3.19) in lepton specific models,

$$
\begin{aligned}
-2\,\mathrm{Tr}\,G_e^{2\dagger}G_e^2 - 6\,\mathrm{Tr}\,G_u^{2\dagger}G_u^2 - 6\,\mathrm{Tr}\,G_d^{2\dagger}G_d^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\
\Rightarrow -6\,\mathrm{Tr}\,G_u^{2\dagger}G_u^2 - 6\,\mathrm{Tr}\,G_d^{2\dagger}G_d^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\
\Rightarrow 6\frac{M_{\mathrm{w}}^2}{v^2} + 3\frac{M_{\mathrm{z}}^2}{v^2} + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5 - \frac{12}{v^2}\left[(m_u^2 + m_c^2 + m_t^2) + (m_d^2 + m_s^2 + m_b^2)\right]\csc^2\beta &= 0
\end{aligned}
$$

Multiplying both sides of the above equation by $\frac{v^2}{4}$ we get ;

$$
\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{v^2}{4}\left(6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5\right) = 3\left[\,\left(m_u^2 + m_c^2 + m_t^2\right) + \left(m_d^2 + m_s^2 + m_b^2\right)\,\right]\csc^2\beta \tag{3.56}
$$

- **Flipped 2HDM**

Here Φ_2 couples to up-type quarks and charged leptons, Φ_1 couples to down-type quarks. Thus ;

$$
G_{1e} = 0 \quad , \quad G_{2d} = 0 \quad , \quad G_{1u} = 0\,. \tag{3.57}
$$

Even here the last two of the Veltman conditions is invariably satisfied.

For flipped two Higgs doublet models Eq. (3.18) is simplified as,

$$
\begin{aligned}
-2\,\mathrm{Tr}\,G_e^{1\dagger}G_e^1 - 6\,\mathrm{Tr}\,G_u^{1\dagger}G_u^1 - 6\,\mathrm{Tr}\,G_d^{1\dagger}G_d^1 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\
\Rightarrow -6\,\mathrm{Tr}\,G_d^{1\dagger}G_d^1 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\
\Rightarrow 6\frac{M_{\mathrm{w}}^2}{v^2} + 3\frac{M_{\mathrm{z}}^2}{v^2} + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5 - \frac{12}{v^2}\left[m_d^2 + m_s^2 + m_b^2\right]\sec^2\beta &= 0
\end{aligned}
$$

Multiplying both sides of the above equation by $\frac{v^2}{4}$ we get ;

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{v^2}{4}\left(6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5\right) = 3\left(m_d^2 + m_s^2 + m_b^2\right)\sec^2\beta\,. \tag{3.58}$$

Similarly for Eq. (3.19) we solve as:

$$\begin{aligned} -2\,\mathrm{Tr}\,G_e^{2\dagger}G_e^2 - 6\,\mathrm{Tr}\,G_u^{2\dagger}G_u^2 - 6\,\mathrm{Tr}\,G_d^{2\dagger}G_d^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\ \Rightarrow -2\,\mathrm{Tr}\,G_e^{2\dagger}G_e^2 - 6\,\mathrm{Tr}\,G_u^{2\dagger}G_u^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5 &= 0 \\ \Rightarrow 6\frac{M_{\rm w}^2}{v^2} + 3\frac{M_{\rm z}^2}{v^2} + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5 - \frac{4}{v^2}\left[m_e^2 + m_\mu^2 + m_\tau^2\right]\csc^2\beta - \frac{12}{v^2}\left[m_u^2 + m_c^2 + m_t^2\right]\csc^2\beta &= 0 \end{aligned}$$

Multiplying both sides of the above equation by $\frac{v^2}{4}$ we get ;

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{v^2}{4}\left(6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5\right) = \left[\left(m_e^2 + m_\mu^2 + m_\tau^2\right) + 3\left(m_u^2 + m_c^2 + m_t^2\right)\right]\csc^2\beta\,. \tag{3.59}$$

Thus we see that Eq. (3.20) and Eq. (3.21) are not physically relevant as they are identically satisfied for each type. We therefore deal with the first two of the Veltman conditions which have been expressed for the four types of two Higgs doublet models obeying a $U(1)$ symmetry. Henceforth we will identify Eq. (3.18) as Veltman Condition 1 (or VC1) and Eq. (3.19) as Veltman Condition 2 (or VC2). For the sake of convenience we enlist below VC1 and VC2.

- **Type - I 2HDM**
  1. Veltman Condition 1

$$\frac{3}{2}M_{\rm w}^2 + \frac{3}{4}M_{\rm z}^2 + \frac{v^2}{4}\left(6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5\right) = 0\,. \tag{3.60}$$

  2. Veltman Condition 2

$$\begin{aligned} \frac{3}{2}M_w^2 + \frac{3}{4}M_z^2 + \frac{v^2}{4}\left(6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5\right) &= \left[\left(m_e^2 + m_\mu^2 + m_\tau^2\right) + 3\left(m_u^2 + m_c^2 + m_t^2\right)\right. \\ &\quad \left. +3\left(m_d^2 + m_s^2 + m_b^2\right)\right]\csc^2\beta\,. \end{aligned} \tag{3.61}$$

- **Type - II 2HDM**
  1. Veltman Condition 1

$$\begin{aligned} \frac{3}{2}M_{\rm w}^2 + \frac{3}{4}M_{\rm z}^2 + \frac{v^2}{4}\left(6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5\right) &= \left[\left(m_e^2 + m_\mu^2 + m_\tau^2\right)\right. \\ &\quad \left. +3\left(m_d^2 + m_s^2 + m_b^2\right)\right]\sec^2\beta\,. \end{aligned} \tag{3.62}$$

2. Veltman Condition 2

$$\frac{3}{2}M_{\rm w}^2+\frac{3}{4}M_{\rm z}^2+\frac{v^2}{4}\left(6\lambda_2+10\lambda_3+\lambda_4+\lambda_5\right)=3\left(m_u^2+m_c^2+m_t^2\right)\csc^2\beta\,. \tag{3.63}$$

- **Lepton Specific 2HDM**

1. Veltman Condition 1

$$\frac{3}{2}M_{\rm w}^2+\frac{3}{4}M_{\rm z}^2+\frac{v^2}{4}\left(6\lambda_1+10\lambda_3+\lambda_4+\lambda_5\right)=\left(m_e^2+m_\mu^2+m_\tau^2\right)\sec^2\beta\,. \tag{3.64}$$

2. Veltman Condition 2

$$\begin{aligned}\frac{3}{2}M_w^2+\frac{3}{4}M_z^2+\frac{v^2}{4}\left(6\lambda_2+10\lambda_3+\lambda_4+\lambda_5\right) &= 3\Big[\left(m_u^2+m_c^2+m_t^2\right) \\ &\quad +\left(m_d^2+m_s^2+m_b^2\right)\Big]\csc^2\beta\,.\end{aligned} \tag{3.65}$$

- **Flipped 2HDM**

1. Veltman Condition 1

$$\frac{3}{2}M_{\rm w}^2+\frac{3}{4}M_{\rm z}^2+\frac{v^2}{4}\left(6\lambda_1+10\lambda_3+\lambda_4+\lambda_5\right)=3\left(m_d^2+m_s^2+m_b^2\right)\sec^2\beta\,. \tag{3.66}$$

2. Veltman Condition 2

$$\begin{aligned}\frac{3}{2}M_w^2+\frac{3}{4}M_{\rm z}^2+\frac{v^2}{4}\left(6\lambda_2+10\lambda_3+\lambda_4+\lambda_5\right) &= \Big[\left(m_e^2+m_\mu^2+m_\tau^2\right) \\ &\quad +3\left(m_u^2+m_c^2+m_t^2\right)\Big]\csc^2\beta\,.\end{aligned} \tag{3.67}$$

We have written the Veltman conditions 1 and 2 such that the left hand side of the Veltman condition 1 and Veltman condition 2 are the same for all the four types of two Higgs doublet models. The right hand sides are model dependent. Moreover the left hand side contains the various quartic couplings apart from the masses of the gauge bosons. In order to customize the Veltman conditions in terms of the masses of the physical Higgs bosons to get valuable information from them we need to express the quartic couplings in terms of the physical Higgs boson masses. With reference to the 2HDM scalar potential given in Eq. (3.17), we've v_1, v_2 and the five λ parameters as the independent parameters. They are related to the four physical Higgs boson masses in the alignment limit $\left(\beta-\alpha\sim\frac{\pi}{2}\right)$ (to be discussed in detail in Section 4.1) under the $U(1)$ symmetry by the following relations [24] as discussed in Section 1.2.

$$\lambda_1 = \frac{1}{2v^2 c_\beta^2} m_H^2 - \frac{\lambda_5}{4}(\tan^2\beta - 1)\,, \tag{3.68}$$

$$\lambda_2 = \frac{1}{2v^2 s_\beta^2} m_H^2 - \frac{\lambda_5}{4}\left(\frac{1}{\tan^2\beta} - 1\right)\,, \tag{3.69}$$

$$\lambda_3 = -\frac{1}{2v^2}(m_H^2 - m_h^2) - \frac{\lambda_5}{4}\,, \tag{3.70}$$

$$\lambda_4 = \frac{2}{v^2} m_\xi^2\,, \qquad \lambda_5 = \lambda_6 = \frac{2}{v^2} m_A^2\,. \tag{3.71}$$

Again using here the short-hand notation for the trigonometric functions i.e., $c_\alpha \equiv \cos\alpha$ and $s_\alpha \equiv \sin\alpha$, and likewise for β. Thus, an alternative way of counting the independent parameters is through the four masses, the two angles α and β, the electroweak vev, v and the parameter λ_5, which appears on the rhs of the above equations. In this set of eight parameters, v is $\sqrt{v_1^2 + v_2^2} = 246$GeV and so is the lightest CP-even Higgs mass m_h as 125 GeV.

Using relations Eq. (3.68)- Eq. (3.71) we can express the left hand side of the Veltman conditions 1 and 2 in terms of the masses of the physical Higgs bosons, β, v and λ_5.

LHS of first Veltman Condition:

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{5}{4}m_h^2 + \frac{1}{2}m_\xi^2 + \frac{1}{4}m_H^2\left(3\tan^2\beta - 2\right) - \frac{3v^2}{8}\lambda_5\tan^2\beta\,, \tag{3.72}$$

LHS of second Veltman Condition:

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{5}{4}m_h^2 + \frac{1}{2}m_\xi^2 + \frac{1}{4}m_H^2\left(3\cot^2\beta - 2\right) - \frac{3v^2}{8}\lambda_5\cot^2\beta\,. \tag{3.73}$$

The Veltman conditions for the four types of 2HDMs have been tabulated in table 3.1. The Yukawa matrices which vanish in each model are listed in the second column. We note here that although naturalness conditions in specific 2HDMs have been studied earlier on a few occasions [25, 26], they were not done in the SM-like scenario, nor expressed in terms of the physical masses for the different types as in here.

Model	zero Yukawa	VC1	VC2
		$6M_W^2+3M_Z^2+5m_h^2$ $+2m_\xi^2+m_H^2(3\tan^2\beta-2)$ $-\frac{3v^2}{2}\lambda_5\tan^2\beta=$	$6M_W^2+3M_Z^2+5m_h^2$ $+2m_\xi^2+m_H^2\left(3\cot^2\beta-2\right)$ $-\frac{3v^2}{2}\lambda_5\cot^2\beta=$
Type I	$G_{1e}\,,G_{1d}\,,G_{1u}$	0	$4\left[\sum m_e^2+3\sum m_u^2+3\sum m_d^2\right]\csc^2\beta$
Type II	$G_{2e}\,,G_{2d}\,,G_{1u}$	$4\left[\sum m_e^2+3\sum m_d^2\right]\sec^2\beta$	$12\sum m_u^2\csc^2\beta$
LS	$G_{2e}\,,G_{1d}\,,G_{1u}$	$4\sum m_e^2\sec^2\beta$	$12\left[\sum m_u^2+\sum m_d^2\right]\csc^2\beta$
Flipped	$G_{1e}\,,G_{2d}\,,G_{1u}$	$12\sum m_d^2\sec^2\beta$	$4\left[\sum m_e^2+3\sum m_u^2\right]\csc^2\beta$

Table 3.1: Veltman conditions for the different 2HDMs

References

[1] H. Georgi, S. Glashow, Phys. Rev. Lett. **32**, 438 (1974); H. Georgi, H. Quinn, S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).

[2] A. Linde, Pis'ma Zh. Eksp. Teor. Fiz. **19**, (1974) 320; Sov. Phys. JETP Lett. **19**, 183 (1974); M. Veltman, Rockefeller Univ. preprint, 1974, unpublished; Phys. Rev. Lett. **34**, 777 (1975)

[3] M. Veltman, Acta Phys. Pol. **B8**, 475 (1977); Phys. Lett. **70B**, 253 (1977).

[4] T. Appelquist, R. Shankar, Nucl. Phys. **B158**, 317 (1979); T. Appelquist, C. Bernhard, Phys. Rev.**D22**, 200 (1980); Yale preprint, July 1980; A. c. Longhitano, Phys. Rev. **D22**, 1166 (1980).

[5] W. Bardeen, K. Shizuya, Phys. Rev. **D18**, 1969 (1978).

[6] R. Jackiw, K. Johnson, Phys. Rev. **D8**, 2386 (1973).

[7] S. Weinberg, Phys. Rev. **D13**, 974 (1976).

[8] K. Wilson, Phys. Rev. **D3** 1818 (1971), esp. pg. 1840; L. Susskind, Phys. Rev. **D20**, 2619 (1979).

[9] S. Dimopoulos, L. Susskind, Nucl. Phys. **B155**, 237 (1979); E. Farhi, L. Susskind, Phys. Rev. **D20**, 3404; (1979); E. Eichten, K. Lane, Phys. Lett. **90B**, 125 (1980); M. Peskin, Saclay preprint 1980.

[10] J. Ellis, M. Gaillard, L. Maiani, B. Zumino, Europhysics Conf. Erice, March 1980; J. Ellis, M. Gaillard, B. Zumino, Phys, Lett. **94B**, 343 (1980).

[11] D. Foerster, H. Nielsen, M. Ninomiya, Phys. Lett. **94B**, 135 (1980).

[12] L. Maiani, G. Parisi, R. Petronzio, Nucl. Phys. **B136**, 402 (1979); J. Iliopoulos, D. Nanopoulos, T. Tomaras, Phys. Lett. **94B**, 141 (1980).

[13] G. 't Hooft, Cargese lectures 1979.

[14] T. Appelquist, T. Carazzone, Phys. Rev. **D11**, 2856 (1975); K. Symanzik, Comm. Math. Phys. **34**, 7(1973).

[15] D. Politzer, S. Wolfram, Phys. Lett. **82B**, 242 (1979); **E83B**, 421 (1979); P. Hung, Phys. Rev. Lett. **42**, 873 (1979); L. Chan, T. Hagiwara, B. Ovrat, Phys. Rev. **D20**, 1982 (1979); Y. P. Yao, Phys. Rev. **D7**, 1647 (1973); L. F. Li, Carnegie-Mellon, Univ. preprint, July 1980.

[16] M. Veltman, Acta Phys. Polon. **B12**, 437, 1981; Y. Nambu, in *"New Trends in Strong Coupling Gauge Theories",* edited by M. Bando, T. Muta and K. Yamawaki (World Scientific, 1989), pg. 3.

[17] S. Weinberg, Phys. Lett. **82B**, 387 (1979).

[18] M. B. Einhorn and D. R. T. Jones, *"The Effective potential and quadratic divergences",* Phys. Rev. D **46** (1992) 5206.

[19] M. S. Al-sarhi, I. Jack and D. R. T. Jones, Z. Phys. C **55** (1992) 283.

[20] Y. Hamada, H. Kawai and K. -y. Oda, Phys. Rev. **D87**, 053009 (2013) [arXiv:1210.2538[hep-ph]].

[21] T. D. Lee, *"A Theory of Spontaneous T Violation,"* Phys. Rev. D **8**, 1226 (1973).

[22] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *"The Higgs Hunter's Guide,"* Front. Phys. **80**, 1 (2000).

[23] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *"Errata for the Higgs Hunter's guide,"* arXiv:hep-ph/9302272.

[24] A. G. Akeroyd, A. Arhrib and E. M. Naimi, *"Note on tree level unitarity in the general two Higgs doublet model,"* Phys. Lett. B **490**, 119 (2000)

[25] B. Grzadkowski and P. Osland, *"Tempered Two-Higgs-Doublet Model,"* Phys. Rev. D **82**, 125026 (2010).

[26] R. Jora, S. Nasri and J. Schechter, *"Naturalness in a simple two Higgs doublet model,"* Int. J. Mod. Phys. A **28**, 1350036 (2013).

Chapter 4

How obese are the non-standard Higgs bosons?

It is the weight not number of
experiments that is to be regarded.

Sir Isaac Newton

In this chapter we aim to find the mass ranges of the non-standard physical Higgs bosons of the 2HDMs. In order to proceed to that we will be first discussing the various limits of the 2HDMs. Playing with the angles α and β lands us upon various limits of the two Higgs doublet models. The limits that we have worked with are the *alignment limit*, the *reverse alignment limit* and the *wrong sign limit*.

4.1 Alignment limit

The standard model Higgs is CP even, neutral and with a mass of around 125 GeV. In two Higgs doublet models there are two physical Higgs bosons that are CP even and neutral with masses either to be determined or assigned. We will identify one of these Higgs bosons as the SM Higgs that is to say that it has the same couplings with the SM particles as the SM Higgs boson. To this end we start with the trilinear gauge-Higgs couplings which arise from the kinetic part of the Lagrangian density. In 2HDM, trilinear gauge-Higgs coupling is,

$$\mathcal{L}^{\text{scalar}}_{\text{kinetic}} = \mid D_\mu \Phi_1 \mid^2 + \mid D_\mu \Phi_2 \mid^2 \supset \frac{g^2}{2} W^+_\mu W^-_\mu \left(v_1 h_1 + v_2 h_2 \right) . \tag{4.1}$$

If we take the combination,

$$H^0 = \frac{1}{v} \left(v_1 h_1 + v_2 h_2 \right) \tag{4.2}$$

and insert it in Eq. (4.1) we get back the exact SM trilinear gauge-Higgs coupling,

$$\mathcal{L}^{\text{scalar}}_{\text{kinetic}} = \mid D_\mu \Phi \mid^2 \supset \frac{vg^2}{2} W^+_\mu W^-_\mu H^0 \, . \tag{4.3}$$

Thus H^0 carries the exact SM-like gauge couplings. Whereas its orthogonal combination,

$$R = \frac{1}{v}\left(v_2 h_1 - v_1 h_2\right) \tag{4.4}$$

will not have any trilinear couplings with the gauge bosons.

Generalizing to the n-Higgs doublet models, the SM-like Higgs bosons is given as,

$$H^0 = \frac{1}{v}\left(v_1 h_1 + v_2 h_2 + \cdots + v_n h_n\right) \tag{4.5}$$

and

$$v = \left(v_1^2 + v_2^2 + \cdots + v_n^2\right)^{1/2} . \tag{4.6}$$

For two Higgs doublet models this combination of H^0 can be obtained in an alternative way. As we have seen before from Eq. (1.19), Eq. (1.21) and Eq. (1.23), the angle of rotation in the CP-even sector is different from that in the charged and pseudoscalar sectors. If we rotate the CP-even sector by the same angle β, we obtain two different scalar combinations H^0 and R as shown below,

$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} . \tag{4.7}$$

It is found that H^0 has exactly the standard model Higgs couplings with the fermions and gauge bosons [1, 2]. By using Eq. (1.23) and Eq. (4.7) we can find the relation between the physical scalars h and H with H^0 and R. The relevant equations are

$$h = \sin(\beta - \alpha)H^0 + \cos(\beta - \alpha)R \tag{4.8}$$

$$H = \cos(\beta - \alpha)H^0 - \sin(\beta - \alpha)R . \tag{4.9}$$

Thus in order for h, the lightest CP-even state to be the Higgs boson of the standard model, we require

$$\sin(\beta - \alpha) \approx 1 , \tag{4.10}$$

which has been called the SM-like or alignment limit [3]. It is to be noted that the SM-like state, H^0, is not guaranteed to be a mass eigenstate in general. H^0 must also carry the SM-like Yukawa couplings. For this let us have a look into the Yukawa Lagrangian density for two Higgs doublet model,

$$\mathcal{L}_Y = \sum_{i=1}^{2} \left[-\bar{Q}_L \tilde{\Phi}_i G_u^i p_R - \bar{Q}_L \Phi_i G_d^i n_R + h.c. \right] , \tag{4.11}$$

where Q_L is the left-handed 3-vectors of isodoublets in the space of generations, $Q_L = (p_L \;\; n_L)^T$ where p_L and n_L are left-handed up-type quarks and down-type quarks respectively in the gauge basis. p_R and n_R are right-handed 3-vectors of singlets, G_u^i and G_d^i etc. are complex 3×3 matrices in generation space containing the Yukawa coupling constants in the up-type and down-type sectors respectively. In writing the above Yukawa Lagrangian we have suppressed the flavour indices. After spontaneous symmetry breaking,

$$\Phi_i = \begin{pmatrix} w_i^+(x) \\ \dfrac{v_i + h_i(x) + iz_i(x)}{\sqrt{2}} \end{pmatrix}, \tag{4.12}$$

$$\tilde{\Phi}_i \quad = i\tau_2 \Phi_i^* \quad = \begin{pmatrix} \dfrac{v_i + h_i(x) - iz_i(x)}{\sqrt{2}} \\ -w_i^-(x) \end{pmatrix} \tag{4.13}$$

where τ_2 is the second Pauli matrix and i takes values 1 and 2 for the 2HDM.

Therefore the mass matrices take the following form in the gauge basis,

$$M_n \;=\; \frac{1}{\sqrt{2}} \sum_{i=1}^{2} v_i G_d^i\,, \tag{4.14}$$

$$M_p \;=\; \frac{1}{\sqrt{2}} \sum_{i=1}^{2} v_i G_u^i\,. \tag{4.15}$$

If we perform a biunitary transformation then the above mass matrices become diagonal as shown below.

$$\begin{aligned} D_d &= U_L^\dagger \cdot M_n \cdot U_R = \mathrm{diag}\,(m_d,\, m_s,\, m_b)\ , \\ D_u &= V_L^\dagger \cdot M_p \cdot V_R = \mathrm{diag}\,(m_u,\, m_c,\, m_t)\ . \end{aligned} \tag{4.16}$$

The matrices U and V relate the quark fields in the gauge basis to those in the mass basis as follows,

$$\begin{aligned} n_L &= U_L d_L, & n_R &= U_R d_R\,, \\ p_L &= V_L u_L, & p_R &= V_R u_R\,. \end{aligned} \tag{4.17}$$

The CP even Yukawa Lagrangian stemming from Eq. (4.11) becomes,

$$\mathcal{L}_Y^{\mathrm{CP-even}} \;=\; -\frac{1}{\sqrt{2}}\bar{p}_L \left(\sum_{i=1}^{2} G_u^i h_i\right) p_R - \frac{1}{\sqrt{2}}\bar{n}_L \left(\sum_{i=1}^{2} G_d^i h_i\right) n_R + \text{h.c.}\,. \tag{4.18}$$

Now $\{h_1,\, h_2\}$ basis is rotated via an orthogonal transformation to a new basis containing the states $\{H^0,\, R\}$ which was earlier defined in Eq. (4.2) and Eq. (4.4).

Thus using Eq. (4.2) and Eq. (4.4) we can write down the Yukawa couplings of the new state H^0 as follows,

$$\begin{aligned} \mathcal{L}_Y^{H^0} &= -\frac{H^0}{v\sqrt{2}}\bar{p}_L \left(\textstyle\sum_{i=1}^{2} G_u^i v_i\right) p_R - \frac{H^0}{v\sqrt{2}}\bar{n}_L \left(\textstyle\sum_{i=1}^{2} G_d^i v_i\right) n_R + \text{h.c.} \\ &= -\frac{H^0}{v}\left[\bar{p}_L M_p p_R + \bar{n}_L M_n n_R\right] + \text{h.c.}\,. \end{aligned} \tag{4.19}$$

Using Eq. (4.16) and Eq. (4.17) we can rewrite the Yukawa Lagrangian of Eq. (4.19) in the mass basis as,

$$\begin{aligned}\mathcal{L}_Y^{H^0} &= -\frac{H^0}{v}\left[\bar{u}_L D_u u_R + \bar{d}_L D_d d_R\right] + \text{h.c.} \\ &\equiv -\frac{H^0}{v}\left[\bar{u} D_u u + \bar{d} D_d d\right] ,\end{aligned} \tag{4.20}$$

where the last step follows from the fact that $D_{u,d}$ are diagonal. Eq. (4.20) shows that H^0 by construction possesses SM-like Yukawa couplings with the SM fermions.

There is yet another limit the *decoupling limit*, the first cousin of the alignment limit. We need to be able to distinguish between these two limits. We take the help of the potential in the Higgs-basis given by Eq. (1.35) to clearly distinguish between these two limits. It allows us to write expressions that facilitate in some cases the discussion of alignment and decoupling limits in the two Higgs doublet models [4]. Let us try to clarify what we mean by alignment limit and decoupling limit. The LHC has shown beyond all doubt that the 125 GeV scalar which has been discovered has SM-like behaviour which means it seems to couple to gauge bosons and fermions very much like the SM Higgs boson would do. Within models with two doublets, this implies that the scalar state with 125 GeV mass needs to be almost aligned with the vacuum expectation value. The question is how does one obtain such aligned regimes in the two Higgs doublet models? For this let us take a look at the CP even mass matrix in the Higgs-basis,

$$\mathcal{M}_{CP-even} = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix} . \tag{4.21}$$

Having an aligned scalar means that there won't be much mixing between the two CP-even states, and this can be achieved in two ways:

- One of the diagonal elements in Eq. (4.21) is much bigger than the other one. Since Z_1 is a quartic coupling and therefore expected not to be large so that the theory remains perturbative at high scales, this forces the (2,2) entry in the matrix to be quite large, and it is simple to show that all extra scalars will be heavy. In this regime, alignment is achieved in the decoupling limit.
- The off-diagonal elements in Eq. (4.21) are much smaller than the diagonal ones. In this regime, the masses of the extra scalars are not necessarily large, and the SM-like behaviour of the 125 GeV state is said to be caused by the alignment limit.

The relevant coupling equations of h or H to gauge bosons are [5, 6]

$$\mid s_{\beta-\alpha} c_{\beta-\alpha} \mid = \frac{\mid Z_6 \mid v^2}{m_H^2 - m_h^2} , \tag{4.22}$$

and

$$Z_1 v^2 = m_h^2 s_{\beta-\alpha}^2 + m_H^2 c_{\beta-\alpha}^2 . \tag{4.23}$$

Assuming that the lightest state is the one that is aligned with the vacuum expectation value, and that it has a mass of 125 GeV, its tree-level couplings are very close to that of the standard model Higgs boson. This limit is attained by setting $c_{\beta-\alpha} \to 0$. For this it is sufficient to have $Z_6 \ll 1$ to be in the alignment limit as seen from Eq. (4.22). In this regime, although the couplings of the 125 GeV Higgs are all SM-like, the other Higgs bosons can in principle be light and therefore be within the reach of the LHC.

To have alignment in the decoupling limit the masses of the non-125 Higgs bosons must not be much larger than 125 GeV. Defining a common mass scale m_{heavy} with $\Phi_{heavy} = H, A$ and $H^{\pm}$ we can write [7],

$$m^2_{heavy} = M^2 + f(\lambda_i)v^2 + \mathcal{O}(\frac{v^4}{M^2})\,, \tag{4.24}$$

where $f(\lambda_i)$ denotes a linear combination of $\lambda_1 \cdots \lambda_5$. In the case when $s_{\beta-\alpha} \to 0$ there is again alignment but now with the heavy CP-even Higgs H, meaning this would correspond to the heavy Higgs scenario. The condition for this regime to occur is still $Z_6 \ll 1$, but now decoupling is not possible, as the non-SM-like Higgs boson masses are not all much larger than 125 GeV, in particular not m_h.

4.1.1 Mass bounds in Alignment limit

We first rewrite the relations between the quartic couplings that appear in the potential in Eq. (1.6) and the masses of the Higgs bosons in the alignment limit. Imposition of the $U(1)$ symmetry forces $\lambda_5 = \lambda_6$. Keeping this in mind and using $\sin(\beta - \alpha) \sim 1$ the set of equations Eq. (1.29) - Eq. (1.33) can be rewritten as [8],

$$\lambda_1 = \frac{1}{2v^2 c^2_\beta} m^2_H - \frac{\lambda_5}{4}(\tan^2\beta - 1)\,, \tag{4.25}$$

$$\lambda_2 = \frac{1}{2v^2 s^2_\beta} m^2_H - \frac{\lambda_5}{4}\left(\frac{1}{\tan^2\beta} - 1\right)\,, \tag{4.26}$$

$$\lambda_3 = -\frac{1}{2v^2}(m^2_H - m^2_h) - \frac{\lambda_5}{4}\,, \tag{4.27}$$

$$\lambda_4 = \frac{2}{v^2} m^2_\xi\,, \qquad \lambda_5 = \lambda_6 = \frac{2}{v^2} m^2_A\,. \tag{4.28}$$

The Veltman conditions as obtained for the four types of 2HDMs in the alignment limit under $U(1)$ symmetry has been tabulated in Table 3.1. Our main result in this chapter deals with the bounds we have obtained for the masses of the heavy and charged Higgs particles. The lighter CP-even h particle is assumed to be the one that has been observed at the LHC, so that $m_h = 125$ GeV, and $v = 246$ GeV. To explain the procedure let us consider the example of the type II model. Same procedure can be adopted for the other types of 2HDMs.

Since we want the bounds on m_H and m_ξ , let us rewrite VC1 and VC2 for the type

II model in a convenient form,

$$m_H^2\left(3\tan^2\beta-2\right)+2m_\xi^2 =4\left[\sum m_e^2+3\sum m_d^2\right]\sec^2\beta-6M_W^2-3M_Z^2-5m_h^2+\lambda_5\frac{3v^2}{2}\tan^2\beta\,, \tag{4.29}$$

$$m_H^2\left(3\cot^2\beta-2\right)+2m_\xi^2 =12\sum m_u^2\csc^2\beta-6M_W^2-3M_Z^2-5m_h^2+\lambda_5\frac{3v^2}{2}\cot^2\beta\,. \tag{4.30}$$

As we can see all but the last term on the right hand side of either equation are experimentally known. However there is an estimate of the magnitude of the last term. The $U(1)$ symmetry implies that $\lambda_5 > 0$, since $\lambda_5 = \lambda_6 = \frac{2}{v^2}m_A^2$ and we impose the restriction of $|\lambda_i| \leq 4\pi$ based on the validity of perturbativity. Comparing with the second equation in Eq. (4.28), we see that this in turn puts a restriction on $m_A \lesssim 617$GeV.

We further restrict the mass ranges by imposing constraints coming from stability, perturbative unitarity, and the oblique electroweak T-parameter. These have been discussed elaborately in Chapter 2. For the sake of recapitulation we note that conditions for stability, i.e. for the scalar potential being bounded from below, were examined in [4, 9, 10], and found to provide lower bounds on certain combinations of the quartic couplings λ_i. On the other hand, the requirement of perturbative unitarity translates into upper limits on combinations of the λ_i, which for two-Higgs models have been derived by many authors [8, 11–13]. One condition coming from perturbative unitarity is

$$\left|3(\lambda_1+\lambda_2+2\lambda_3)\pm\sqrt{9(\lambda_1-\lambda_2)^2+(4\lambda_3+\lambda_4+\lambda_5)^2}\right| \leq 16\pi \tag{4.31}$$

Stability provides the inequalities

$$\lambda_1+\lambda_3>0\,,\qquad \lambda_2+\lambda_3>0\,, \tag{4.32}$$

so that we can write Eq. (4.31) as $|A\pm B|\leq 16\pi$, with $A\,,B\geq 0$. It then follows that

$$0\leq \lambda_1+\lambda_2+2\lambda_3\leq \frac{16\pi}{3}\,. \tag{4.33}$$

In terms of the scalar masses, this reads

$$0<(m_H^2-m_A^2)(\tan^2\beta+\cot^2\beta)+2m_h^2<\frac{32\pi v^2}{3}. \tag{4.34}$$

For $\tan\beta \gg 1$, this inequality implies that m_H and m_A are almost degenerate, a result also found in [14]. In figure 4.1 we have shown this degeneracy by plotting m_A against m_H for different values of $\tan\beta$. It is easy to see from the plots that the degeneracy is more pronounced at higher values of m_A for any value of $\tan\beta$. For these plots we have used the perturbativity condition $|\lambda_i|\leq 4\pi$, which restricts $m_A \lesssim 617$ GeV.

For our purpose we will need another inequality which follows from the condition

$$|2\lambda_3+\lambda_4| \leq 16\pi \tag{4.35}$$

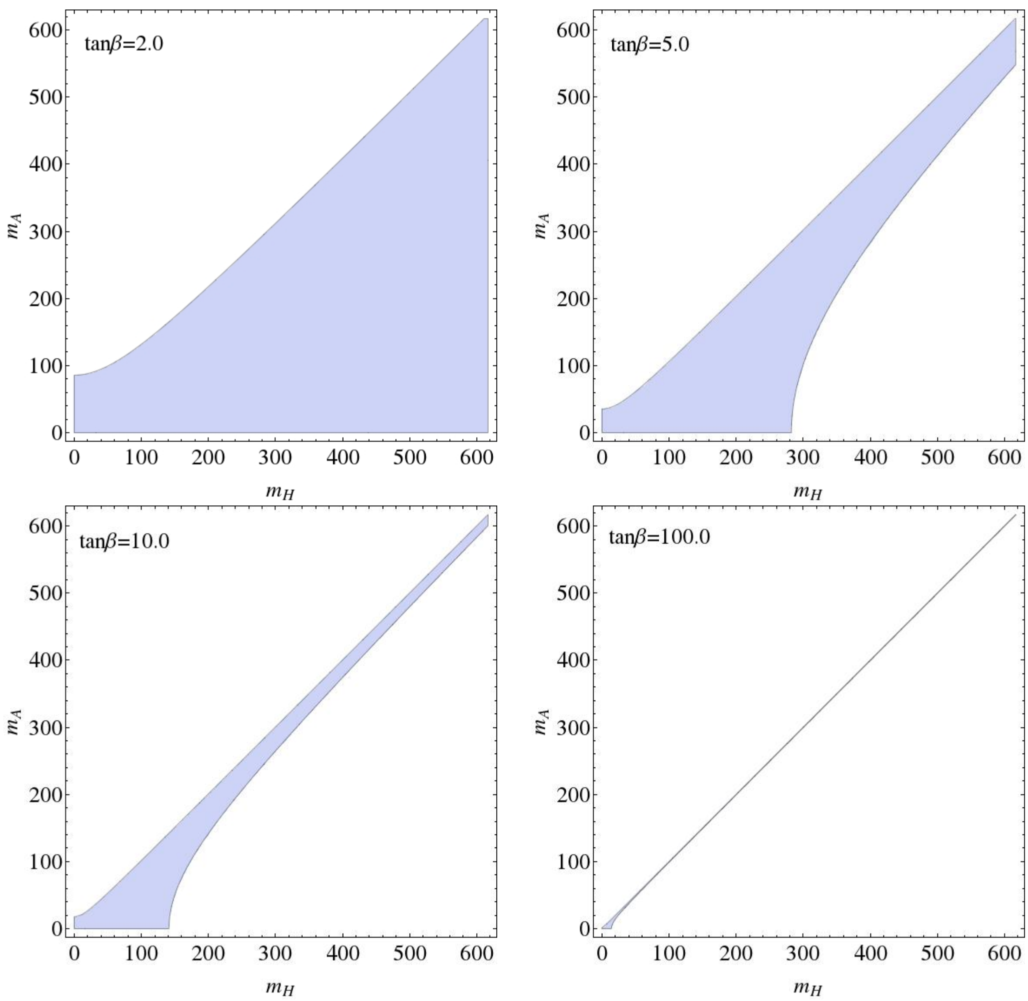


Figure 4.1: Degeneracy of $m_H - m_A$ (in GeV) for progressively increasing $\tan\beta$. The condition $|\lambda_i| \leq 4\pi$ restricts $m_A \lesssim 617$ GeV.

required for perturbative unitarity. Substituting the mass relations of Eq. (4.27) and Eq. (4.28) into this, we get

$$\left|2m_\xi^2 - m_H^2 - m_A^2 + m_h^2\right| \le 16\pi v^2\,. \tag{4.36}$$

Next we take into account the oblique parameter T for the 2HDMs, which has the expression [15, 16]

$$T = \frac{1}{16\pi \sin^2\theta_W M_W^2}\left[F(m_\xi^2, m_H^2) + F(m_\xi^2, m_A^2) - F(m_H^2, m_A^2)\right], \tag{4.37}$$

with

$$F(x,y) = \begin{cases} \frac{x+y}{2} - \frac{xy}{x-y}\ln\frac{x}{y}\,, & x \neq y \\ 0 & x = y \end{cases} \tag{4.38}$$

The T parameter is constrained by the global fit to precision electroweak data to be [17]

$$T = 0.05 \pm 0.12. \tag{4.39}$$

We have the tools ready. For $\tan\beta = 5$, we have plotted the two Veltman conditions for several values of λ_5 and the constraints arising from stability, perturbative unitarity and the T parameter in the m_H vs m_ξ plane for each type of 2HDM. The resulting plot is shown in figure 4.2. VC1 produces ellipses, and VC2 gives a narrow band of hyperbolae. Their crossings which fall inside the band representing the bound from the inequalities from Eq. (4.34) and Eq. (4.36) are the allowed masses. From the plot we can estimate the individual bounds: for all four models, we find approximately 550 GeV $\lesssim m_\xi \lesssim$ 700 GeV, and about 450 GeV $\lesssim m_H \lesssim$ 620 GeV, with a higher m_H implying a higher m_ξ. As mentioned earlier, m_A is close to m_H as a result of Eq. (4.34). We also note that direct searches have put a rough lower bound of $m_\xi > 100$ GeV [18].

4.2 Reverse Alignment limit

This limit is the counter part of the alignment limit when the heavier CP-even Higgs boson is considered to be the SM-like Higgs with mass $m_H = 125$ GeV. This is formulated by rearranging the equations obtained in the previous section. Now H is obtained in terms of H^0 and R using Eq. (4.40) and Eq. (4.41).

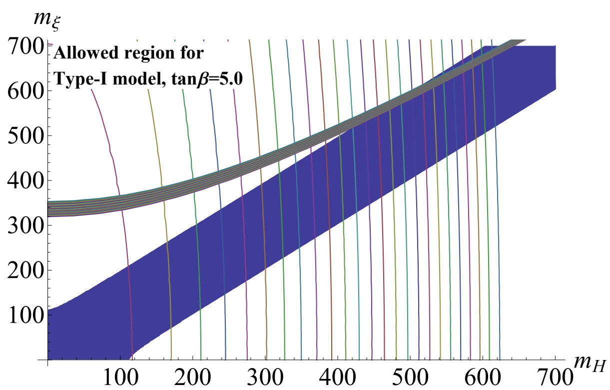
$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \tag{4.40}$$

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}. \tag{4.41}$$

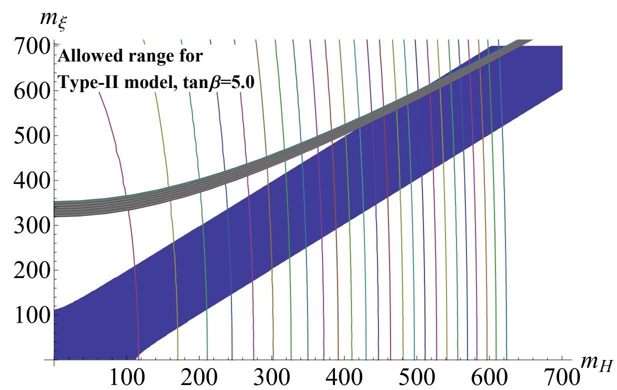
We obtain,

$$H = H^0\cos(\beta-\alpha) - R\sin(\beta-\alpha)\,, \tag{4.42}$$

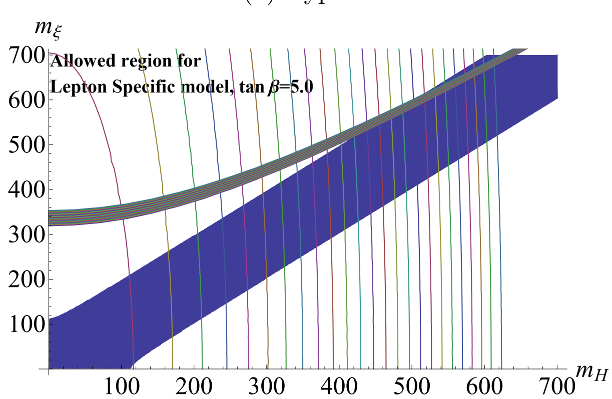
where if H has to be the SM-like Higgs boson, it will have to resemble the properties of H^0 and for that β would have to approximately equal α or $\pi+\alpha$. The results for the



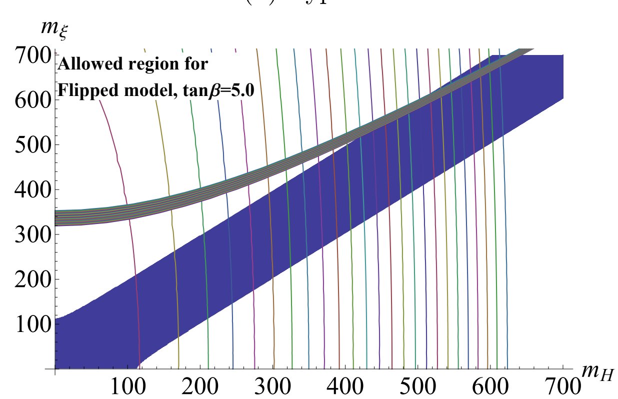
(a) Type I



(b) Type II



(c) Lepton specific



(d) Flipped

Figure 4.2: Allowed mass range in GeV for the charged Higgs and the heavy CP even Higgs in *Alignment limit* for (a) type I, (b) type II, (c) lepton specific and (d) flipped 2HDM. The shaded band is the region allowed by stability, perturbative unitarity and the T parameter constraints. The ellipses and the narrow band of hyperbolae are the two Veltman conditions for different values of λ_5 with $0 < \lambda_5 < 4\pi$ and $\tan\beta = 5$.

mass ranges of the non-standard Higgs bosons with $\beta \approx \alpha$ and $\beta \approx \pi + \alpha$ are identical, so in what follows we will work with $\beta \approx \alpha$ and call it the *Reverse Alignment Limit.*

In the reverse alignment limit under $U(1)$ symmetry, Eq. (1.29) - Eq. (1.33) become,

$$\begin{aligned}
\lambda_1 &= \frac{m_h^2}{2v^2}(\tan^2\beta + 1) - \frac{\lambda_5}{4}(\tan^2\beta - 1)\,, && (4.43)\\
\lambda_2 &= \frac{m_h^2}{2v^2}(\cot^2\beta + 1) - \frac{\lambda_5}{4}(\cot^2\beta - 1)\,, && (4.44)\\
\lambda_3 &= \frac{1}{2v^2}(m_H^2 - m_h^2) - \frac{\lambda_5}{4}\,, && (4.45)\\
\lambda_4 &= \frac{2}{v^2}m_\xi^2\,, && (4.46)\\
\lambda_5 &= \lambda_6 = \frac{2}{v^2}m_A^2\,. && (4.47)
\end{aligned}$$

Setting up the tools for this limit next we need to rewrite the Veltman conditions in the reverse alignment limit. For demonstration we rewrite the Veltman conditions Eq. (4.48) and Eq. (4.49) for type II 2HDM,

$$2\,\mathrm{Tr}\,G_e^1 G_e^{1\dagger} + 6\,\mathrm{Tr}\,G_u^{1\dagger} G_u^1 + 6\,\mathrm{Tr}\,G_d^1 G_d^{1\dagger} = \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5\,, \quad (4.48)$$

$$2\,\mathrm{Tr}\,G_e^2 G_e^{2\dagger} + 6\,\mathrm{Tr}\,G_u^{2\dagger} G_u^2 + 6\,\mathrm{Tr}\,G_d^2 G_d^{2\dagger} = \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5\,. \quad (4.49)$$

Thus for type II the two Veltman conditions in the reverse alignment limit read,

$$\begin{aligned}
m_h^2\left(3\tan^2\beta - 2\right) + 2m_\xi^2 &= 4\left[\sum m_e^2 + 3\sum m_d^2\right]\sec^2\beta - 6M_W^2 - 3M_Z^2 - 5m_H^2\\
&\quad + \lambda_5\frac{3v^2}{2}\tan^2\beta\,, && (4.50)\\
m_h^2\left(3\cot^2\beta - 2\right) + 2m_\xi^2 &= 12\sum m_u^2\csc^2\beta - 6M_W^2 - 3M_Z^2 - 5m_H^2\\
&\quad + \lambda_5\frac{3v^2}{2}\cot^2\beta\,. && (4.51)
\end{aligned}$$

We have plotted the above equalities on the $m_h - m_\xi$ plane for several values of λ_5 and for a fixed value of $\tan\beta$. We have fixed $m_H = 125$ GeV, with $m_h \leq m_H$. On the same graph, we have also plotted the region allowed by stability, perturbative unitarity, and constraints from $\delta\rho$. Among the various conditions arising due to stability and perturbative unitarity as discussed in Section 2.1 and Section 2.2, the following two inequalities in the reverse alignment limit are relevant to this plot:

$$0 \leq \left(m_h^2 - m_A^2\right)\left(\tan^2\beta + \cot^2\beta\right) + 2m_H^2 \leq \frac{32\pi v^2}{3}\,, \quad (4.52)$$

$$\left|2m_\xi^2 - m_h^2 - m_A^2 + m_H^2\right| \leq 16\pi v^2\,. \quad (4.53)$$

These are analogous to similar inequalities found in the alignment limit.

For $\tan\beta = 5$, the plots for all four types of 2HDM are shown in figure 4.3. The gray region covers the points which satisfy the inequalities Eq. (4.52) and Eq. (4.53) in addition to the constraints from $\delta\rho$, the first Veltman condition provides the curves (ellipses) which cross this region, and the second Veltman condition provides the nearly flat hyperbolas above the gray region.

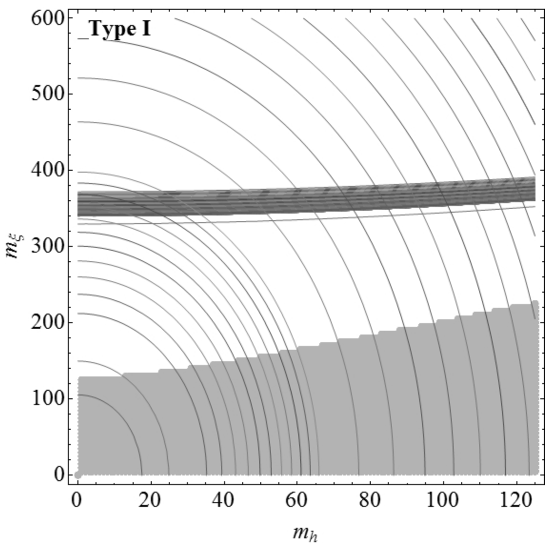
As we can see from the plots in figure 4.3, there is no region on the $m_h - m_\xi$ plane where all the constraints are obeyed. In other words, if we insist on naturalness, as embodied by the Veltman conditions, the reverse alignment limit is not a valid limit for any of the 2HDMs, i.e. the observed Higgs particle cannot be the heavier CP-even neutral scalar in any of the 2HDMs.

It should be mentioned here that allowed mass ranges of scalars in both the alignment limit and the reverse alignment limit were studied in [19]. However, that paper considered an unbroken Z_2 symmetry, not a softly broken symmetry as we have considered. As a result the mass ranges of scalars, as well as the allowed range of $\tan\beta$ found in that paper, are different from the ones we have found.

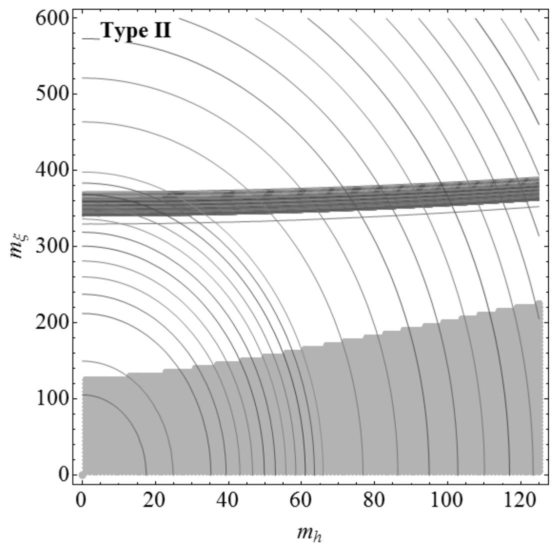
4.3 Wrong sign limit

Wrong sign limit discusses the interesting possibility of a sign change in one of the Higgs Yukawa couplings, $h\bar{D}D$ for down-type fermions or $h\bar{U}U$ for up-type fermions, relative to the Higgs coupling to VV where $V = W^{\pm}$ or Z. In the SM scenario just by measuring the properties of the observed Higgs-like boson the current LHC results cannot differentiate between scenarios where a sign change occurs in the $h\bar{D}D$ Yukawa couplings [20–22]. For example, the coupling of the Higgs to top quarks must have the conventional positive sign relative to the Higgs coupling to the gauge bosons while the absolute value of the couplings of down-type quarks and leptons relative to their SM values are constrained to values, 1.0 ± 0.2, where the sign ambiguity arises from the weak dependence of the gg and $\gamma\gamma$ loops on the Higgs couplings to bottom-quark pairs. This observation was put forward in a recent work [22]. This sign degeneracy in the determination of $h\bar{D}D$ at the LHC has also been emphasized in [23]. With the introduction of new particles beyond the SM ones, one can attempt to distinguish between the same sign and the wrong sign scenarios through the Higgs-diphoton decay width which is affected due to this sign reversal since the sign of $h\bar{D}D$ impacts both the ggh and $\gamma\gamma h$ couplings. We intend to study this in the current section.

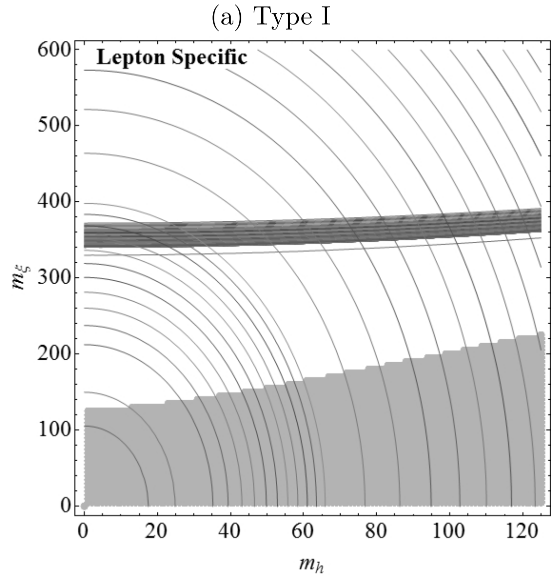
The wrong-sign Yukawa coupling regime [3, 24, 25] is defined as the region of 2HDM parameter space in which at least one of the couplings of the SM-like Higgs to up-type and down-type quarks is opposite in sign to the corresponding coupling of SM-like Higgs to vectors bosons. This is to be distinguished from the Standard Model, where the couplings of h_{SM} to $\bar{f}f$ and vector bosons are of the same sign. The *wrong sign limit* needs to be considered in conjunction with either the alignment limit or the reverse alignment limit. We will now calculate the regions of parameter space when each of these two limits are combined with the wrong sign limit.



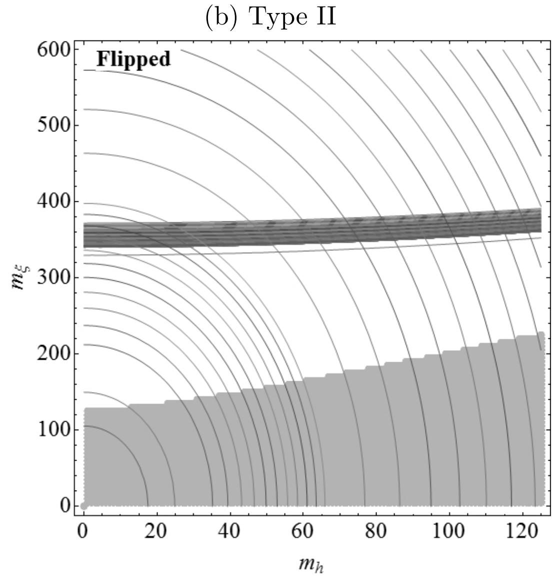
(a) Type I



(b) Type II



(c) Lepton specific



(d) Flipped

Figure 4.3: Mass range (in GeV) plot for the charged Higgs and the light CP even Higgs in *Reverse alignment limit* for (a) type I, (b) type II, (c) lepton specific and (d) flipped 2HDM for $|\lambda_5| \leq 4\pi$ and $\tan\beta = 5$.

The CP-even neutral scalars couple to the up-type and down-type quarks in the various 2HDMs as shown in table 4.1, with the SM couplings of the quarks to the SM Higgs field normalized to unity.

2HDMs	$h\bar{U}U$ (ξ_h^U)	$h\bar{D}D$ (ξ_h^D)	$H\bar{U}U$ (ξ_H^U)	$H\bar{D}D$ (ξ_H^D)
Type I	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$
Type II	$\frac{\cos\alpha}{\sin\beta}$	$-\frac{\sin\alpha}{\cos\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\cos\beta}$
Lepton Specific	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$
Flipped	$\frac{\cos\alpha}{\sin\beta}$	$-\frac{\sin\alpha}{\cos\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\cos\beta}$

Table 4.1: Yukawa couplings for the different 2HDMs

4.3.1 Wrong sign in the Alignment limit

Let us first look at what happens if some Yukawa couplings are of the wrong sign, in the alignment limit. In this case h is the SM Higgs, and its coupling to the vector bosons is $\sin(\beta-\alpha)$ times the corresponding SM value. Then in the convention where $\sin(\beta-\alpha)\geq 0$, the hVV couplings in the 2HDM are always non-negative. In order to analyse the wrong sign regime we write the type-II and Flipped Higgs-fermion Yukawa couplings, normalized with respect to the Standard Model couplings, in the following form:

$$h\bar{D}D: \qquad -\frac{\sin\alpha}{\cos\beta} = -\sin(\beta+\alpha)+\cos(\beta+\alpha)\tan\beta\,, \tag{4.54}$$

$$h\bar{U}U: \qquad \frac{\cos\alpha}{\sin\beta} = \sin(\beta+\alpha)+\cos(\beta+\alpha)\cot\beta\,. \tag{4.55}$$

In the case when $\sin(\beta+\alpha)=1$, the $h\bar{D}D$ coupling normalized to its SM value is equal to -1, while the normalized $h\bar{U}U$ coupling is $+1$. We note that in this limiting case, $\sin(\beta-\alpha)=-\cos 2\beta$, which implies that the wrong-sign $h\bar{D}D$ Yukawa coupling can only be achieved for values of $\tan\beta>1$.

Likewise, in the case of $\sin(\beta+\alpha)=-1$, the $h\bar{U}U$ coupling normalized to its SM value is equal to -1, whereas the normalized $h\bar{D}D$ coupling is $+1$. Then $\sin(\beta-\alpha)=$

$\cos 2\beta$, which implies that the wrong-sign $h\bar{U}U$ couplings can occur only if $\tan\beta < 1$. In the type-I and lepton specific 2HDM, both the $h\bar{D}D$ and $h\bar{U}U$ couplings are given by Eq. (4.55). Thus for $\sin(\beta+\alpha) = -1$, both the normalized $h\bar{D}D$ and $h\bar{U}U$ couplings are equal to -1, which is only possible if $\tan\beta < 1$. Thus realistically only the $h\bar{D}D$ coupling of the type-II and flipped 2HDM can be of the wrong sign, since $\tan\beta > 1$. We have arrived at the lower bound of $\tan\beta$ from the experimental findings as well. Although there is no consensus on the value of $\tan\beta$ except that it should be larger than unity. This is based on constraints coming from $Z \to b\bar{b}$, $B_q\bar{B}_q$ mixing [26], muon $g-2$ in lepton specific 2HDM [27] or using $b \to s\gamma$ in type I and flipped models [28].

Let us therefore consider a type II model with a wrong sign $h\bar{D}D$ coupling. The wrong sign limit approaches the alignment limit for $\tan\beta \approx 17$ as was displayed in [24,25] for the allowed parameter space of the type II CP-conserving 2HDM, based on the 8 TeV run of the LHC. For this model, we will plot the values of the pair (m_H, m_ξ) allowed by the naturalness conditions as well as the constraints imposed by perturbativity, stability, tree-level unitarity, and the ρ parameter. We will do this for four different values of $\tan\beta$ around the 'critical' value of 17. By choosing a small enough α we can ensure that for all these choices, both $\sin(\beta-\alpha) \approx 1$ and $\sin(\beta+\alpha) \approx 1$, as needed for the alignment limit and the wrong sign coupling.

In figure 4.4 we have plotted the Veltman conditions on the $m_H - m_\xi$ plane for Type II 2HDM for the four choices of $\tan\beta$, for different values of m_A constrained by $|\lambda_5| \leq 4\pi$. These plots are further constrained by conditions coming from stability of the potential, perturbative unitarity, and experimental bounds on $\delta\rho$. We have also taken $m_h = 125$ GeV. One can estimate from the plots that for $\tan\beta = 17$ that the range of m_H is approximately (250, 330) GeV, and that of m_ξ is approximately (260, 310) GeV. At higher values of $\tan\beta$, both ranges become narrower and move down on the mass scale.

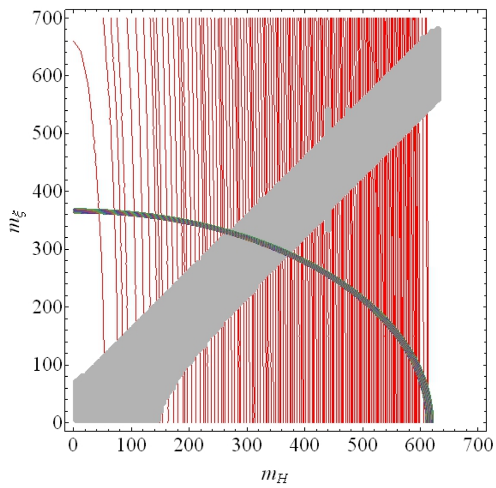
4.3.2 Wrong Sign and Reverse alignment limit

Let us now consider the case of wrong sign Yukawa couplings in the reverse alignment limit. The heavier CP-even neutral scalar H corresponds to the SM Higgs in the reverse alignment limit, with a coupling to vector bosons which is $\cos(\beta-\alpha)$ times the corresponding SM value. In the convention where $\cos(\beta-\alpha) \geq 0$, the HVV couplings in the 2HDM are always non-negative. To analyse the wrong-sign coupling regime, we write the Yukawa couplings in the type-II and Flipped 2HDMs in the following form:

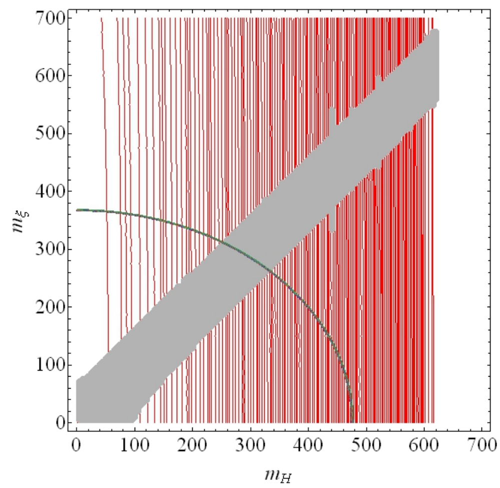
$$H\bar{D}D: \qquad \frac{\cos\alpha}{\cos\beta} = \cos(\beta+\alpha) + \sin(\beta+\alpha)\tan\beta\,, \tag{4.56}$$

$$H\bar{U}U: \qquad \frac{\sin\alpha}{\sin\beta} = -\cos(\beta+\alpha) + \sin(\beta+\alpha)\cot\beta\,. \tag{4.57}$$

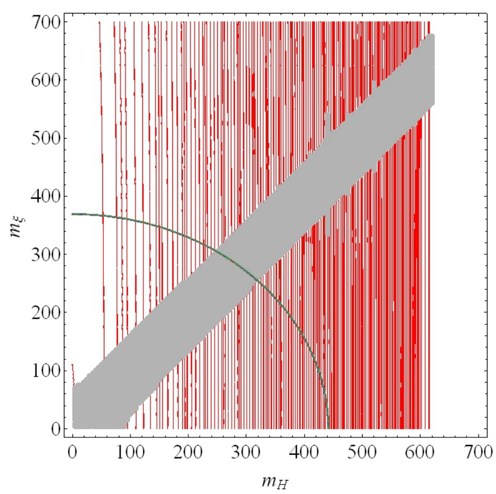
In the case when $\cos(\beta+\alpha) = -1$, the $H\bar{D}D$ coupling normalized to its SM value is equal to -1, whereas the normalized $H\bar{U}U$ coupling is $+1$. Thus in this case, when the reverse alignment limit is taken in conjunction with the wrong sign limit, we have



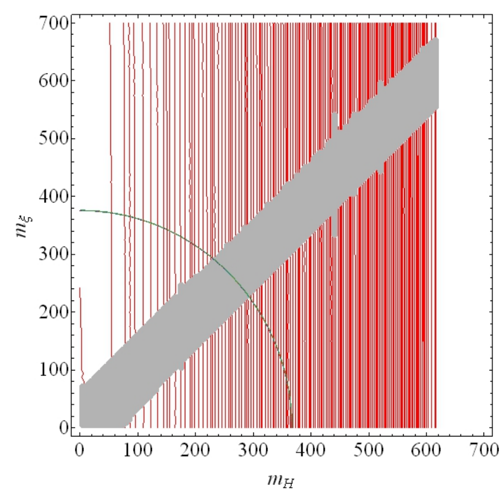
(a) $\tan\beta = 10$



(b) $\tan\beta = 17$



(c) $\tan\beta = 20$



(d) $\tan\beta = 30$

Figure 4.4: Allowed mass range in GeV for the charged Higgs and the heavy CP even Higgs when approaching *wrong sign* and *alignment limits* simultaneously for (a) $\tan\beta = 10$ (b) $\tan\beta = 17$ (c) $\tan\beta = 20$ and (d) $\tan\beta = 30$ for $|\lambda_5| \leq 4\pi$ and Type II 2HDM.

$\alpha \approx \beta \approx \frac{\pi}{2}$. It turns out there is no point on the $m_h - m_\xi$ plane which satisfies the Veltman conditions as well as the bounds coming from unitarity, stability and the ρ-parameter. In figure 4.5 only the first Veltman condition has been plotted, and it

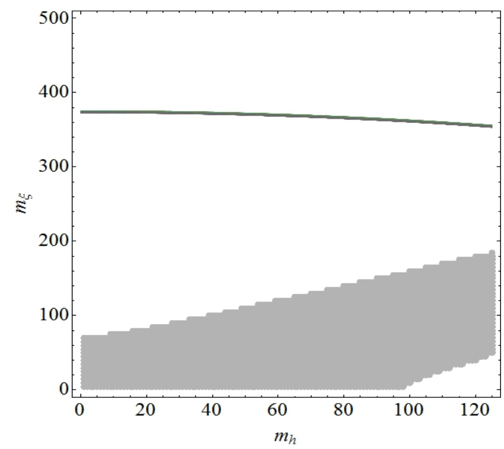


Figure 4.5: Veltman conditions are not satisfied for any (m_h, m_ξ) satisfying unitarity and other bounds, in the reverse alignment limit with wrong sign Yukawa couplings.

does not cross the grey region corresponding to the bounds. The other Veltman condition does not show up in this picture at all, it is not satisfied for any point in this plot.

On the other hand, in the case when $\cos(\beta+\alpha) = 1$, the $H\bar{U}U$ coupling normalized to its SM value is equal to -1, while the normalized $H\bar{D}D$ coupling is $+1$. In this limiting case, $\cos(\beta - \alpha) = \cos 2\beta$, which implies that the wrong-sign $H\bar{U}U$ couplings can only be achieved for $\tan\beta < 1$ for the type II and Flipped 2HDMs.

In the type-I and lepton specific 2HDMs, both the $H\bar{D}D$ and $H\bar{U}U$ couplings are given by Eq. (4.57). Thus, for $\cos(\beta + \alpha) = 1$, both the normalized $H\bar{D}D$ and $H\bar{U}U$ couplings are equal to -1, which is only possible if $\tan\beta < 1$.

Since $\tan\beta > 1$, we see that the wrong-sign Yukawa coupling is incompatible with the reverse alignment limit in all of the four types of 2HDMs.

4.4 Higgs decay

Higgs decays have been of vital importance and here we will consider its decay into a pair of photons. They have been studied to find possible signatures of physics beyond the SM. Since the diphoton decay channel provides a clean final-state topology thus the mass of the decaying object can be reconstructed with high precision. The diphoton decay is mediated by loop diagrams containing charged particles. The decay amplitude gets its highest contribution from the top quark loop and the W boson loop diagrams. They contribute with opposite sign. Figure 4.6 shows the Feynman diagrams for $h \to \gamma\gamma$ and $h \to Z\gamma$ decays in the SM. Gluon-gluon fusion (ggh) [29] is the primary production mechanism of the Higgs boson at the LHC with additional smaller contributions from vector boson fusion (VBF) [30] and production in association with a W or Z boson (Vh) [31] or a $t\bar{t}$ pair (tth) [32, 33].

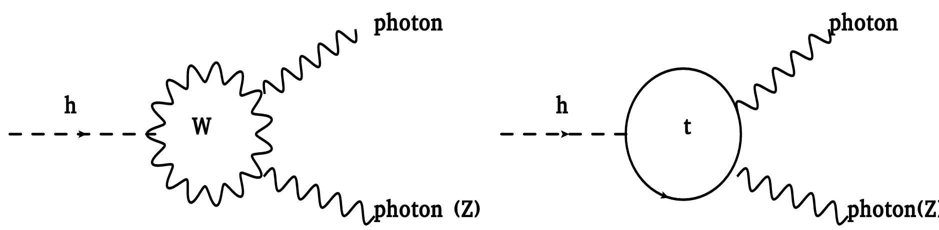


Figure 4.6: Feynman diagram for $h \to \gamma\gamma$ and $h \to Z\gamma$ decays in the Standard model where the loop particles are the $W^{\pm}$ gauge bosons and the top quark.

The LHC is sensitive to the Higgs-diphoton decay states, for masses in the range 114 - 130 GeV. Both LHC and Tevatron experiments observed an excess of events in this channel, consistent with the production of a Higgs boson with a mass of about 125 GeV, with a local significance which is close to 3σ [34–36]. An excess in the hZZ decay channel was observed at the ATLAS experiment in this mass range [37]. Even the CMS provides a similar but less significant result in this line [38]. These two search channels are highly efficient in probing the presence of a Higgs boson in the narrow mass range around 125 GeV. The results of both these experiments give a central value of the rate of production of ZZ similar to the SM one, while the central value of the diphoton production rate appears to be enhanced by 1.5 to 2 times the SM one. The ATLAS result [39] at $\sqrt{s}$= 7 TeV and $\sqrt{s}$= 8 TeV and the CMS data [40] collected at $\sqrt{s}$= 13 TeV in the recent past add to the conclusion that the diphoton decay width shows an excess as compared to the SM prediction. This excess observed has triggered us to study the Higgs-diphoton decay in the light of 2HDMs where there are extra contributions in the loop. It may also turn out that the excess results are just the product of a statistical fluctuation.

Since the diphoton rate is enhanced we can account for this enhancement as an increase in the partial diphoton decay width of the Higgs, but without significantly varying the total width or production cross sections with respect to their SM values. Since the Higgs coupling to photons is induced at the loop-level, such an enhancement of the diphoton decay width may be reasoned with the presence of colorless charged particles with significant couplings to the Higgs boson that will add to the dominant SM contribution from the $W^{\pm}$ boson loop. On the other hand, SM fermions which receive their mass via a Yukawa coupling to the Higgs, give sub-leading corrections which suppress the diphoton partial width. Therefore, a modified diphoton rate is suggestive of the presence of new charged particles and probes physics beyond the SM.

The literature of Higgs decay suggests that a large number of people have studied the effects of new particles in the diphoton decay widths of the Higgs as well as in the gluon fusion production channel [41–75].

4.4.1 Study of Higgs-diphoton decay width in 2HDMs

As already mentioned the decay width can be enhanced or reduced in the 2HDMs due to loop effects. In the alignment limit, the couplings of h with the fermions and gauge bosons will be exactly like in the Standard Model. The production cross-section of h will therefore be as expected in the Standard Model. All the tree level decay widths of h will also have the SM values for the same reason. Loop induced decays like $h \to \gamma\gamma$ and $h \to Z\gamma$ will however have additional contributions from the extra charged scalars ($\xi^{\pm}$) present in the model. Thus the decay widths will be different from the SM in general.

On the other hand, if h has wrong sign Yukawa couplings to the down-type quarks then the bottom quarks will contribute with a relative negative sign in the loops, and the $h \to \gamma\gamma$ decay width will be different from the SM, as well as from 2HDMs in the usual alignment limit.

The Higgs-diphoton decay width is calculated using the formula [76]

$$\Gamma(h \to \gamma\gamma) = \frac{G_\mu \alpha^2 m_h^3}{128\sqrt{2}\pi^3} \left| \sum_f N_c Q_f^2 g_{hff} A_{1/2}^h(\tau_f) + g_{hVV} A_1^h(\tau_W) + \frac{m_W^2 \lambda_{h\xi^+\xi^-}}{2c_W^2 M_{\xi^\pm}^2} A_0^h(\tau_{\xi^\pm}) \right|^2 . \tag{4.58}$$

The trilinear $\lambda_{h\xi^+\xi^-}$ couplings to charged Higgs bosons is given by

$$\lambda_{h\xi^+\xi^-} = \cos 2\beta \sin(\beta+\alpha) + 2c_W^2 \sin(\beta-\alpha) \tag{4.59}$$
$$= \lambda_{hAA} + 2c_W^2 g_{hVV} , \tag{4.60}$$

where $c_W = \cos\theta_W$, with θ_W being the Weinberg angle.

The reduced couplings g_{hff} and g_{hVV} of the Higgs boson to fermions and W bosons are $g_{htt} = \dfrac{\cos\alpha}{\sin\beta}$, $g_{hbb} = -\dfrac{\sin\alpha}{\cos\beta}$ and $g_{hWW} = \sin(\beta-\alpha)$. It is to be noted that the decay rate is independent of the type of the 2HDM.

In the case of the CP even Higgs boson h, the amplitudes A_i at lowest order for the spin 1, spin $\frac{1}{2}$ and spin 0 particle contributions are given by [77]

$$A_{1/2}^h = -2\tau[1 + (1-\tau) f(\tau)] \tag{4.61}$$
$$A_1^h = 2 + 3\tau + 3\tau(2-\tau) f(\tau) \tag{4.62}$$
$$A_0^h = -\tau[1 - \tau f(\tau)] . \tag{4.63}$$

Here

$$\tau_x = 4m_x^2/m_h^2 \tag{4.64}$$

and

$$f(\tau) = \begin{cases} \arcsin^2 \sqrt{1/\tau} , & \tau \geq 1 \\ -\dfrac{1}{4}\left[\log \dfrac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 , & \tau < 1 \end{cases} \tag{4.65}$$

The decay width formula given in Eq. Eq. (4.58) takes a much simplified expression with the use of the above equations,

$$\Gamma(h \to \gamma\gamma) = \frac{G_\mu \alpha^2 m_h^3}{128\sqrt{2}\pi^3} \left| A_W^h + \frac{4}{3} A_t^h \pm \frac{1}{3} A_b^h + \kappa A_\xi^h \right|^2 , \tag{4.66}$$

where the '+' sign before A_b^h is for when the $h\bar{b}b$ Yukawa coupling has the same sign as the hVV coupling and the '−' sign is for the wrong sign of the Yukawa coupling, and κ is defined as

$$\kappa = \frac{1}{m_\xi^2}(m_A^2 - m_\xi^2 - \frac{1}{2} m_h^2) . \tag{4.67}$$

Due to the imposition of $U(1)$ symmetry λ_5 is related to m_A by $\lambda_5 = \frac{2}{v^2} m_A^2$ and thus appears in the definition of κ in Eq. (4.67). For a more general potential the expression for κ involves λ_5.

The relative diphoton decay width is given as,

$$\mu_{\gamma\gamma} = \frac{\sigma\,(pp \to h)}{\sigma^{SM}\,(pp \to h)} . \frac{BR\,(h \to \gamma\gamma)}{BR^{SM}\,(h \to \gamma\gamma)} . \tag{4.68}$$

which takes the following form in the alignment limit where the production cross-section is just like the SM one,

$$\mu_{\gamma\gamma} = \frac{\Gamma\,(h \to \gamma\gamma)}{\Gamma^{SM}\,(h \to \gamma\gamma)} = \frac{\left| F_W + \frac{4}{3} F_t \pm \frac{1}{3} F_b + \kappa F_\xi \right|^2}{\left| F_W + \frac{4}{3} F_t \pm \frac{1}{3} F_b \right|^2} . \tag{4.69}$$

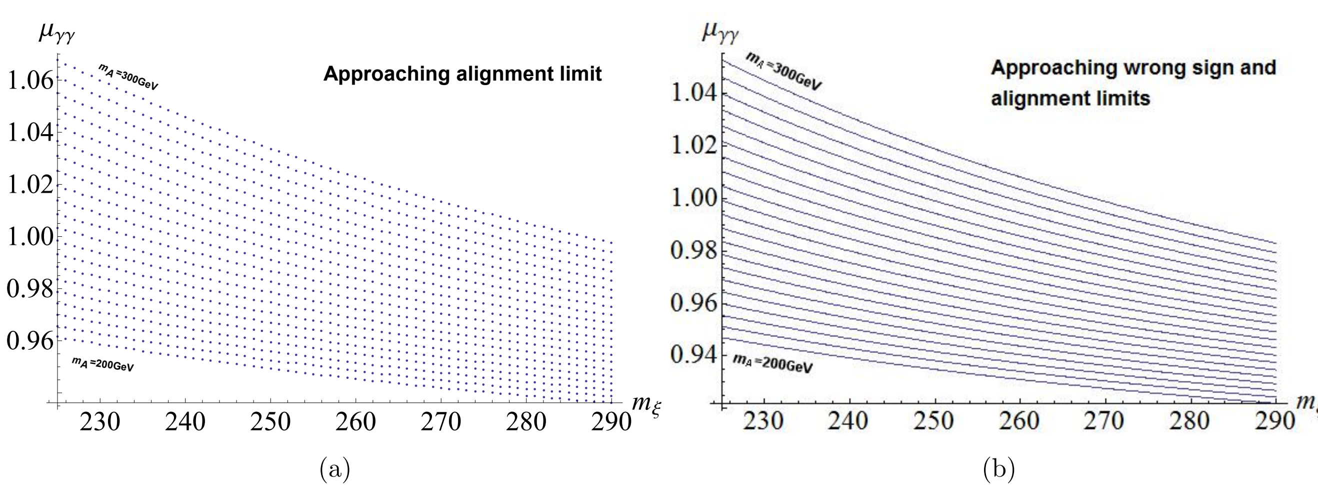


Figure 4.7: Diphoton decay width of the SM-like Higgs particle (normalized to SM) as a function of the charged Higgs mass in GeV at $\tan\beta = 17$, for (a) same sign and (b) wrong sign, of down-type Yukawa couplings.

In figure 4.7 we have plotted the $h \to \gamma\gamma$ decay width in 2HDMs in the alignment limit, normalized with respect to the SM value, against the mass of the charged Higgs particle, and for different values of the mass of the CP-odd scalar. Figure 4.7a shows the decay width for the case where the $h\bar{q}q$ Yukawa coupling has the same sign as the hVV coupling, whereas figure 4.7b is for the decay width corresponding to the case where the Yukawa coupling of h to the down-type quarks is of the opposite sign to the hVV coupling. The first case i.e. the diphoton decay width in the alignment limit has been plotted, although for smaller values of $\tan\beta$ and without the use of the Veltman conditions (and thus for a much larger range of m_ξ), in [14].

As we have seen in Section 4.3.1, $\tan\beta$ takes a higher value when we simultaneously choose the alignment limit and the wrong sign limit. The critical value $\tan\beta = 17$, and a small but non-zero value of α , namely $\alpha \simeq 0.035$, were chosen for both the plots. The plots are not noticeably different for other high values of $\tan\beta$ or other similar values of α . The decay width does not depend on the type of 2HDM once the masses of the charged Higgs particle and the CP-odd Higgs particle are fixed. However, the range of allowed masses depends on the type of 2HDM being considered. We have chosen the ranges 225 GeV$\leq m_\xi \leq$290 GeV and 200 GeV$\leq m_A \leq$ 300 GeV which cover the allowed ranges for all four types for $\tan\beta = 17$. Although it is clear from the plot it is perhaps worth pointing out that when m_A is small, for example $m_A \simeq 200$ GeV, the diphoton decay width deviates from the SM value by 5-7% for all values of m_ξ . The deviation is noticeable for many other values of m_A also, as can be easily seen from the plots. On the other hand, for specific choices of $(m_A\,, m_\xi\,)$ the $h \to \gamma\gamma$ decay width is the same as for the SM, so the non-observation of a deviation does not rule out 2HDMs.

The two plots are similar, but not identical. The decay width when the $h\bar{D}D$ Yukawa coupling is of the 'wrong sign' is smaller than the decay width for the case when it is of the same sign (as hVV couplings) by about 1.5%, as can be seen from the ratio of the decay widths, displayed in figure 4.8. In Chapter D we have jotted down the formulae for the decay width of the $h \to ZZ$ process for the sake of completeness.

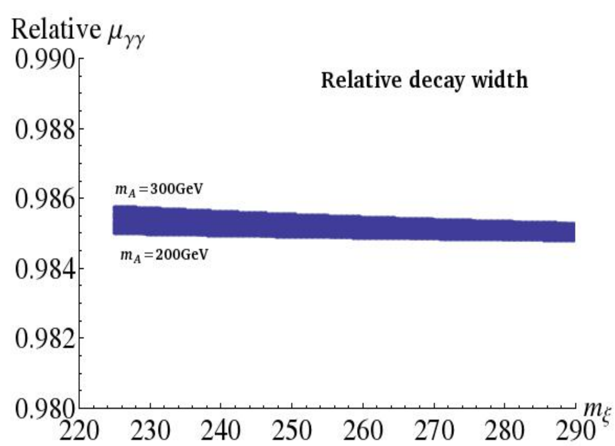


Figure 4.8: $h\gamma\gamma$ decay width for 'wrong sign' $h\bar{D}D$ coupling relative to the case with 'same sign' Yukawa couplings.

References

[1] G. C. Branco, W. Grimus and L. Lavoura, *"Relating the scalar flavor changing neutral couplings to the CKM matrix,"* Phys. Lett. B **380**, 119 (1996)

[2] J. F. Gunion and H. E. Haber, *"The CP conserving two Higgs doublet model: The Approach to the decoupling limit,"* Phys. Rev. D **67**, 075019 (2003)

[3] P. M. Ferreira, J. F. Gunion, H. E. Haber and R. Santos, *"Probing wrong-sign Yukawa couplings at the LHC and a future linear collider,"* Phys. Rev. D **89**, no. 11, 115003 (2014) [arXiv:1403.4736 [hep-ph]].

[4] J. F. Gunion and H. E. Haber, *"The CP conserving two Higgs doublet model: The Approach to the decoupling limit,"* Phys. Rev. D 67 (2003) 075019 doi:10.1103/PhysRevD.67.075019 [hep-ph/0207010].

[5] P. Ferreira, H. E. Haber, and E. Santos, Phys. Rev. D92, 033003 (2015), 1505.04001, [Erratum: Phys. Rev. D94, no.5, 059903 (2016)].

[6] S. Gori, H. E. Haber, and E. Santos, JHEP 06, 110 (2017), 1703.05873.

[7] S. Kanemura, Y. Okada, E. Senaha, and C. P. Yuan, Phys. Rev. D70, 115002 (2004), hep-ph/0408364.

[8] A. G. Akeroyd, A. Arhrib and E. M. Naimi, *"Note on tree level unitarity in the general two Higgs doublet model,"* Phys. Lett. B **490**, 119 (2000)

[9] M. Sher, *"Electroweak Higgs Potentials and Vacuum Stability,"* Phys. Rept. **179**, 273 (1989).

[10] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, *"Theory and phenomenology of two-Higgs-doublet models,"* Phys. Rept. **516**, 1 (2012)

[11] J. Maalampi, J. Sirkka and I. Vilja, *"Tree level unitarity and triviality bounds for two Higgs models,"* Phys. Lett. B **265**, 371 (1991).

[12] S. Kanemura, T. Kubota and E. Takasugi, *"Lee-Quigg-Thacker bounds for Higgs boson masses in a two doublet model,"* Phys. Lett. B **313**, 155 (1993).

[13] J. Horejsi and M. Kladiva, *"Tree-unitarity bounds for THDM Higgs masses revisited,"* Eur. Phys. J. C **46**, 81 (2006).

[14] G. Bhattacharyya, D. Das, P. B. Pal and M. N. Rebelo, *"Scalar sector properties of two-Higgs-doublet models with a global U(1) symmetry,"* JHEP **1310**, 081 (2013).

[15] H. J. He, N. Polonsky and S. f. Su, *"Extra families, Higgs spectrum and oblique corrections,"* Phys. Rev. D **64**, 053004 (2001).

[16] W. Grimus, L. Lavoura, O. M. Ogreid and P. Osland, *"A Precision constraint on multi-Higgs-doublet models,"* J. Phys. G **35**, 075001 (2008).

[17] M. Baak and R. Kogler, *"The global electroweak Standard Model fit after the Higgs discovery"*, arXiv:1306.0571 [hep-ph].

[18] J. Beringer *et al.* [Particle Data Group Collaboration], *Review of Particle Physics (RPP),* Phys. Rev. D **86**, 010001 (2012) and 2013 partial update for the 2014 edition.

[19] B. Coleppa, F. Kling and S. Su, *"Constraining Type II 2HDM in Light of LHC Higgs Searches,"* JHEP **1401**, 161 (2014).

[20] J. R. Espinosa, C. Grojean, M. Mühlleitner and M. Trott, JHEP 1212 (2012) 045 [arXiv:1207.1717 [hep-ph]].

[21] A. Falkowski, F. Riva and A. Urbano, JHEP 1311 (2013) 111 [arXiv:1303.1812 [hep-ph]].

[22] G. Belanger, B. Dumont, U. Ellwanger, J. F. Gunion and S. Kraml, Phys. Rev. D 88, 075008 (2013) [arXiv:1306.2941 [hep-ph]].

[23] A. Celis, V. Ilisie and A. Pich, JHEP 1312, 095 (2013) [arXiv:1310.7941 [hep-ph]].

[24] P. M. Ferreira, R. Guedes, M. O. P. Sampaio and R. Santos, *"Wrong sign and symmetric limits and non-decoupling in 2HDMs,"* JHEP **1412**, 067 (2014).

[25] P. M. Ferreira, R. Guedes, J. F. Gunion, H. E. Haber, M. O. P. Sampaio and R. Santos, *"The Wrong Sign limit in the 2HDM,"* arXiv:1410.1926 [hep-ph].

[26] A. Arhrib, R. Benbrik, C. H. Chen, R. Guedes, and R. Santos, *"Double neutral Higgs production in the two-Higgs doublet model at the LHC"*, J. High Energy Phys. 08 (2009)035.

[27] J. Cao, P. Wan, L. Wu, and J. M. Yang, *"Lepton-specific two-Higgs doublet model: Experimental constraints and implication on Higgs phenomenology"*, Phys. Rev. D 80, 071701 (2009).

[28] J. h. Park, *"Lepton non-universality at LEP and charged Higgs"*, J. High Energy Phys. 10 (2006) 077.

[29] H. M. Georgi, S. L. Glashow, M. E. Machacek, D. V. Nanopoulos, *"Higgs Bosons from two-gluon annihilation in proton–proton collisions",* Phys. Rev. Lett. 40, 692 (1978), doi:10.1103/PhysRevLett.40.692.

[30] R. N. Cahn, S. D. Ellis, R. Kleiss, W. J. Stirling, *"Transverse momentum signatures for heavy Higgs bosons",* Phys. Rev. D 35, 1626 (1987), doi:10.1103/PhysRevD.35.1626.

[31] S. L. Glashow, D. V. Nanopoulos, A. Yildiz, *"Associated production of Higgs bosons and Z particles",* Phys. Rev. D 18, 1724 (1978), doi:10.1103/PhysRevD.18.1724.

[32] R. Raitio, W. W. Wada, *"Higgs-boson production at large transverse momentum in quantum chromodynamics",* Phys. Rev. D 19, 941 (1979), doi:10.1103/PhysRevD.19.941.

[33] Z. Kunszt, *"Associated production of heavy Higgs boson with top quarks",* Nucl. Phys. B 247, 339 (1984), doi:10.1016/0550-3213(84)90553-4.

[34] ATLAS collaboration, G. Aad et al., *"Search for the standard model Higgs boson in the diphoton decay channel with 4.9 fb^{-1} of pp collisions at $\sqrt{s}$ = 7 TeV with ATLAS",* Phys. Rev. Lett. 108 (2012) 111803 [arXiv:1202.1414].

[35] CMS collaboration, S. Chatrchyan et al., *"Search for the standard model Higgs boson decaying into two photons in pp collisions at $\sqrt{s}$=7 TeV",* Phys. Lett. B 710 (2012) 403, [arXiv:1202.1487].

[36] CMS collaboration, *"A search using multivariate techniques for a standard model Higgs boson decaying into two photons",* PAS-HIG-12-001 (2012).

[37] ATLAS collaboration, G. Aad et al., *"Search for the standard model Higgs boson in the decay channel $H \to ZZ^* \to 4l$ with 4.8 fb^{-1} of pp collision data at $\sqrt{s}$ = 7 TeV with ATLAS",* Phys. Lett. B 710 (2012) 383 [arXiv:1202.1415].

[38] CMS collaboration, S. Chatrchyan et al., *"Search for the standard model Higgs boson in the decay channel $H \to ZZ \to$ 4 leptons in pp collisions at $\sqrt{s}$ = 7 TeV",* Phys. Rev. Lett. 108, 111804 (2012) [arXiv:1202.1997].

[39] G. Aad et al. (ATLAS Collaboration), *"Measurement of Higgs boson production in the diphoton decay channel in pp collisions at center-of-mass energies of 7 and 8 TeV with the ATLAS detector",* Phys. Rev. D 90, 112015.

[40] The CMS collaboration, Sirunyan, A. M., Tumasyan, A. et al., *"Measurements of Higgs boson properties in the diphoton decay channel in proton-proton collisions at $\sqrt{s}$=13 TeV",* J. High Energ. Phys. (2018) 2018: 185. https://doi.org/10.1007/JHEP11(2018)185.

[41] F. J. Petriello, *"Kaluza-Klein effects on Higgs physics in universal extra dimensions",* JHEP 05 (2002) 003 [hep-ph/0204067].

[42] T. Han, H. E. Logan, B. McElrath and L. T. Wang, *"Loop induced decays of the little Higgs: $H \to gg, \gamma\gamma$",* Phys. Lett. B 563 (2003) 191 [Erratum ibid. B 603 (2004) 257-259] [hep-ph/0302188].

[43] C. R. Chen, K. Tobe and C. P. Yuan, *"Higgs boson production and decay in little Higgs models with T-parity"*, Phys. Lett. B 640 (2006) 263 [hep-ph/0602211].

[44] A. Djouadi and G. Moreau, *"Higgs production at the LHC in warped extra-dimensional models"*, Phys. Lett. B 660 (2008) 67 [arXiv:0707.3800].

[45] A. Falkowski, *"Pseudo-goldstone Higgs production via gluon fusion"*, Phys. Rev. D 77 (2008) 055018 [arXiv:0711.0828].

[46] N. Maru, *"Finite gluon fusion amplitude in the gauge-Higgs unification"*, Mod. Phys. Lett. A 23 (2008) 2737 [arXiv:0803.0380].

[47] G. Cacciapaglia, A. Deandrea and J. Llodra-Perez, *"Higgs $\to \gamma\gamma$ beyond the standard model"*, JHEP 06 (2009) 054 [arXiv:0901.0927].

[48] R. Boughezal and F. Petriello, *"Color-octet scalar effects on Higgs boson production in gluon fusion"*, Phys. Rev. D 81 (2010) 114033 [arXiv:1003.2046].

[49] S. Casagrande, F. Goertz, U. Haisch, M. Neubert and T. Pfoh, *"The custodial Randall-Sundrum model: from precision tests to Higgs physics"*, JHEP 09 (2010) 014 [arXiv:1005.4315].

[50] A. Azatov, M. Toharia and L. Zhu, *"Higgs production from gluon fusion in warped extra dimensions"*, Phys. Rev. D 82 (2010) 056004 [arXiv:1006.5939].

[51] J. Cao, Z. Heng, T. Liu and J. M. Yang, *"Di-photon Higgs signal at the LHC: a comparative study for different supersymmetric models"*, Phys. Lett. B 703 (2011) 462 [arXiv:1103.0631].

[52] E. Furlan, *"Gluon-fusion Higgs production at NNLO for a non-standard Higgs sector"*, JHEP 10 (2011) 115 [arXiv:1106.4024].

[53] A. Alves et al., *"Probing 3-3-1 models in diphoton Higgs boson decay"*, Phys. Rev. D 84 (2011) 115004 [arXiv:1109.0238].

[54] A. Azatov and J. Galloway, *"Light custodians and Higgs physics in composite models"*, Phys. Rev. D 85 (2012) 055013 [arXiv:1110.5646].

[55] Y. Bai, J. Fan and J. L. Hewett, *"Hiding a heavy Higgs boson at the 7 TeV LHC"*, arXiv:1112.1964.

[56] B. A. Dobrescu, G. D. Kribs and A. Martin, *"Higgs underproduction at the LHC"*, Phys. Rev. D 85 (2012) 074031 [arXiv:1112.2208].

[57] A. Arvanitaki and G. Villadoro, *"A non standard model Higgs at the LHC as a sign of naturalness"*, JHEP 02 (2012) 144 [arXiv:1112.4835].

[58] S. Kanemura and K. Yagyu, *"Radiative corrections to electroweak parameters in the Higgs triplet model and implication with the recent Higgs boson searches"*, Phys. Rev. D 85 (2012) 115009 [arXiv:1201.6287].

[59] N. D. Christensen, T. Han and S. Su, *"MSSM Higgs bosons at The LHC"*, arXiv:1203.3207.

[60] V. Barger, M. Ishida and W. Y. Keung, *"Total width of 125 GeV Higgs boson"*, Phys. Rev. Lett. 108 (2012) 261801 [arXiv:1203.3456].

[61] M. Carena, S. Casagrande, F. Goertz, U. Haisch and M. Neubert, *"Higgs production in a warped extra dimension"*, arXiv:1204.0008.

[62] P. Draper and D. McKeen, *"Diphotons from Tetraphotons in the decay of a 125 GeV Higgs at the LHC"*, Phys. Rev. D 85 (2012) 115023 [arXiv:1204.1061].

[63] K. Kumar, R. Vega-Morales and F. Yu, *"Effects from new colored states and the Higgs portal on gluon fusion and Higgs decays"*, arXiv:1205.4244.

[64] S. Dawson and E. Furlan, *"A Higgs conundrum with vector fermions"*, arXiv:1205.4733.

[65] A. Arhrib, R. Benbrik and C. H. Chen, *"$H \to \gamma\gamma$ in the complex two Higgs doublet model"*, arXiv:1205.5536.

[66] A. Akeroyd and S. Moretti, *"Enhancement of $H \to \gamma\gamma$ from doubly charged scalars in the Higgs triplet model"*, Phys. Rev.D**86**, 035015, arXiv:1206.0535.

[67] A. Djouadi, *"Squark effects on Higgs boson production and decay at the LHC"*, Phys. Lett. B 435 (1998) 101 [hep-ph/9806315].

[68] R. Dermisek and I. Low, *"Probing the stop sector and the sanity of the MSSM with the Higgs boson at the LHC"*, Phys. Rev. D 77 (2008) 035012 [hep-ph/0701235].

[69] I. Low and S. Shalgar, *"Implications of the Higgs discovery in the MSSM golden region"*, JHEP 04 (2009) 091 [arXiv:0901.0266].

[70] I. Low, R. Rattazzi and A. Vichi, *"Theoretical constraints on the higgs effective couplings"*, JHEP 04 (2010) 126 [arXiv:0907.5413].

[71] I. Low and A. Vichi, *"On the production of a composite Higgs boson"*, Phys. Rev. D 84 (2011) 045019 [arXiv:1010.2753].

[72] M. Carena, S. Gori, N.R. Shah and C.E. Wagner, *"A 125 GeV SM-like Higgs in the MSSM and the $\gamma\gamma$ rate"*, JHEP 03 (2012) 014 [arXiv:1112.3336].

[73] B. Batell, S. Gori and L. T. Wang, *"Exploring the Higgs portal with 10/fb at the LHC"*, JHEP 06 (2012) 172 [arXiv:1112.5180].

[74] M. Carena, S. Gori, N. R. Shah, C. E. Wagner and L. T. Wang, *"Light stau phenomenology and the Higgs $\gamma\gamma$ rate"*, JHEP 07 (2012) 175 [arXiv:1205.5842].

[75] M. Carena, I. Low and C. E. M. Wagner, *"Implications of a modified Higgs to diphoton decay width"*, JHEP 08 (2012) 060.

[76] A. Djouadi, *"The Anatomy of electro-weak symmetry breaking. II. The Higgs bosons in the minimal supersymmetric model,"* Phys. Rept. **459**, 1 (2008) [hep-ph/0503173].

[77] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *"The Higgs Hunter's Guide,"* Front. Phys. **80**, 1 (2000).

Chapter 5

Is *Higgsium* a possibility?

Probable impossibilities are to be
preferred to improbable possibilities.

Aristotle

For long physicists have been bothered with the hierarchy problem [1–7]. Unlike the quarks, leptons and the intermediate weak gauge bosons any symmetry does not protect the mass of the Higgs boson from radiative corrections. The radiative corrections to the mass of the Higgs boson are proportional to the mass squared and the new physics scale might destroy the electroweak scale of the Standard model. The existing mass hierarchy $M_W/M_{GUT} \sim 10^{-14}$ thus breaks leading to the hierarchy problem. A lot of work has been done on this hierarchy problem where physicists have explored the physics at higher dimensions and have gone beyond the Standard Model to find an explanation for the hierarchy problem. Hierarchy problem involves the quadratic dependence of the low energy physics on the unknown high energy physics. A new physics at an intermediate scale might be a solution to this problem.

Triviality problem [1, 4, 8–10] is yet another problem that the SM faces. It is supported by strong evidence that a field theory involving only a scalar Higgs boson is trivial in four space-time dimensions. But the situation for realistic models including other particles in addition to the Higgs boson is not known in general. Since the Higgs boson plays a central role in the Standard Model, the question of triviality in Higgs models is of great importance. Keeping these two problems in mind we have gone beyond the standard model and introduced a second Higgs doublet and also have introduced a new physics at a higher scale ($\mathcal{M} \sim$1 TeV).

We have considered 2HDMs with a softly broken global U(1) symmetry [11–17], with the parameters so chosen as to make the 2HDM "SM-like." An approximate custodial $SU(2)_C$ symmetry [18–20] has also been imposed on the total Lagrangian density. The new higher dimensional operators are made to respect this $SU(2)_C$ symmetry up to hypercharge and Yukawa coupling violations. There will be operators that break the custodial symmetry but their coefficients are taken to be naturally small.

2HDM and the new physics scale modify the relation $M_W = M_Z \cos\theta_W$, which is parametrized by the ρ parameter. ρ parameter is defined as $\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W}$. The PDG quotes $\rho_0 = 1.00039 \pm 0.00019$ for the global fit [21] of precision electro-weak observables. Thus new physics must be integrated out to naturally preserve $\rho_0 \approx 1$. In order to integrate out the quanta of the new physics the scale of the masses of the new quanta, $\mathcal{M}$, is chosen to be sufficiently higher than the scale of electro-weak symmetry breaking ($v \sim 246$ GeV). Thus for phenomenological reasons, we set the new physics scale at $\mathcal{M} \sim 1$TeV. In the low energy regime, the unknown new physics at high energy manifests itself as non-renormalizable local operators, of dimension $D > 4$ in addition to the 2HDM operators. These local operators are constructed of 2HDM fields invariant under the gauge symmetry of the 2HDM. Though this approach is model independent but it is messy in the sense that the new physics is parametrized in terms of several arbitrary parameters, the coefficients of higher dimension operators, and nothing is known a priori about these coefficients.

A gross approximate of $\mathcal{M}$ can be obtained from low energy experiments. $K^0 - \bar{K}_0$ experiment puts flavor changing neutral current bounds on $\mathcal{M}$ and sets it to be $\geq 10^4$ TeV. The Minimal Flavour Violation hypothesis [22–31] relaxes the bound on $\mathcal{M}$. Thus $\mathcal{M}$ can be set to a few TeV while naturally avoiding flavor changing neutral currents if Minimal Flavour Violation hypothesis is adopted.

Bound states of particles have drawn the attention of many physicists. Formation of bound states of Higgs bosons has been discussed in the literature for quite some time. Much before the discovery of the Higgs boson at the LHC, the formation of two Higgs bound state in the Higgs model, or equivalently in the Higgs sector of the minimal Standard Model, has been investigated using different methods. The results obtained were interesting, but failed to be consistent with the Standard Model Higgs boson when it was actually discovered. We thus attempt to study the bound state formation of the heavier CP even neutral Higgs boson present in the model. The repulsive interaction of the quartic coupling and the attractive interaction determined by the cubic coupling compete to form the bound state. For large enough coupling the exchange interaction is strong enough to produce binding. Another approach to determine if a bound state is formed or not is from the study of the formation and decay times of the bound state.

We have looked for a necessary condition on the coupling for which a non-relativistic (NR) bound state may form and for this we have followed the method of [32] which proposes to study the formation of the bound state in a non-relativistic effective theory for Higgs-Higgs interactions. We have referred to the Higgs-Higgs bound state as *'Higgsium'*, a name that has been borrowed from the same paper [32].

Previously various physicists have adopted different methods to look for bound states of the Higgs particle of the SM. In the N/D method [33, 34] which was used to calculate the bound state of the Higgs boson in the Standard Model, the elastic scattering amplitude is written as $N(s)/D(s)$ where $N(s)$ has only left hand cuts and $D(s)$ has only right hand singularities. $N(s)$ is approximated by the Born amplitude which is the appropriate s-wave projection of the sum of the four point scattering amplitudes of the particle in picture which forms the bound state. Bound states for s-wave occur when $D(s) = s$, for $0 < s < 4m_H^2$. The N/D method had been studied to account for the bound state of two particles (not necessarily the Higgs particle) in [35, 36]. It was found that for the Standard Model Higgs particle, bound states occur only if $m_H > 1.3$ TeV. Since this is an order of magnitude higher than the observed mass of the Higgs particle, we have to conclude that the Higgs does not form a bound state with itself. Furthermore, even for such heavy Higgs bosons the binding was weak. For example at $m_H = 2$ TeV, the binding energy was found to be 150 GeV.

The study of relativistic two-particle bound states in SM involves another method, the variational method within the Hamiltonian formalism of quantum field theory [37–39]. This method can be extended to accommodate three-particle systems [40–42]. In principle, the variational method does not depend on the coupling strength, which is in contrast with the perturbation theory. The perturbation theory becomes increasingly doubtful as the coupling becomes stronger. This fact is relevant and the perturbation theory becomes questionable since the Higgs self-coupling approaches a strong regime as the Higgs boson mass becomes large.

When the Higgs boson mass is approximately 700 GeV [1, 43] the theory behaves like a strongly coupled one and the perturation theory ceases to act. Gunion and others treated the possibility of heavier Higgs to be discovered at the LHC and hence applied the variational method rather than the perturbative method.

Leo and Darewych in their work [37–39] found that two-Higgs bound states which they called “Higgsonium” would appear only for rather obese minimal Standard Model Higgs particles with mass $m_H > 894$ GeV. This was quite similar to the 810 GeV estimate [44] which was obtained by using a phenomenological Yukawa potential to describe the Higgs-Higgs interaction.

Bethe and Salpeter approached the bound state problem in a different way. We briefly mention it here for the sake of completeness. The original paper of Bethe and Salpeter [45] dealt with the bound-state problem for two interacting Fermi-Dirac particles where they applied the relativistic S-matrix formalism of Feynman. The bound state was described by a wave function depending on separate times for each of the two particles forming the bound state. Integral equations for this wave function were derived with kernels in the form of an expansion in powers of g^2, the dimensionless coupling constant for the interaction. Each term in these expansions gave Lorentz-invariant equations. The validity and physical significance of these equations were discussed. In extreme non-relativistic approximation and to lowest order in g^2 they reduced to the appropriate Schrödinger equation.

Grienstein [32] too attempted to find conditions for the bound state of the SM Higgs particle and found that this state was not likely to form for the light Higgs particle.

5.1 Formalising 2HDM with the new physics scale

The Lagrangian density of the two Higgs doublet model containing two Higgs doublets ϕ_1 and ϕ_2 of hypercharge $\frac{1}{2}$ is given by

$$\mathcal{L}^4_{\phi_{1,2}} = (D^\mu\phi_1)^\dagger(D_\mu\phi_1) + (D^\mu\phi_2)^\dagger(D_\mu\phi_2) - V(\phi_{1,2}) + \text{h.c.}\,, \tag{5.1}$$

where the covariant derivative is

$$D_\mu = \partial_\mu - ig_1 B_\mu - ig_2\frac{\sigma^I}{2}W^I_\mu\,. \tag{5.2}$$

σ^I are the Pauli matrices and W^I_μ and B_μ are SU(2) and U(1) gauge bosons operators. The scalar potential [1,46] as introduced in Eq. (1.6) under $U(1)$ symmetry is,

$$\begin{aligned} V(\phi_{1,2}) \;=\; & \lambda_1\left(|\phi_1|^2 - \frac{v_1^2}{2}\right)^2 + \lambda_2\left(|\phi_2|^2 - \frac{v_2^2}{2}\right)^2 \\ & +\lambda_3\left(|\phi_1|^2 + |\phi_2|^2 - \frac{v_1^2+v_2^2}{2}\right)^2 \\ & +\lambda_4\left(|\phi_1|^2|\phi_2|^2 - |\phi_1^\dagger\phi_2|^2\right) \\ & +\lambda_5\left|\phi_1^\dagger\phi_2 - \frac{v_1v_2}{2}\right|^2\,, \end{aligned} \tag{5.3}$$

where the λ_i are considered to be real parameters. As already mentioned except for a soft breaking term $\lambda_5 v_1 v_2 \Re(\phi_1^\dagger\phi_2)$ this potential is invariant under the $U(1)$ symmetry to avoid flavor-changing neutral currents (FCNCs) [47,48]. Additional dimension four terms, including one allowed by a softly broken Z_2 symmetry [49] are also set to zero by this U(1) symmetry. One such term was $\lambda_6(\frac{1}{2i}(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1))^2$.

We work with the assumption that there is new physics at 1 TeV, imposed on the 2HDM, which together may provide a solution to the problems of hierarchy and triviality. The new physics demonstrates itself as non-renormalizable local operators of dimension $D > 4$ added to the 2HDM lagrangian density in the resulting low energy effective theory. These $D > 4$ operators are constructed out of 2HDM fields invariant under the $SU(3)\times SU(2)\times U(1)$ gauge symmetry. The effective Lagrangian density of this extended 2HDM can be written as

$$\mathcal{L}_{\phi_{1,2}} = \mathcal{L}^4_{\phi_{1,2}} + \frac{\mathcal{L}^6_{\phi_{1,2}}}{\mathcal{M}^2} + \mathcal{O}\left(\frac{v_1^4}{\mathcal{M}^4}\right) + \mathcal{O}\left(\frac{v_2^4}{\mathcal{M}^4}\right)\,, \tag{5.4}$$

where the dimension six operators that preserve the symmetries of the 2HDM (here U(1) symmetry) and custodial $SU(2)_C$ [50] in the Higgs sector are given by

$$\begin{aligned} \mathcal{L}^6_{\phi_{1,2}} \;\to\; \mathcal{L}^6_C = & \; C^1_{\phi_1}\partial^\mu(\phi_1^\dagger\phi_1)\partial_\mu(\phi_1^\dagger\phi_1) + C^2_{\phi_1}(\phi_1^\dagger\phi_1)(D_\mu\phi_1)^\dagger(D^\mu\phi_1) - \frac{\lambda_7}{3!}(\phi_1^\dagger\phi_1)^3 \\ & +C^3_{\phi_2}\partial^\mu(\phi_2^\dagger\phi_2)\partial_\mu(\phi_2^\dagger\phi_2) + C^4_{\phi_2}(\phi_2^\dagger\phi_2)(D_\mu\phi_2)^\dagger(D^\mu\phi_2) - \frac{\lambda_8}{3!}(\phi_2^\dagger\phi_2)^3 \\ & +C^5_{\phi_{1,2}}\partial^\mu(\phi_1^\dagger\phi_2)^\dagger\partial_\mu(\phi_1^\dagger\phi_2) - \frac{1}{2}\phi_1^\dagger\phi_2\phi_2^\dagger\phi_1(\lambda_9\phi_1^\dagger\phi_1 + \lambda_9'\phi_2^\dagger\phi_2)\,. \end{aligned} \tag{5.5}$$

In Chapter E we discuss how the fields transform under custodial symmetry.

The scalar fields are now expanded about their vacuum expectation values v_1 and v_2,

$$\phi_1(x) = \frac{U_1(x)}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 + h(x) \end{pmatrix}, \tag{5.6}$$

$$\phi_2(x) = \frac{U_2(x)}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 + H(x) \end{pmatrix}. \tag{5.7}$$

Here h and H are the CP-even Higgs fields, with $\langle h(x)\rangle = 0$ and $\langle H(x)\rangle = 0$, and $U_i(x) = e^{i\xi_i^a(x)\sigma_a/v_i}$, $i = 1,2$. The six fields ξ_i^a include the three Goldstone bosons which get eaten by the $W^\pm$ and Z bosons to make them massive, while the other three combine to become the charged Higgs bosons and the CP-odd Higgs boson. The vevs v_1 and v_2, and therefore $v = \sqrt{v_1^2 + v_2^2}$, will be taken to be small compared to $\mathcal{M}$, the scale of new physics. In the unitarity gauge, the gauge transformation has been used to remove the Goldstone bosons from the Lagrangian. This simplifies our task. The charged scalars and the CP-odd scalar will still remain in the Lagrangian, but we can neglect their contribution for the bound state calculations.

In order to normalize the kinetic term to have a coefficient of $\frac{1}{2}$, we redefine the fields as

$$h \to \frac{h'}{(1+2C_h^K)^{1/2}} \tag{5.8}$$

$$H \to \frac{H'}{(1+2C_H^K)^{1/2}}, \tag{5.9}$$

where

$$C_h^K = (v_1^2/\mathcal{M}^2)(C_{\phi_1}^1 + \frac{1}{4}C_{\phi_1}^2), \tag{5.10}$$

$$C_H^K = (v_2^2/\mathcal{M}^2)(C_{\phi_2}^3 + \frac{1}{4}C_{\phi_2}^4). \tag{5.11}$$

The potential is written in terms of the rescaled fields where the self couplings of the CP neutral Higgs fields are clearly focussed. We call this potential V_{eff}. Though we will be discussing the possibility of bound state formation of the heavy CP-even Higgs field, but still for the sake of completeness we will write the self couplings of the light CP-even Higgs field too. In terms of the rescaled fields, the terms in the effective potential which are of interest to us can be written as

$$\begin{aligned} V_{\mathit{eff}}(h',H') \supset\ & \frac{1}{2}m_h^2 h'^2 + \frac{1}{2}m_H^2 H'^2 + v_1\frac{\lambda_{10}^{\mathit{eff}}}{3!}h'^3 + \frac{\lambda_{11}^{\mathit{eff}}}{4!}h'^4 + +v_2\frac{\lambda_{12}^{\mathit{eff}}}{3!}H'^3 + \frac{\lambda_{13}^{\mathit{eff}}}{4!}H'^4 \\ & + \frac{\lambda_{14}^{\mathit{eff}}}{2!}v_2 h'h'H' + \frac{\lambda_{15}^{\mathit{eff}}}{2!}v_1 H'H'h' + \frac{\lambda_{16}^{\mathit{eff}}}{2!2!}h'h'H'H'. \end{aligned} \tag{5.12}$$

The mass terms and the coupling constants are related to the original λ_i. Since we are interested in the bound state formation of H therefore we write down the cubic and quartic self couplings and also the mass of H in terms of the original λ_i and evaluate their relative strengths.

$$
\begin{aligned}
m_H^2 &= (1-2C_H^K)(2v_2^2(\lambda_2+\lambda_3)+\frac{\lambda_5}{2}v_1^2)+\frac{5\lambda_8}{8}\frac{v_2^4}{\mathcal{M}^2}+\frac{\lambda_9}{8}\frac{v_1^4}{\mathcal{M}^2}+\frac{3\lambda_9'}{4}\frac{v_1^2v_2^2}{\mathcal{M}^2} \\
&\quad +\mathcal{O}(\frac{v^4}{\mathcal{M}^4}) && (5.13) \\
\lambda_{12}^{eff} &= 6(\lambda_2+\lambda_3)(1-3C_H^K)+\frac{5\lambda_8}{2}\frac{v_2^2}{\mathcal{M}^2}+\frac{3\lambda_9'}{2}\frac{v_1^2}{\mathcal{M}^2}+\mathcal{O}(\frac{v^4}{\mathcal{M}^4}) && (5.14) \\
\lambda_{13}^{eff} &= 6(\lambda_2+\lambda_3)(1-4C_H^K)+\frac{15\lambda_8}{2}\frac{v_2^2}{\mathcal{M}^2}+\frac{3\lambda_9'}{2}\frac{v_1^2}{\mathcal{M}^2}+\mathcal{O}(\frac{v^4}{\mathcal{M}^4}) && (5.15) \\
\lambda_{14}^{eff} &= (2\lambda_3+\lambda_5)(1-2C_h^K)(1-C_H^K)+\frac{3\lambda_9}{2}\frac{v_1^2}{\mathcal{M}^2}+\frac{\lambda_9'}{2}\frac{v_2^2}{\mathcal{M}^2}+\mathcal{O}(\frac{v^4}{\mathcal{M}^4}) && (5.16) \\
\lambda_{15}^{eff} &= (2\lambda_3+\lambda_5)(1-2C_H^K)(1-C_h^K)+\frac{\lambda_9}{2}\frac{v_1^2}{\mathcal{M}^2}+\frac{3\lambda_9'}{2}\frac{v_2^2}{\mathcal{M}^2}+\mathcal{O}(\frac{v^4}{\mathcal{M}^4}) && (5.17) \\
\lambda_{16}^{eff} &= (2\lambda_3+\lambda_5)(1-2C_h^K)(1-2C_H^K)+\frac{3\lambda_9}{2}\frac{v_1^2}{\mathcal{M}^2}+\frac{3\lambda_9'}{2}\frac{v_2^2}{\mathcal{M}^2}+\mathcal{O}(\frac{v^4}{\mathcal{M}^4}) && (5.18)
\end{aligned}
$$

In passing we comment that the $h-H$ bound state formation involves much complicated calculations and so we leave that for another occasion.

We should mention here that for a single Higgs particle, the effective field theory may also be written as a nonlinear realization analogous to the σ and π fields of QCD as described by a chiral Lagrangian [32]. It is not obvious to us how to write a non-linear realization involving neutral and charged Higgs fields along with the necessary Goldstone bosons, nor is it clear whether that will help in looking for bound states. So we will stick to the linear realization.

5.2 Effective couplings

In order to discuss the low energy effective 2HDM theory the momentum modes heavier than the Higgs needs to be integrated out. As the top Yukawa coupling is fairly large due to reasonably large top mass, we need to estimate the effects of the top quark on the possibility of a bound state of H. When the top quark mass is much heavier than the Higgs mass, it is integrated out which in turn modifies coupling constants and masses of the Higgs particles.

We will use this approximation even when the Higgs is slightly heavier than the top. As mentioned in [32], for the case of a single Higgs field, the approximation is known to work better than one would expect when $m_h < 2m_t$. This is because there is no non-analytic dependence on the mass, the Higgs being the pseudo-goldstone boson of spontaneously broken scale invariance [51–53].

However, when the mass of the Higgs particle is more than $2m_t$, we cannot integrate out the top quark. We will work in the *Alignment limit* and thus we must set

m_h = 125 GeV. The remaining heavier CP-even Higgs can have any mass above 125 GeV restricted by constraints coming from perturbative unitarity and stability. We have further restricted its mass by the use of Naturalness conditions, and the bounds were found to be 450 GeV $\leq m_H \leq$ 620 GeV for $\tan\beta$ =5. It is worth mentioning here that though these limits on m_H are for type - II 2HDM but the other types of 2HDMs also exhibit the mass ranges for H in the close vicinity of these limits. Moreover when these mass ranges were evaluated the then recent value of ρ - parameter was used [54]. We will usually work with these limits, but also consider the possibility that the heavier Higgs has a mass smaller than $2m_t$. We will not consider the situation where the heavier CP-even Higgs is identified with the Standard Model Higgs (Reverse Alignment limit), for reasons discussed in Section 4.2.

Parameter choice has been crucial in any calculation and we are not exempt from that. We choose our parameters accordingly. $\tan\beta$ being a crucial parameter in 2HDMs is set to a reasonable value of 5 since it has to be larger than unity as evident from constraints coming from $Z \to b\bar{b}$, $B_q\bar{B}_q$ mixing [55], muon $g-2$ in lepton specific 2HDM [56] or using $b \to s\gamma$ in type I and flipped models [57]. We further choose v = 246 GeV, m_t = 174 GeV and the new physics scale $\mathcal{M}$ to be 1TeV. We broadly categorize the heavier Higgs boson mass as $m_H < 2m_t$ and $m_H > 2m_t$. For $m_H < 2m_t$ the top quark is integrated out while for $m_H > 2m_t$, the effect of top quark is retained in the theory.

Integrating out the top quark: $m_H \lesssim 2m_t$

The top mass term and couplings to the Higgs bosons are given by

$$\mathcal{L}_Y = -\frac{m_t}{v}\xi^t_H \bar{t}tH\,. \tag{5.19}$$

where ξ stands for the Yukawa coupling of H with the fermion indicated in the superscript. The values of ξ for up-type and down-type quarks are displayed in table 4.1.

Feynman graphs that contribute to modifications of the Higgs self-couplings are shown in figure 5.1. The solid line denotes a top quark, the external dashed lines denote the heavy CP-even neutral Higgs boson.

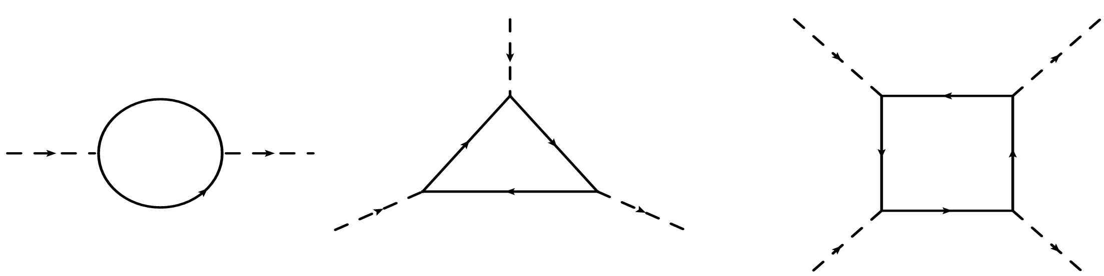


Figure 5.1: t-quark loop corrections to Higgs self couplings.

Calculations are performed up to the lowest order in p^2/m_t^2. These corrections further modify the effective potential of the Higgs scalar field H. Let us take for example, the 1-loop correction to the four point function of H which requires the four point amplitude with the top quark circulating in the loop. This four point amplitude has the form,

$$i\mathcal{A}_4(s,t,u) = -6N_c\left(\frac{m_t}{v}\xi_H^t\right)^4\int\frac{d^dk}{(2\pi)^d}\,\mathrm{Tr}\left[\frac{(\not{k}+m_t)(\not{k}+\not{a}+m_t)(\not{k}+\not{b}+m_t)(\not{k}+\not{c}+m_t)}{(k^2-m_t^2)((k+a)^2-m_t^2)((k+b)^2-m_t^2)((k+c)^2-m_t^2)}\right], \tag{5.20}$$

where a, b and c are the invariants of the external momenta and k is the internal momentum.

For leading order in $p^2/m_t^2 \to 0$, the amplitude is given by

$$\begin{aligned} i\mathcal{A}_4^0(s,t,u) &= -24N_c\left(\frac{m_t}{v}\xi_H^t\right)^4\int\frac{d^dk}{(2\pi)^d}\frac{(m_t^4+6k^2m_t^2+k^4)}{(k^2-m_t^2)^4} \\ &= -\frac{iN_c}{16\pi^2}\left(\frac{m_t}{v}\xi_H^t\right)^4\left(\frac{24}{\epsilon}-64+24\log\left(\frac{\mu^2}{m_t^2}\right)\right). \end{aligned} \tag{5.21}$$

The leading order term gives a factor of $-\frac{4N_c}{\pi^2}\left(\frac{m_t}{v}\xi_H^t\right)^4$. Similar calculations have been done for the other effective couplings and mass terms. Thus the expressions for the effective couplings and mass terms given by Eq. (5.13) - Eq. (5.18) get modified as

$$\begin{aligned} m_H^2 &= (1-2C_H^K)(2v_2^2(\lambda_2+\lambda_3)+\frac{\lambda_5}{2}v_1^2)+\frac{5\lambda_8}{8}\frac{v_2^4}{\mathcal{M}^2}+\frac{\lambda_9}{8}\frac{v_1^4}{\mathcal{M}^2}+\frac{3\lambda_9'}{4}\frac{v_1^2v_2^2}{\mathcal{M}^2}+\frac{N_c}{4\pi^2}\frac{m_t^4}{v^2}(\xi_H^t)^2 \\ &\quad +\mathcal{O}(\frac{v^4}{\mathcal{M}^4}) \end{aligned} \tag{5.22}$$

$$\lambda_{12}^{eff} = 6(\lambda_2+\lambda_3)(1-3C_H^K)+\frac{5\lambda_8}{2}\frac{v_2^2}{\mathcal{M}^2}+\frac{3\lambda_9'}{2}\frac{v_1^2}{\mathcal{M}^2}-\frac{N_c}{\pi^2}\frac{m_t^4}{v^4}(\xi_H^t)^3+\mathcal{O}(\frac{v^4}{\mathcal{M}^4}) \tag{5.23}$$

$$\lambda_{13}^{eff} = 6(\lambda_2+\lambda_3)(1-4C_H^K)+\frac{15\lambda_8}{2}\frac{v_2^2}{\mathcal{M}^2}+\frac{3\lambda_9'}{2}\frac{v_1^2}{\mathcal{M}^2}-\frac{4N_c}{\pi^2}\frac{m_t^4}{v^4}(\xi_H^t)^4+\mathcal{O}(\frac{v^4}{\mathcal{M}^4}) \tag{5.24}$$

$$\begin{aligned} \lambda_{14}^{eff} &= (2\lambda_3+\lambda_5)(1-2C_h^K)(1-C_H^K)+\frac{3\lambda_9}{2}\frac{v_1^2}{\mathcal{M}^2}+\frac{\lambda_9'}{2}\frac{v_2^2}{\mathcal{M}^2}-\frac{N_c}{6\pi^2}\frac{m_t^4}{v^4}(\xi_h^t)^2(\xi_H^t) \\ &\quad +\mathcal{O}(\frac{v^4}{\mathcal{M}^4}) \end{aligned} \tag{5.25}$$

$$\begin{aligned} \lambda_{15}^{eff} &= (2\lambda_3+\lambda_5)(1-2C_H^K)(1-C_h^K)+\frac{\lambda_9}{2}\frac{v_1^2}{\mathcal{M}^2}+\frac{3\lambda_9'}{2}\frac{v_2^2}{\mathcal{M}^2}-\frac{N_c}{6\pi^2}\frac{m_t^4}{v^4}(\xi_h^t)(\xi_H^t)^2 \\ &\quad +\mathcal{O}(\frac{v^4}{\mathcal{M}^4}) \end{aligned} \tag{5.26}$$

$$\begin{aligned} \lambda_{16}^{eff} &= (2\lambda_3+\lambda_5)(1-2C_h^K)(1-2C_H^K)+\frac{3\lambda_9}{2}\frac{v_1^2}{\mathcal{M}^2}+\frac{3\lambda_9'}{2}\frac{v_2^2}{\mathcal{M}^2}-\frac{4N_c}{24\pi^2}\frac{m_t^4}{v^4}(\xi_h^t)^2(\xi_H^t)^2 \\ &\quad +\mathcal{O}(\frac{v^4}{\mathcal{M}^4}) \end{aligned} \tag{5.27}$$

where N_c stands for the three colors of the top quark. Contributions of other quarks as the loop particle have been ignored here because of the very large difference in the masses of the top quark and the other quarks. These are the effective low energy couplings which have been obtained by integrating out the heavier momentum modes.

5.3 Will bound states be formed?

Will H-H bound states be formed at all? This answer to this vital question is what we have tried to look for. We adopt two approaches. One from the consideration of the relative strengths of the cubic and quartic couplings of the Higgs boson and other from the production and decay times.

5.3.1 Relative strengths of couplings

In lieu of this we consider the non-relativistic Schrödinger equation,

$$[-\nabla_r^2 + V(r) - E]\psi(r) = 0\,. \tag{5.28}$$

The above potential has contributions from a Yukawa exchange and a contact interaction,

$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r} + \kappa\delta^3(r)\,, \tag{5.29}$$

where, g denotes the Yukawa exchange coupling constant and κ denotes the contact interaction coupling constant for the two CP even neutral Higgs bosons. As a non-relativistic approximation of the Higgs bosons self interactions $g \sim 3$ point coupling and $\kappa \sim 4$ point coupling. Thus from the potential in Eq. (5.29) we can conclude that the attractive contact interaction and the repulsive contact interaction are governed by the cubic and quartic couplings respectively. If the attractive interaction overpowers the repulsive interaction, the formation of a bound state becomes feasible. Let us treat the possibility of bound state formation by evaluating the relative strengths of the cubic and quartic self couplings.

Due to the D = 6 operators the three and four point contact interactions and the Higgs mass m_H gain corrections in the effective potential. Eliminating the self-couplings λ_1 , λ_2 and λ_3 in favour of the Higgs mass m_H , we can write for the effective cubic and quartic Higgs-self couplings,

$$\begin{aligned}
\lambda_{12}^{eff} &= 3(1 - C_H^K)\frac{m_H^2}{v_2^2} - \frac{3\lambda_5}{2}\frac{v_1^2}{v_2^2}(1 - 3C_H^K) + \frac{5\lambda_8}{8}\frac{v_2^2}{\mathcal{M}^2} - \frac{3\lambda_9}{8}\frac{v_1^4}{\mathcal{M}^2 v_2^2}(1 - C_H^K) \\
&\quad - \frac{3\lambda_9'}{4}\frac{v_1^2}{\mathcal{M}^2} - (7 - 3C_H^K)\frac{N_c}{4\pi^2}\left(\frac{m_t^4}{v^4}\right)\left(\xi_H^t\right)^3\,, \qquad (5.30)\\
\lambda_{13}^{eff} &= 3(1 - 2C_H^K)\frac{m_H^2}{v_2^2} - \frac{3\lambda_5}{2}\frac{v_1^2}{v_2^2}(1 - 4C_H^K) + \frac{45\lambda_8}{8}\frac{v_2^2}{\mathcal{M}^2} - \frac{3\lambda_9}{8}\frac{v_1^4}{\mathcal{M}^2 v_2^2}(1 - 2C_H^K) \\
&\quad - \frac{3\lambda_9'}{4}\frac{v_1^2}{\mathcal{M}^2} - (19 - 6C_H^K)\frac{N_c}{4\pi^2}\left(\frac{m_t^4}{v^4}\right)\left(\xi_H^t\right)^4\,. \qquad (5.31)
\end{aligned}$$

When the cancellation of quadratic divergences is used as a criterion of restriction, m_H turns out to be heavier than $2m_t$ as seen in Chapter 4. However, the idea of a $H-H$ bound state is not restricted by the consideration of naturalness, so it is worthwhile to check on the possibility of bound state formation even when $m_h \leq m_H \leq 2m_t$. Consequently the top quark is integrated out. Using Eq. (5.11) in Eq. (5.30) and Eq. (5.31) we calculate the cubic and quartic couplings of the H field as,

$$\begin{aligned}
\lambda_{12}^{eff} &= \frac{3m_H^2}{v^2\tan^2\beta}(1+\tan^2\beta) - \frac{3m_H^2}{\mathcal{M}^2}\left(C_{\phi_2}^3 + \frac{1}{4}C_{\phi_2}^4\right) - \frac{3\lambda_5}{2\tan^2\beta} + \frac{5\lambda_8}{8}\frac{v^2\tan^2\beta}{\mathcal{M}^2(1+\tan^2\beta)} \\
&\quad -\frac{3\lambda_9}{8\mathcal{M}^2}\frac{v^2}{(1+\tan^2\beta)\tan^2\beta} - \frac{3\lambda_9'}{4}\frac{v^2}{\mathcal{M}^2(1+\tan^2\beta)} + \frac{7N_c}{4\pi^2}\left(\frac{m_t^4}{v^4}\right)\cot^3\beta\,, \qquad (5.32) \\
\lambda_{13}^{eff} &= \frac{3m_H^2}{v^2\tan^2\beta}(1+\tan^2\beta) - \frac{6m_H^2}{\mathcal{M}^2}\left(C_{\phi_2}^3 + \frac{1}{4}C_{\phi_2}^4\right) - \frac{3\lambda_5}{2\tan^2\beta} + \frac{45\lambda_8}{8}\frac{v^2\tan^2\beta}{\mathcal{M}^2(1+\tan^2\beta)} \\
&\quad -\frac{3\lambda_9}{8\mathcal{M}^2}\frac{v^2}{(1+\tan^2\beta)\tan^2\beta} - \frac{3\lambda_9'}{4}\frac{v^2}{\mathcal{M}^2(1+\tan^2\beta)} - \frac{19N_c}{4\pi^2}\left(\frac{m_t^4}{v^4}\right)\cot^4\beta\,. \qquad (5.33)
\end{aligned}$$

We have put $\xi_H^t \approx -\cot\beta$, which is its value for all types of 2HDMs in the alignment limit. Letting $(C_{\phi_2}^3 + \frac{1}{4}C_{\phi_2}^4) \sim 1$, for $v = 246$ GeV, $m_t = 174$ GeV, $\tan\beta = 5$ and keeping the λ's well within the perturbative bounds by choosing $\lambda_i \sim 1$, we have evaluated the strengths of the cubic and quartic couplings from Eq. (5.32) and Eq. (5.33) for $m_H =$ 300 GeV (chosen arbitrarily within the range). It was found that $|\lambda_{12}^{eff}| - |\lambda_{13}^{eff}| = -0.02$, i.e. $|\lambda_{12}^{eff}| \approx |\lambda_{13}^{eff}|$ at the level of accuracy we are considering. We conclude that in this case of a not too heavy H, an $H-H$ bound state may form, but it is also likely to have a very short lifetime.

Naturalness arguments coupled with unitarity, perturbativity and constraints from the T-parameter lead to a heavy H with a mass between 450 GeV and 620 GeV. In this case we cannot integrate out the top quark. For the cubic and quartic couplings we find

$$\begin{aligned}
\lambda_{12}^{eff} &= 3(1-C_H^K)\frac{m_H^2}{v_2^2} - \frac{3\lambda_5}{2}\frac{v_1^2}{v_2^2}(1-3C_H^K) + \frac{5\lambda_8}{8}\frac{v_2^2}{\mathcal{M}^2} - \frac{3\lambda_9}{8}\frac{v_1^4}{\mathcal{M}^2 v_2^2}(1-C_H^K) - \frac{3\lambda_9'}{4}\frac{v_1^2}{\mathcal{M}^2} \\
&= \frac{3m_H^2}{v^2\tan^2\beta}(1+\tan^2\beta) - \frac{3m_H^2}{\mathcal{M}^2}(C_{\phi_2}^3 + \frac{1}{4}C_{\phi_2}^4) - \frac{3\lambda_5}{2\tan^2\beta} + \frac{5\lambda_8}{8}\frac{v^2\tan^2\beta}{\mathcal{M}^2(1+\tan^2\beta)} \\
&\quad -\frac{3\lambda_9}{8\mathcal{M}^2}\frac{v^2}{(1+\tan^2\beta)\tan^2\beta} - \frac{3\lambda_9'}{4}\frac{v^2}{\mathcal{M}^2(1+\tan^2\beta)}\,, \qquad (5.34) \\
\lambda_{13}^{eff} &= 3(1-2C_H^K)\frac{m_H^2}{v_2^2} - \frac{3\lambda_5}{2}\frac{v_1^2}{v_2^2}(1-4C_H^K) + \frac{45\lambda_8}{8}\frac{v_2^2}{\mathcal{M}^2} - \frac{3\lambda_9}{8}\frac{v_1^4}{\mathcal{M}^2 v_2^2}(1-2C_H^K) - \frac{3\lambda_9'}{4}\frac{v_1^2}{\mathcal{M}^2} \\
&= \frac{3m_H^2}{v^2\tan^2\beta}(1+\tan^2\beta) - \frac{6m_H^2}{\mathcal{M}^2}(C_{\phi_2}^3 + \frac{1}{4}C_{\phi_2}^4) - \frac{3\lambda_5}{2\tan^2\beta} + \frac{45\lambda_8}{8}\frac{v^2\tan^2\beta}{\mathcal{M}^2(1+\tan^2\beta)} \\
&\quad -\frac{3\lambda_9}{8\mathcal{M}^2}\frac{v^2}{(1+\tan^2\beta)\tan^2\beta} - \frac{3\lambda_9'}{4}\frac{v^2}{\mathcal{M}^2(1+\tan^2\beta)}\,. \qquad (5.35)
\end{aligned}$$

Letting $(C_{\phi_2}^3 + \frac{1}{4}C_{\phi_2}^4) \sim 1$, for $v = 246$ GeV, $m_t = 174$ GeV, $\tan\beta = 5$ and keeping the λ's well within the perturbative bounds by choosing $\lambda_i \sim 1$ the strengths of the cubic and quartic couplings are evaluated from Eq. (5.34) and Eq. (5.35) for $m_H = 450$ GeV, 500 GeV and 620 GeV. 450 GeV and 620 GeV are the lower and upper limits for m_H as found in Section 4.1.1 and 500 GeV is an intermediate value. It is found that $|\lambda_{12}^{eff}| - |\lambda_{13}^{eff}| = 0.317$, 0.459 and 0.87 respectively for $m_H = 450$ GeV, 500 GeV and 620 GeV. Thus formation of $H - H$ bound state is likely for obese Higgs bosons and the likelihood increases as the Higgs becomes massive.

5.3.2 HIGGSIUM: Production and Decay

When two H bosons approach each other with a relative velocity u_H to form a $H - H$ bound state with characteristic radius R_0^H, then the formation time of the $H-H$ bound state can be approximated by $\tau_f^H \sim \frac{4R_0^H}{u_H}$. This is roughly the period of oscillation for s-wave states [58].

For a non-relativistic bound state we can approximate the relative momenta of H by $p_H \sim m_H u_H$ so that

$$\begin{aligned} \tau_f^H &\sim \frac{4R_0^H}{u_H} \\ &\sim \frac{4}{m_H u_H^2} \, . \end{aligned} \tag{5.36}$$

The predominant decay channel/s for $m_h \leq m_H \leq 2m_t$ is $H \to b\bar{b}$ and for $m_H > 2m_t$ is $H \to b\bar{b}$ and $H \to t\bar{t}$. We take these decays as dictating the decay rate of Higgsium. Below we calculate the decay width neglecting the effects of new physics operators.

Case I: $m_h < m_H < 2m_t$

Neglecting the effect of the new physics operators, decay width and decay time for $H \to b\bar{b}$ are,

$$\Gamma_b^H = \frac{m_H (\xi_H^b)^2}{8\pi} (1 - 4\frac{m_b^2}{m_H^2})^{3/2} \approx \frac{m_H (\xi_H^b)^2}{8\pi} \, , \tag{5.37}$$

and

$$\tau_b^H = \frac{1}{\Gamma_b^H} = \frac{8\pi}{m_H (\xi_H^b)^2} \, . \tag{5.38}$$

Since $m_b^2 \ll m_H^2$ we have neglected $4\frac{m_b^2}{m_H^2}$ in comparison to 1. Let us find an estimate of τ_b^H for the given range of m_H. For a reasonable choice of $\tan\beta = 5$, τ_b^H for $m_H = 130$ and 350 GeV (chosen arbitrarily within the range) for various types of 2HDMs in the alignment limit have been tabulated below in table 5.1. We have used the conversion 1 GeV$^{-1} = 6.58 \times 10^{-25}$ sec to find the decay time in seconds.

2HDMs	ξ_H^b	τ_b^H	τ_b^H ($m_H = 130$ GeV) in secs	τ_b^H ($m_H = 350$ GeV) in secs
Type I	$-\cot\beta$	$\frac{8\pi}{m_H \cot^2\beta}$	3.18×10^{-24}	1.18×10^{-24}
Type II	$\tan\beta$	$\frac{8\pi}{m_H \tan^2\beta}$	5.09×10^{-27}	1.89×10^{-27}
Lepton Specific	$-\cot\beta$	$\frac{8\pi}{m_H \cot^2\beta}$	3.18×10^{-24}	1.18×10^{-24}
Flipped	$\tan\beta$	$\frac{8\pi}{m_H \tan^2\beta}$	5.09×10^{-27}	1.89×10^{-27}

Table 5.1: Decay time (in seconds) of H when $m_h < m_H < 2m_t$ for the different 2HDMs.

For the $H-H$ bound state to be formed, the formation time of the $H-H$ bound state must be smaller than the decay time of H. In other words,

$$\begin{aligned} \tau_f^H &< \tau_b^H \\ \Rightarrow \frac{4}{m_H u_H^2} &< \frac{8\pi}{m_H (\xi_H^b)^2} \\ \Rightarrow \quad u_H &> \frac{\xi_H^b}{\sqrt{2\pi}} . \end{aligned} \tag{5.39}$$

Now we proceed in two ways. First we fix a value of $\tan\beta$ consistent with observations [55–57] and find the range of u_H for which a bound state may form. Next we fix a non-relativistic value of u_H and find the range for $\tan\beta$.

Let us fix $\tan\beta = 5$ and use Eq. (5.39) for various types of two Higgs doublet models in the alignment limit to find the range of u_H. Next we fix $u_H = 0.01c$ and find the range of $\tan\beta$. The results are displayed in table 5.2.

2HDMs	ξ_H^b	Limit of u_H for $\tan\beta = 5$	Limit of $\tan\beta$ for u_H =0.01c
Type I	$-\cot\beta$	$u_H > 0.08c$	$\tan\beta$ >39.89
Type II	$\tan\beta$	$u_H > 1.99c$	$\tan\beta < 0.025$
Lepton Specific	$-\cot\beta$	$u_H > 0.08c$	$\tan\beta$ >39.89
Flipped	$\tan\beta$	$u_H > 1.99c$	$\tan\beta < 0.025$

Table 5.2: Limits for relative velocity for $\tan\beta = 5$ and $\tan\beta$ for $u_H = 0.01c$ when $m_h < m_H < 2m_t$.

When $\tan\beta = 5$, u_H for the type II and flipped 2HDMs is not sensible. Similarly, when we set $u_H = 0.01c$, the bound on $\tan\beta$ is far too low. Thus we conclude that these two types of 2HDMs do not seem to allow the formation of $H-H$ bound states. In the other two types of 2HDMs the formation of $H-H$ bound state is not very easy. The bounds on $\tan\beta$ and u_H seem to bear values in the vicinity of their limits.

Case II: $m_H > 2m_t$

In the case of a heavy H particle with $m_H > 2m_t$, the predominant decay channels are $H \to b\bar{b}$ and $H \to t\bar{t}$. Neglecting the effects of new physics operators the decay width of H into $t\bar{t}$ pair is,

$$\Gamma_t^H = \frac{m_H(\xi_H^t)^2}{8\pi}(1-4\frac{m_t^2}{m_H^2})^{3/2}. \tag{5.40}$$

The total decay width is then approximately $\Gamma^H = \Gamma_b^H + \Gamma_t^H$ where the expression for Γ_b^H is given in Eq. (5.37), and the decay time is the inverse of the total decay width, $\tau^H = (\Gamma^H)^{-1}$.

We now estimate τ^H for the two extreme values of m_H (450 and 620 GeVs) found in the alignment limit when Naturalness was taken into account for $\tan\beta = 5$. We also display the lower limits of the relative velocity using the logic that the formation time of the bound state must be shorter than the decay time of the parent particle if the bound state is to form. For all types of 2HDMs in alignment limit the relative $Ht\bar{t}$ coupling is $\xi_H^t = -\cot\beta$ and as we have seen in the first case ξ_H^b is type dependent. Thus overall the decay time is type dependent. The results are displayed in the table 5.3.

2HDMs	τ^H in secs (m_H=450 GeV)	$u_H > \frac{2}{\sqrt{m_H\tau^H}}$	τ^H in secs (m_H=620 GeV)	$u_H > \frac{2}{\sqrt{m_H\tau^H}}$
Type I	7.25×10^{-25}	u_H >0.09c	4.24×10^{-25}	u_H >0.1c
Type II	1.47×10^{-27}	u_H >1.99c	1.06×10^{-27}	u_H >1.99c
Lepton Specific	7.25×10^{-25}	u_H >0.09c	4.24×10^{-25}	u_H >0.1c
Flipped	1.47×10^{-27}	u_H >1.99c	1.06×10^{-27}	u_H >1.99c

Table 5.3: Decay time (in seconds) and relative velocity for $m_H = 450$ GeV and 620 GeV and $\tan\beta = 5$ for the different 2HDMs.

It is clear that $H-H$ bound state will not form in the Type II and flipped 2HDMs, but may form in Type I and lepton specific models. But even that conclusion is not a strong one, as the range of parameters for bound state formation are at the edge of the allowed values.

5.4 Outlook

As already discussed the peak at 125 GeV indicates that it is due to the Higgs predicted by the Standard model. In two Higgs doublet model, the alignment limit assigns the lighter CP even Higgs boson as the SM Higgs. We have thus studied the bound state formation of the heavier CP even non-standard Higgs boson whose mass spectra is flexible. We have imposed Naturalness conditions and have restricted the mass of H within bounds. If the Naturalness criteria is withdrawn and the potential is only subjected to stability and perturbative unitarity constraints then m_H is much more flexible and the entire spectrum can be studied for the possibility of the bound state formation.

The possibility of formation of h-H bound state is an interesting topic to study in the future. We would like to study the variation of effective cubic (attractive) and quartic (repulsive) coupling strengths with $\lambda_i's$ maintaining the perturbative unitarity condition. Here we have studied the bound state formation in the non-relativistic regime. The bound state equation in the more general, fully relativistic case can also be attempted to solve for in the future.

The mass of the bound state and its life time have not been addressed to in this study. Another important point is the detectability of the bound state. In future works we could attempt to address these questions.

References

[1] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *"The Higgs Hunter's Guide,"* Front. Phys. **80**, 1 (2000).

[2] G. 't Hooft, *"Recent developments in Gauge Theories",* Proceedings of the NATO Advanced Summer Institute, Cargese, 1979, edited by G. 't Hooft et al. (Plenum, New York, 1980) p.135; L. Suskind, Phys.Rep.104(1984) **181**.

[3] H. E. Haber and G. L. Kane, Phys. Rep. **117** (1985) 75.

[4] *"The Standard Model Higgs Boson",* edited by M. B. Einhorn, (North Holland, Amsterdam, 1991).

[5] D. Jouadi, *"The anatomy of electroweak symmetry breaking Tome II: The Higgs bosons in the Minimal Supersymmetric Model",* Physics Reports Volume 459, Issues 1–6, April 2008, Pages 1-241, doi.org/10.1016/j.physrep.2007.10.005.

[6] F. Jegerlehner, *"The hierarchy problem of the electroweak Standard Model revisited",* DESY $\sim$ 13-093, HU-EP-13/25, arXiv:1305.6652v2[hep-ph].

[7] F. Jegerlehner, *"The hierarchy problem and the cosmological constant problem revisited - A new view on the SM of particle physics",* DESY $\sim$ 18-211, HU-EP-18/38, arXiv:1812.03863[hep-ph].

[8] M. Sher, Phys. Rep. **179** (1989) 273.

[9] K. G. Wilson, Phys. Rev. **B4** (1971) 3184; K. G. Wilson and J. Kogut, Phys. Rep. **12** (1974) 75.

[10] N. Christiansen, A. Eichhorn, *"An asymptotically safe solution to the U(1) triviality problem",* Physics Letters B Volume 770, 10 July 2017, Pages 154-160, doi.org/10.1016/j.physletb.2017.04.047.

[11] R. D. Peccei and H. R. Quinn, *"CP Conservation in the Presence of Instantons,"* Phys. Rev. Lett. 38, 1440 (1977).

[12] R. D. Peccei and H. R. Quinn, *"CP Conservation in the Presence of Instantons,"* Phys. Rev. D 16,1791 (1977).

[13] N. Turok and J. Zadrozny, *"Electroweak baryogenesis in the two doublet model,"* Nucl. Phys. B358, 471 (1991).

[14] E. Ma, *"Verifiable radiative seesaw mechanism of neutrino mass and dark matter,"* Phys. Rev. D 73, 077301 (2006).

[15] E. Ma, *"Utility of a special second scalar doublet,"* Mod. Phys.Lett. A 23, 647 (2008).

[16] R. Barbieri, L. J. Hall, and V. S. Rychkov, *"Improved naturalness with a heavy Higgs: An alternative road to LHC physics,"* Phys. Rev. D 74, 015007 (2006).

[17] P. M. Ferreira and D. R. T. Jones, *"Bounds on scalar masses in two Higgs doublet models,"* J. High Energy Phys. 08 (2009) 069.

[18] L. Susskind, Phys. Rev. D20, 2619 (1979).

[19] S. Weinberg, Phys. Rev. D19, 1277 (1979).

[20] P. Sikivie, L. Susskind, M. B. Voloshin, and V. I. Zakharov, Nucl. Phys. B173, 189 (1980).

[21] The Review of Particle Physics (2018), M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).

[22] M. Dugan, B. Grinstein, and L. J. Hall, Nucl. Phys. B255, 413 (1985).

[23] R. S. Chivukula and H. Georgi, Phys. Lett. B188, 99 (1987).

[24] L. J. Hall and L. Randall, Phys. Rev. Lett. 65, 2939 (1990).

[25] G. D'Ambrosio, G. F. Giudice, G. Isidori, and A. Strumia, Nucl. Phys. B645, 155 (2002), hep-ph/0207036.

[26] V. Cirigliano, B. Grinstein, G. Isidori, and M. B. Wise, Nucl. Phys. B728, 121 (2005), hep-ph/0507001.

[27] A. Ali and D. London, Eur. Phys. J. C9, 687 (1999), hep-ph/9903535.

[28] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager, and L. Silvestrini, Phys. Lett. B500, 161 (2001), hep-ph/0007085.

[29] C. Bobeth, T. Ewerth, F. Kruger, and J. Urban, Phys. Rev. D66, 074021 (2002), hep- ph/0204225.

[30] A. J. Buras, Acta Phys. Polon. B34, 5615 (2003), hep-ph/0310208.

[31] G. C. Branco, A. J. Buras, S. Jager, S. Uhlig, and A. Weiler, (2006), hep-ph/0609067.

[32] B. Grinstein and M. Trott, *"A Higgs-Higgs bound state due to new physics at a TeV",* Phys. Rev. D **76**, 073002 (2007)

[33] G. F. Chew and S. Mandelstam, Phys. Rev. 199 (1960) 467.

[34] R. N. Cahn and M. Suzuki, Phys. Lett. B134, 115 (1984).

[35] H. G. Rosdolsky, *"Bound-state conditions from the N/D equations"*, Nuovo Cimento A(1965-1970), February 1970, Volume 65, Issue 3, pp 491–510.

[36] T. Akiba, S. Saito, F. Takagi, *"Z_1=0 and the conditions from the N/D method"*, Nuovo Cimento A, September 1965, Volume 63, Issue 1, pp 316–330.

[37] L. Di Leo and J. W. Darewych, Can. J. Phys. 70, 412 (1992).

[38] L. Xiao, J. W. Darewych, and M. Horbatsch, Phys. Rev. A 42, 69 (1990).

[39] J. W. Darewych, M. Horbatsch, and R. Koniuk, Phys. Rev. D 45, 675 (1992).

[40] L. Di Leo and J. W. Darewych, Can. J. Phys. 71, 365 (1993).

[41] J. W. Darewych, A. G. Sitenko, I. V. Simenog, and A. I. Sitnichenko, Phys. Rev. C 47, 1885 (1993).

[42] W. C. Berseth and J. W. Darewych, Phys. Lett. A 157, 361 (1991).

[43] B. W. Lee, C. Quigg, and H. B. Thacker, Phys. Rev. D 16, 1519 (1977); Phys. Rev. Lett. 38, 883 (1977).

[44] J. A. Grifols, Phys. Lett. B 264, 149 (1991).

[45] E. E. Salpeter and H. A. Bethe, Physical Review, Volume 84, Number 6 (1951).

[46] T. D. Lee, *"A Theory of Spontaneous T Violation,"* Phys. Rev. D **8**, 1226 (1973).

[47] S. L. Glashow and S. Weinberg, *"Natural Conservation Laws for Neutral Currents,"* Phys. Rev. D **15**, 1958 (1977).

[48] E. A. Paschos, *"Diagonal Neutral Currents,"* Phys. Rev. D **15**, 1966 (1977).

[49] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *"Errata for the Higgs Hunter's guide,"* arXiv:hep-ph/9302272.

[50] Solberg, M .A . J. High Energ. Phys. (2018) 2018: 163.

[51] B. Grinstein, L. J. Hall, and L. Randall, Phys. Lett. B211, 363 (1988).

[52] S. Dawson and H. E. Haber, Int. J. Mod. Phys. A7, 107 (1992).

[53] R. S. Chivukula, A. G. Cohen, H. Georgi, and A. V. Manohar, Phys. Lett. B222, 258 (1989).

[54] W. M. Yao et al., Journal of Physics G 33, 1+ (2006).

[55] A. Arhrib, R. Benbrik, C. H. Chen, R. Guedes, and R. Santos, *"Double neutral Higgs production in the two-Higgs doublet model at the LHC"*, J. High Energy Phys. 08 (2009)035.

[56] J. Cao, P. Wan, L. Wu, and J. M. Yang, *"Lepton-specific two-Higgs doublet model: Experimental constraints and implication on Higgs phenomenology"*, Phys. Rev. D 80, 071701 (2009).

[57] J. h. Park, *"Lepton non-universality at LEP and charged Higgs"*, J. High Energy Phys. 10 (2006) 077.

[58] M. J. Strassler and M. E. Peskin, Phys. Rev. D43, 1500 (1991).

Chapter 6

Yet another model - the HTM

> Innovation is taking two things that already exist and putting them together in a new way.
>
> *Tom Freston*

The SM predicted neutrinos to be massless but various experiments have proven that neutrinos have mass how small it may be. The problem of neutrino mass generation has been addressed by yet another model, the Higgs Triplet Model abbreviated as HTM [1–5] with a non-minimal Higgs sector. In Higgs triplet models an $SU(2)_L$ triplet of scalar particles with hypercharge $Y = 2$ denoted by Δ is present in addition to the SM particles. The experimental findings about the masses of the neutrinos indirectly put some bounds on the parameter of the model that gives rise to the neutrino masses. The vacuum expectation value (vev) of a neutral Higgs boson in an isospin triplet representation is the source for the mass of the neutrinos and thus it has to be small and is assumed to be less than 1 GeV. The non-conservation of lepton number which is explicitly broken in the scalar potential of the Higgs triplet model by a trilinear coupling μ is protected by symmetry and is naturally small which also assures the smallness of neutrino masses.

Higgs triplet model is a new physics beyond the SM. Thus its scalar sector is much richer than that of the SM. The model predicts a doubly charged Higgs boson ($H^{\pm\pm}$) and a singly charged Higgs boson ($H^{\pm}$), for which direct searches are being carried out at the LHC [6, 7]. Apart from these there is a CP odd neutral scalar, A^0 and two CP even neutral scalars, H and h like the two Higgs doublet model. The vector bosons absorb the rest of the degrees of freedom. In a large part of the parameter space of the HTM the lightest CP-even scalar, h, has essentially the same couplings to the fermions and vector bosons as the Higgs boson of the SM [8–10].

The charged scalars of the model, $H^{\pm\pm}$ and $H^{\pm}$ contribute to the loop induced $h \to \gamma\gamma$ decay width in addition to the contribution from the top quarks and the W bosons. Thus this additional contribution may result in a decay width which matches the LHC decay width results [11, 12]. λ_1 which is a quartic coupling in the potential of the Higgs triplet model to be discussed in the upcoming section, controls the contribution of $H^{\pm\pm}$ to the Higgs diphoton decay width. The case of $\lambda_1 > 0$ leads to destructive interference between the combined SM contribution (from W and fermion loops) and the contribution from $H^{\pm\pm}$ as was studied in [13]. Later on the case for $\lambda_1 < 0$ was studied in [14] which leads to constructive interference.

Production channels for $H^{\pm\pm}$ were extensively studied in [15–21]. Some of the most discussed production channels are $q\overline{q} \to \gamma^*, Z^* \to H^{++}H^{--}$ and $q^{'}\overline{q} \to W^* \to H^{\pm\pm}H^{\mp}$. Quartic terms in the scalar potential induce a mass splitting between $H^{\pm\pm}$ and $H^{\pm}$, which can be of either sign. If $m_{H^{\pm\pm}} > m_{H^{\pm}}$ then a new decay channel becomes available for $H^{\pm\pm}$, namely $H^{\pm\pm} \to H^{\pm}W^*$. Another scenario is the case of $m_{H^{\pm}} > m_{H^{\pm\pm}}$, which would give rise to a new decay channel for the singly charged scalar, namely $H^{\pm} \to H^{\pm\pm}W^*$. This decay of singly charged Higgs would give rise to an alternative way to produce $H^{\pm\pm}$ in pairs, namely by the production mechanism $q^{'}\overline{q} \to W^* \to H^{\pm\pm}H^{\mp}$ followed by $H^{\mp} \to H^{\pm\pm}W^*$. The effect of this additional sources of production of the contributing non-standard charged particles to the loop induced Higgs decays has been discussed in this chapter.

Higgs triplet model can also be considered in the light of the *Wrong sign limit*. This limit was discussed in Section 4.3for 2HDMs where some of the Yukawa couplings of the SM like Higgs boson are of the opposite sign to that of the vector boson couplings (wrong sign). The scalar spectrum of the model gets constrained by unitarity and the stability of the potential. The oblique T-parameter and Higgs decay branching ratios, in particular $h \to \gamma\gamma$ largely depend on the scalar spectrum of the model and thus constraints coming from their experimental values also restrict the scalar spectrum.

6.1 Higgs Triplet model - Modelling

As mentioned earlier in a Higgs triplet model a $Y = 2$ complex $SU(2)_L$ isospin triplet of scalar fields, $\mathbf{T} =(T_1, T_2, T_3)$, is added to the SM Lagrangian. Without the introduction of $SU(2)_R$ singlet neutrinos, Majorana masses can be obtained by the observed neutrinos in Higgs triplet model. The gauge invariant Yukawa interaction written below accomplishes the task.

$$\mathcal{L} = h_{ll'} L_l^T C i\tau_2 \Delta L_{l'} + h.c. \ , \tag{6.1}$$

where $h_{ll'}$ $\left(l, l' = e, \mu, \tau\right)$ is a complex and symmetric coupling, C is the Dirac charge conjugation operator, τ_2 is the second Pauli matrix, $L_l = (\nu_{lL}, l_L)^T$ is a left-handed lepton doublet, and Δ is a 2×2 representation of the $Y = 2$ complex triplet fields.

$$\Delta = \mathbf{T}.\tau = T_1\tau_1 + T_2\tau_2 + T_3\tau_3 = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} \ , \tag{6.2}$$

where $T_1 = \left(\Delta^{++} + \Delta^0\right)/2$, $T_2 = i\left(\Delta^{++} - \Delta^0\right)/2$, and $T_3 = \Delta^+/\sqrt{2}$. Now $\langle\Delta^0\rangle = \frac{v_\Delta}{\sqrt{2}}$ results in the following neutrino mass matrix:

$$m_{ll'} = 2h_{ll'}\langle\Delta^0\rangle = \sqrt{2}h_{ll'}v_\Delta \ . \tag{6.3}$$

When $\Phi = \left(\phi^+, \phi^0\right)^T$ defines the usual SM Higgs doublet, the Higgs Triplet scalar potential as defined in [22, 23] is

$$\begin{aligned} V(\Phi,\Delta) = {} & -m_\Phi^2 \Phi^\dagger\Phi + \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^2 + M_\Delta^2 \operatorname{Tr}\Delta^\dagger\Delta + \left(\mu\Phi^T i\tau_2\Delta^\dagger\Phi + h.c.\right) \\ & + \lambda_1\left(\Phi^\dagger\Phi\right)\operatorname{Tr}\Delta^\dagger\Delta + \lambda_2\left(\operatorname{Tr}\Delta^\dagger\Delta\right)^2 + \lambda_3\operatorname{Tr}\left(\Delta^\dagger\Delta\right)^2 + \lambda_4\Phi^\dagger\Delta\Delta^\dagger\Phi \ . \end{aligned} \tag{6.4}$$

In this expression $-m_\Phi^2$ is negative to ensure non-zero vev of the neutral component of the scalar doublet while M_Δ^2 is positive. Here, $\langle\phi^0\rangle = v/\sqrt{2}$ which spontaneously breaks the $SU(2)_L \otimes U(1)_Y$ to $U(1)_Q$. v_Δ is obtained from the minimisation of V and for small v_Δ/v the expression for the triplet vev is,

$$v_\Delta \simeq \frac{\mu v^2}{\sqrt{2}\left(M_\Delta^2 + v^2\left(\lambda_1+\lambda_4\right)/2\right)} \ . \tag{6.5}$$

When the triplet scalars are heavy i.e., $M_\Delta \gg v$ then v_Δ can be approximated as $v_\Delta \simeq \mu v^2/(\sqrt{2}M_\Delta^2)$. Even if μ is of the order of the electroweak scale, v_Δ will be naturally small and this is sometimes called the "Type II seesaw mechanism". Such heavy triplet scalars would be beyond the search limits of LHC and thus much interest has been drawn towards light triplet scalars ($M_\Delta \approx v$) which are within the discovery reach of LHC. This would lead to v_Δ being approximately equal to μ. It is to be noted that v_Δ has to be small basically for two reasons. First with reference to Eq. (6.3) where the neutrino mass matrix is directly proportional to v_Δ and thus to preserve the smallness of neutrino masses v_Δ has to be naturally small. Secondly the case of $v_\Delta < 0.1$ MeV is assumed in the ongoing searches at the LHC, for which the branching ratios of the triplet scalars to leptonic final states (e.g. $H^{\pm\pm} \to l^\pm l^\pm$) would be $\sim 100\%$. Since $v_\Delta \sim \mu$ for light triplet scalars then μ must also be small (compared to the electroweak scale) for the scenario of $v_\Delta < 0.1$ MeV.

The ρ parameter ($\rho = M_W^2/M_Z^2\cos^2\theta_W$) puts an upper bound on v_Δ as we will just discuss. In the SM $\rho = 1$ at tree-level and all the new physics models must respect this value to a fraction of a percentage. In the Higgs triplet model

$$\rho \equiv 1 + \delta\rho = \frac{1+2x^2}{1+4x^2} \ , \tag{6.6}$$

where $x = v_\Delta/v$ and carries the extra contribution due to new physics. The measurement $\rho \approx 1$ leads to the bound $v_\Delta/v \lesssim 0.03$, or $v_\Delta \lesssim 8$GeV when v is essentially equal to the vev of the Higgs boson of the SM (i.e. $v \approx 246$ GeV).

Coming to the physical mass eigenstates, mostly $\Delta^{\pm\pm}$ comprises the doubly charged scalar of the model, $H^{\pm\pm}$. However the remaining eigenstates are in general mixtures of the doublet and triplet fields. But since such mixing is proportional to the triplet vev therefore it is small even if v_Δ assumes its largest value of a few GeV. The lighter CP-even Higgs h is predominantly composed of the doublet field and plays the role of the SM Higgs boson. While the heavier CP-even Higgs H, the singly charged Higgs $H^\pm$ and the CP-odd Higgs A^0 all have predominant contribution from the triplet fields. In obtaining the mass eigenstates from the charge eigenstates we come across two angles, α' and β'. α' is the mixing angle in the CP-even sector and β' is the mixing angle in the charged Higgs sector. Their expressions follow below.

$$\sin\alpha' \sim 2v_\Delta/v \ , \ \tan\beta' = \sqrt{2}v_\Delta/v \ . \tag{6.7}$$

Neglecting the small off-diagonal elements in the CP-even mass matrix, the approximate expressions for the squared masses of h and H are as follows:

$$m_h^2 = \frac{\lambda}{2}v^2 \ , \tag{6.8}$$

which is the same as in the SM and

$$m_H^2 = M_\Delta^2 + (\frac{\lambda_1}{2} + \frac{\lambda_4}{2})v^2 + 3(\lambda_2 + \lambda_3)v_\Delta^2 \ . \tag{6.9}$$

The squared masses of the doubly charged scalar $H^{\pm\pm}$, singly charged scalar $H^\pm$ and CP-odd scalar A^0 are,

$$m_{H^{\pm\pm}}^2 = M_\Delta^2 + \frac{\lambda_1}{2}v^2 + \lambda_2 v_\Delta^2 \ , \tag{6.10}$$

$$m_{H^\pm}^2 = M_\Delta^2 + (\frac{\lambda_1}{2} + \frac{\lambda_4}{4})v^2 + (\lambda_2 + \sqrt{2}\lambda_3)v_\Delta^2 \ , \tag{6.11}$$

and

$$m_{A^0}^2 = M_\Delta^2 + (\frac{\lambda_1}{2} + \frac{\lambda_4}{2})v^2 + (\lambda_2 + \lambda_3)v_\Delta^2 \ . \tag{6.12}$$

As we can figure out from the above equations there is a common term in the expressions for the masses of H, $H^{\pm\pm}$, $H^\pm$ and A^0 which is $M_\Delta^2 + \frac{\lambda_1}{2}v^2$. Thus when terms proportional to the small parameter v_Δ is neglected then $m_{A^0} = m_H$. In this situation there are only two possible mass hierarchies for the non-standard scalars. As can be seen the magnitude of the mass splitting is controlled by λ_4 and the two possible mass hierarchies are,

$$\begin{aligned} m_{H^{\pm\pm}} &> m_{H^\pm} > m_{A^0}, m_H \text{ for } \lambda_4 < 0 \ , \\ m_{H^{\pm\pm}} &< m_{H^\pm} < m_{A^0}, m_H \text{ for } \lambda_4 > 0 \ . \end{aligned} \tag{6.13}$$

6.2 Constraints on the scalar potential

Analogous to the SM and the 2HDM, the Higgs triplet model too is restricted by stability and unitarity bounds. The condition for the scalar potential in Eq. (6.4) to be bounded from below are [24] :

$$\lambda \geq 0,\;\; \lambda_2+\lambda_3 \geq 0,\;\; \lambda_2+\frac{\lambda_3}{2} \geq 0,\;\; \lambda_1+\sqrt{\lambda\left(\lambda_2+\lambda_3\right)} \geq 0,\;\; \lambda_1+\lambda_4+\sqrt{\lambda\left(\lambda_2+\lambda_3\right)} \geq 0 \tag{6.14}$$

$$\text{and}\quad \left[|\lambda_4|\sqrt{\lambda_2+\lambda_3}-\lambda_3\sqrt{\lambda} \geq 0,\;\; or,\;\; 2\lambda_1+\lambda_4+\sqrt{\left(2\lambda\lambda_3-\lambda_4^2\right)\left(\frac{2\lambda_2}{\lambda_3}+1\right)} \geq 0\right].$$

Hence the conditions from vacuum stability.

The scattering matrix, S-matrix having 2-particle states as rows and columns has the scattering amplitudes involving longitudinal gauge bosons and Higgs bosons as its elements. The eigenvalues of this matrix are restricted by $|a_0| < 1$, where a_0 is the $l = 0$ partial wave amplitude. These conditions translate into upper limits on combinations of Higgs quartic couplings, which for multi-Higgs models have been derived by different authors. For Higgs triplet model these have been derived in [10] and are enlisted below:

$$|\left(\lambda+4\lambda_2+8\lambda_3\right) \pm \sqrt{\left(\lambda-4\lambda_2-8\lambda_3\right)^2+16\lambda_4^2}| \leq 64\pi, \tag{6.15}$$

$$|\left(3\lambda+16\lambda_2+12\lambda_3\right) \pm \sqrt{\left(3\lambda-16\lambda_2-12\lambda_3\right)^2+24\left(2\lambda_1+\lambda_4\right)^2}| \leq 64\pi, \tag{6.16}$$

$$|\lambda| \leq 32\pi, \tag{6.17}$$

$$|2\lambda_1+3\lambda_4| \leq 32\pi, \tag{6.18}$$

$$|2\lambda_1-\lambda_4| \leq 32\pi, \tag{6.19}$$

$$|\lambda_1| \leq 16\pi, \tag{6.20}$$

$$|\lambda_1+\lambda_4| \leq 16\pi, \tag{6.21}$$

$$|2\lambda_2-\lambda_3| \leq 16\pi, \tag{6.22}$$

$$|\lambda_2| \leq 8\pi, \tag{6.23}$$

$$|\lambda_2+\lambda_3| \leq 8\pi. \tag{6.24}$$

Perturbativity constrains the quartic couplings to be within $[-4\pi, 4\pi]$.

New Physics contribution to the electroweak T-parameter is given by [25, 26],

$$\Delta T = \frac{1}{4\pi\sin^2\theta_W m_W^2}\left[F(m_{H^\pm}^2, m_A^2)+F(m_{H^{\pm\pm}}^2, m_{H^\pm}^2)\right] , \tag{6.25}$$

where, θ_W is the Weinberg angle and m_W is the W-boson mass.
The function $F(x, y)$ is defined as,

$$F(x,y) = \frac{x+y}{2} - \frac{xy}{x-y}\ln(\frac{x}{y}) . \tag{6.26}$$

Experimentally the new Physics contribution to the T-parameter is given in [11] and this translates into mass bounds on the non-standard scalar masses in HTM.

The combined constraints from vacuum stability, unitarity, perturbativity and the electroweak T-parameter confine the parameter space for the masses of the non-standard scalars of the Higgs Triplet model. Since the masses of the non-standard Higgs bosons are correlated thus mass bound on one of these automatically puts a bound on other masses too. These have been discussed recently in [27].

6.3 Diphoton decay in the Higgs Triplet model

BSM Physics literature has shown the impact of singly charged scalars on the decay $h \to \gamma\gamma$ as for e.g. in the context of the minimal supersymmetric SM (MSSM) [28], two-Higgs Doublet Model [29–31] and next-to-MSSM [32]. The contribution of doubly charged scalars to this decay has received comparatively little attention. This was dealt in the Little Higgs Model [33], but due to the structure of the scalar potential the magnitude of the contribution from $H^{\pm\pm}$ was shown to be much smaller than $H^{\pm}$. The contribution from $H^{\pm\pm}$ was studied in the HTM in [13,34], and was shown to give a sizeable contribution to $h \to \gamma\gamma$.

The loop induced diphoton decay width has contribution from the fermions, the W bosons and the charged Higgs of the model. The decay width is [35]

$$\Gamma\left(h\to\gamma\gamma\right) = \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\,\Big|\sum_f N_c Q_f^2 g_{hff} A_{1/2}^h\left(\tau_f\right) + g_{hWW} A_1^h\left(\tau_W\right) + \tilde{g}_{hH^\pm H^\mp} A_0^h\left(\tau_{H^\pm}\right) + 4\tilde{g}_{hH^{\pm\pm}H^{\mp\mp}} A_0^h\left(\tau_{H^{\pm\pm}}\right)\Big|^2 \ . \tag{6.27}$$

In the above equation, α is the fine structure constant, N_c is the color quantum number which is 3 for quarks, Q_f is the electric charge of the fermion in the loop and

$$\tau_i = \frac{4m_i^2}{m_h^2}, \quad i = f, W, H^\pm, H^{\pm\pm} \ . \tag{6.28}$$

The loop functions $A_{1/2}$ (for fermions), A_1 (for W-bosons) and A_0 (for the charged scalars) are defined below:

$$A_{1/2}\left(\tau_x\right) = -2\tau_x\left\{1+(1-\tau_x)\mathcal{F}\left(\tau_x\right)\right\} \ , \tag{6.29}$$

$$A_1\left(\tau_x\right) = 2+3\tau_x+3\tau_x\left(2-\tau_x\right)\mathcal{F}\left(\tau_x\right) \ , \tag{6.30}$$

$$A_0\left(\tau_x\right) = -\tau_x\left\{1-\tau_x\mathcal{F}\left(\tau_x\right)\right\} \ , \tag{6.31}$$

$$\text{with,} \quad \mathcal{F}\left(\tau_x\right) = \begin{cases} \left[\sin^{-1}\left(\sqrt{\frac{1}{\tau_x}}\right)\right]^2 & \text{for } \tau_x \geq 1, \\ -\frac{1}{4}\left[\ln\left(\frac{1+\sqrt{1-\tau_x}}{1-\sqrt{1-\tau_x}}\right) - i\pi\right]^2 & \text{for } \tau_x < 1. \end{cases} \tag{6.32}$$

The relative decay width is

$$\mu_{\gamma\gamma} = \frac{\sigma\left(pp \to h\right)^{HTM} \times \Gamma\left(h \to \gamma\gamma\right)^{HTM}}{\sigma\left(pp \to h\right)^{SM} \times \Gamma\left(h \to \gamma\gamma\right)^{SM}} \,. \tag{6.33}$$

At the time when this work was done, the then recent bounds on the relative decay width was $\mu_{\gamma\gamma} = 1.16^{+0.20}_{-0.18}$ [12]. Since the lighter CP even Higgs boson h of the Higgs Triplet model is considered to be the SM like Higgs boson so, its production cross-section from gluon-gluon fusion is the same for HTM and SM. For the contribution from the fermion loops we will only keep the term with the top and bottom quarks, which are dominant. There is an enhancement factor of four for $H^{\pm\pm}$ relative to $H^{\pm}$. This is due to the electric charge. The couplings of h to the vector bosons and fermions relative to the values in the SM are as follows:

$$g_{ht\bar{t}} = \cos\alpha^{'} / \cos\beta^{'} \ , \tag{6.34}$$

$$g_{hb\bar{b}} = \cos\alpha^{'} / \cos\beta^{'} \ , \tag{6.35}$$

$$g_{hWW} = \cos\alpha^{'} + 2\sin\alpha^{'} v_{\Delta}/v \ , \tag{6.36}$$

$$g_{hZZ} = \cos\alpha^{'} + 4\sin\alpha^{'} v_{\Delta}/v \ . \tag{6.37}$$

From Eq. (6.7), it follows that $\cos\alpha^{'} = \sqrt{\left(1 - 4v_{\Delta}^2/v^2\right)} \sim 1$ and $\cos\beta^{'} = \sqrt{\left(1 - 2v_{\Delta}^2/v^2\right)} \sim 1$ and thus the above couplings of h are essentially the same as that of the SM Higgs boson because $v_{\Delta} \ll v$.

The scalar trilinear couplings parametrized by $g_{hH^{++}H^{--}}$ and $g_{hH^{+}H^{-}}$ are written below explicitly in terms of the parameters of the scalar potential (Eq. (6.4)) [10]:

$$g_{hH^{++}H^{--}} = -\left\{2\lambda_2 v_{\Delta} s_{\alpha'} + \lambda_1 v c_{\alpha'}\right\} \ , \tag{6.38}$$

$$\begin{aligned} g_{hH^{+}H^{-}} = & -\frac{1}{2}\left\{\left[4v_{\Delta}\left(\lambda_2+\lambda_3\right)c_{\beta'}^2 + 2v_{\Delta}\lambda_1 s_{\beta'}^2 - \sqrt{2}\lambda_4 v c_{\beta'} s_{\beta'}\right] s_{\alpha'} \right. \\ & \left. + \left[\lambda v s_{\beta'}^2 + \left(2\lambda_1+\lambda_4\right) v c_{\beta'}^2 + \left(4\mu - \sqrt{2}\lambda_4 v_{\Delta}\right) c_{\beta'} s_{\beta'}\right] c_{\alpha'}\right\} \ , \end{aligned} \tag{6.39}$$

where $s_{\alpha'} = \sin\alpha^{'}$ and so on. The scalar trilinear couplings are rescaled for the ease of calculation as follows:

$$\tilde{g}_{hH^{++}H^{--}} = -\frac{m_W}{g m_{H^{\pm\pm}}^2} g_{hH^{++}H^{--}} \ , \tag{6.40}$$

$$\tilde{g}_{hH^{+}H^{-}} = -\frac{m_W}{g m_{H^{\pm}}^2} g_{hH^{+}H^{-}} \ . \tag{6.41}$$

As argued earlier v_{Δ} is taken to be small and when the terms associated with it are neglected in Eq. (6.38) and Eq. (6.39) then these trilinear couplings take the form [10, 36]:

$$g_{hH^{++}H^{--}} \approx -\lambda_1 v \ , \tag{6.42}$$

$$g_{hH^{+}H^{-}} \approx -\left(\lambda_1 + \frac{\lambda_4}{2}\right) v \ . \tag{6.43}$$

The contribution from the $H^{\pm\pm}$ loop interferes constructively with that of the W boson loop for $\lambda_1 < 0$, while for $\lambda_1 > 0$ the interference is destructive. For $\lambda_1 \sim 10$ the contribution from the doubly charged Higgs and the W boson loop nearly cancel each other. The $H^\pm$ loop is usually sub-dominant.

The coupling λ_1 determines the value of $g_{hH^{++}H^{--}}$ and $g_{hH^{+}H^{-}}$. The main constraint on λ_1 comes from the requirement of the stability of the scalar potential Eq. (6.14). One of those constraints is $\lambda_1 + \sqrt{\lambda(\lambda_2+\lambda_3)} \geq 0$. If λ_2 and λ_3 are taken to be zero, the combined constraints on λ_1 from perturbative unitarity in scalar-scalar scattering and from stability of the potential require λ_1 >0. However, if λ_2 and λ_3 are chosen to be positive then negative values of λ_1 are allowed. Now the question is whether positive values of λ_2 and λ_3 will affect the trilinear couplings and the masses of the triplet scalars or not? The answer is no. This is easily verifiable from Eq. (6.9) - Eq. (6.12) where λ_2 and λ_3 are associated with the sufficiently small parameter v_Δ. One of the simplest choice that can be made is by letting $\lambda_2 = \lambda_3$. Then using $\lambda_1 + \sqrt{\lambda(\lambda_2+\lambda_3)} \geq 0$, λ_2 can be determined as a function of λ_1 and λ. Eq. (6.8) fixes λ at 0.516 when $m_h = 125$ GeV and $v = 246$ GeV. As for example for $\lambda_1 = -1$, $\lambda_2 \geq 0.97$ and for $\lambda_1 = -2$, $\lambda_2 \geq 3.9$.

6.4 What happens when $m_{H^\pm} > m_{H^{\pm\pm}}$?

As we have discussed in the last part of the Section 6.1 for $\lambda_4 > 0$, $m_{H^\pm} > m_{H^{\pm\pm}}$. So, $H^\pm$ may decay to $H^{\pm\pm}$. The decay rate for $H^\pm \to H^{\pm\pm}W^*$ after summing over all fermion states for $W^* \to f'\overline{f}$, excluding the top quark is given below:

$$\Gamma\left(H^\pm \to H^{\pm\pm}W^* \to H^{\pm\pm}f'\overline{f}\right) \simeq \frac{9G_F^2 m_W^4 m_{H^\pm}}{4\pi^3} \int_0^{1-\kappa_{H^{\pm\pm}}} dx_2 \int_{1-x_2-\kappa_{H^{\pm\pm}}}^{1-\frac{\kappa_{H^{\pm\pm}}}{1-x_2}} dx_1 F_{H^{\pm\pm}W}\left(x_1,x_2\right) \ , \tag{6.44}$$

where $\kappa_{H^{\pm\pm}} \equiv m_{H^{\pm\pm}}/m_{H^\pm}$. The analytical expression for $F_{ij}\left(x_1,x_2\right)$ is given by [37, 38],

$$F_{ij}\left(x_1,x_2\right) = \frac{\left(1-x_1\right)\left(1-x_2\right)-\kappa_i}{\left(1-x_1-x_2-\kappa_i+\kappa_j\right)^2+\kappa_j\gamma_j} \ , \tag{6.45}$$

with $\gamma_j = \Gamma_j^2/m_{H^\pm}^2$.

This decay mode is independent of v_Δ. As long as the mass splitting between $m_{H^\pm}$ and $m_{H^{\pm\pm}}$ is above the mass of the charmed hadrons (~ 2 GeV), f and f' can be taken to be massless to a good approximation.

The other possible decays for $H^\pm$ are $H^\pm \to l^\pm \nu_{l'}$, $H^\pm \to W^\pm Z$, $H^\pm \to W^\pm h$, $H^\pm \to \bar{t}b$ and to other lighter quarks. The branching ratio BR($H^\pm \to H^{\pm\pm} W^*$) will be maximised with respect to v_Δ if $\Gamma(H^\pm \to l^\pm \nu_{l'}) = \Gamma(H^\pm \to W^\pm Z) + \Gamma(H^\pm \to W^\pm h) + \Gamma(H^\pm \to \bar{t}b)$ which is realized for $v_\Delta \simeq 0.1$ MeV.

From the production mechanism $q^{'}\overline{q} \to W^* \to H^{\pm\pm} H^{\mp}$, the decay mode $H^\pm \to H^{\pm\pm} W^*$ would give rise to pair production of $H^{\pm\pm}$, with a cross section which can be comparable to that of the standard pair-production mechanism $q\overline{q} \to \gamma^*, Z^* \to H^{++}H^{--}$. Thus the detection prospects of $H^{\pm\pm}$ in the four-lepton channel could increase significantly. In addition to the above, the decays $H \to H^\pm W^*$ and $A^0 \to H^\pm W^*$ would also provide an additional source of $H^\pm$, which can subsequently decay to $H^{\pm\pm}$.

6.5 Numerical analysis

As has been discussed in the previous section that $H^{\pm\pm}$ has additional production channels above the standard one, thus this must affect the loop induced Higgs diphoton decay width. Now one may be inquisitive that since the additional production of $H^{\pm\pm}$ results from the decay of $H^\pm$ and thus the diphoton decay width may or may not experience an overall increment. But due to a factor of four in case of the contribution from $H^{\pm\pm}$ as compared to the contribution from $H^\pm$ in Eq. (6.27) an overall increment in the decay rate is expected.

The choice of parameters made along with the justification of the choices are enlisted below.

- $\lambda_4 > 0$, for obtaining the mass hierarchy, $m_{H^{\pm\pm}} < m_{H^\pm} < m_{A^0}, m_H$ which is in turn needed for the decay of $H^\pm$ to $H^{\pm\pm}$.
- $\lambda_1 < 0$ for constructive interference between the combined SM contribution (from W boson and fermion loops) and contribution from $H^{\pm\pm}$.
- We choose $m_{H^\pm} = 250$ GeV and $m_{H^{\pm\pm}} = 200$ GeV since it has already been mentioned in [27] that lower masses of $H^\pm$ and $H^{\pm\pm}$ give an enhancement to diphoton decay width with respect to the SM as compared to heavier charged scalars. Moreover at lower masses, the non-standard scalars are not degenerate. Degeneracy encroaches at higher masses of the scalar particles which is undesirable for the present numerical analysis.
- $v_\Delta = 0.1$ MeV and thus the vev of the doublet $v \sim 246$ GeV as constrained from the ρ parameter.

With reference to Eq. (6.34) - Eq. (6.36), $g_{ht\bar{t}}$, $g_{hb\bar{b}}$ and g_{hWW} are ~ 1 for $v_\Delta = 0.1$ MeV and $v \sim 246$ GeV as discussed.

Now, from Eq. (6.28),

$$\tau_W = \left(\frac{2m_W}{m_h}\right)^2 = 1.65 > 1 \ , \ \text{for } m_W = 80.385 \text{ GeV} \tag{6.46}$$

$$\tau_t = \left(\frac{2m_t}{m_h}\right)^2 = 7.66 > 1 \ , \ \text{for } m_t = 173.0 \text{ GeV} \tag{6.47}$$

$$\tau_b = \left(\frac{2m_b}{m_h}\right)^2 = 0.00447 < 1 \ , \ \text{for } m_b = 4.18 \text{ GeV} \tag{6.48}$$

$$\tau_{H^\pm} = \left(\frac{2m_{H^\pm}}{m_h}\right)^2 = 16 > 1 \ , \ \text{for } m_{H^\pm} = 250.0 \text{ GeV} \tag{6.49}$$

$$\tau_{H^{\pm\pm}} = \left(\frac{2m_{H^{\pm\pm}}}{m_h}\right)^2 = 10.24 > 1 \ , \ \text{for } m_{H^{\pm\pm}} = 200.0 \text{ GeV}. \tag{6.50}$$

Using $\frac{m_W}{g} = \frac{v}{2}$ and Eq. (6.40)- Eq. (6.43) we can rephrase the $hH^{++}H^{--}$ and hH^+H^- coupling as,

$$\begin{aligned}\tilde{g}_{hH^{++}H^{--}} &= -\frac{m_W}{gm^2_{H^{\pm\pm}}} g_{hH^{++}H^{--}} \\ &= -\frac{m_W}{gm^2_{H^{\pm\pm}}} \times (-\lambda_1 v) \\ &= \frac{v}{2m^2_{H^{\pm\pm}}} \times (\lambda_1 v) \\ &= \frac{\lambda_1}{2m^2_{H^{\pm\pm}}} v^2 \ .\end{aligned} \tag{6.51}$$

$$\begin{aligned}\tilde{g}_{hH^+H^-} &= -\frac{m_W}{gm^2_{H^\pm}} g_{hH^+H^-} \\ &= -\frac{m_W}{gm^2_{H^\pm}} \times \left(-\left(\lambda_1 + \frac{\lambda_4}{2}\right) v\right) \\ &= \frac{v}{2m^2_{H^\pm}} \times \left(\lambda_1 + \frac{\lambda_4}{2}\right) v \\ &= \frac{\left(\lambda_1 + \frac{\lambda_4}{2}\right)}{2m^2_{H^\pm}} v^2 \ .\end{aligned} \tag{6.52}$$

Now, that the platter is ready we rewrite the diphoton decay width as mentioned in Eq. (6.33) under the fact that h is the SM like Higgs.

$$\begin{aligned}\mu_{\gamma\gamma} &= \frac{\sigma\,(pp \to h)^{HTM} \times \Gamma\,(h \to \gamma\gamma)^{HTM}}{\sigma\,(pp \to h)^{SM} \times \Gamma\,(h \to \gamma\gamma)^{SM}} \\ &\approx \frac{\Gamma\,(h \to \gamma\gamma)^{HTM}}{\Gamma\,(h \to \gamma\gamma)^{SM}} \ \text{ since } \sigma\,(pp \to h)^{HTM} \sim \sigma\,(pp \to h)^{SM} \ .\end{aligned} \tag{6.53}$$

From eq.(Eq. (6.27)) we have,

$$\begin{aligned}
\Gamma\left(h\to\gamma\gamma\right)^{HTM} &= \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\Big|\sum_f N_c Q_f^2 g_{hff} A_{1/2}^h\left(\tau_f\right) + g_{hWW} A_1^h\left(\tau_W\right) \\
&\quad + \tilde{g}_{hH^\pm H^\mp} A_0^h\left(\tau_{H^\pm}\right) + 4\tilde{g}_{hH^{\pm\pm}H^{\mp\mp}} A_0^h\left(\tau_{H^{\pm\pm}}\right)\Big|^2 \qquad (6.54)\\
&= \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\Big|3\times\left(\frac{2}{3}\right)^2\times 1\times A_{1/2}^h\left(\tau_t\right) + 3\times\left(\frac{-1}{3}\right)^2\times 1\times A_{1/2}^h\left(\tau_b\right) \\
&\quad + 1\times A_1^h\left(\tau_W\right) + \frac{\left(\lambda_1+\frac{\lambda_4}{2}\right)}{2m_{H^\pm}^2}v^2 A_0^h\left(\tau_{H^\pm}\right) + 4\times\frac{\lambda_1}{2m_{H^{\pm\pm}}^2}v^2 A_0^h\left(\tau_{H^{\pm\pm}}\right)\Big|^2 \\
&= \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\Big|\frac{4}{3}A_{1/2}^h\left(\tau_t\right) + \frac{1}{3}A_{1/2}^h\left(\tau_b\right) + A_1^h\left(\tau_W\right) \\
&\quad + \frac{\left(\lambda_1+\frac{\lambda_4}{2}\right)}{2m_{H^\pm}^2}v^2 A_0^h\left(\tau_{H^\pm}\right) + 4\times\frac{\lambda_1}{2m_{H^{\pm\pm}}^2}v^2 A_0^h\left(\tau_{H^{\pm\pm}}\right)\Big|^2
\end{aligned}$$

$$\begin{aligned}
\text{and }\ \Gamma\left(h\to\gamma\gamma\right)^{SM} &= \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\Big|\sum_f N_c Q_f^2 g_{hff} A_{1/2}^h\left(\tau_f\right) + g_{hWW} A_1^h\left(\tau_W\right)\Big|^2 \qquad (6.55)\\
&= \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\Big|3\times\left(\frac{2}{3}\right)^2\times 1\times A_{1/2}^h\left(\tau_t\right) + 3\times\left(\frac{-1}{3}\right)^2\times 1\times A_{1/2}^h\left(\tau_b\right) \\
&\quad + 1\times A_1^h\left(\tau_W\right)\Big|^2 \\
&= \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\Big|\frac{4}{3}A_{1/2}^h\left(\tau_t\right) + \frac{1}{3}A_{1/2}^h\left(\tau_b\right) + A_1^h\left(\tau_W\right)\Big|^2 .
\end{aligned}$$

$$\therefore \mu_{\gamma\gamma} = \frac{\left|\ \frac{4}{3}A_{1/2}^h\left(\tau_t\right) + \frac{1}{3}A_{1/2}^h\left(\tau_b\right) + A_1^h\left(\tau_W\right) + \frac{\left(\lambda_1+\frac{\lambda_4}{2}\right)}{2m_{H^\pm}^2}v^2 A_0^h\left(\tau_{H^\pm}\right) + 4\times\frac{\lambda_1}{2m_{H^{\pm\pm}}^2}v^2 A_0^h\left(\tau_{H^{\pm\pm}}\right)\ \right|^2}{\left|\ \frac{4}{3}A_{1/2}^h\left(\tau_t\right) + \frac{1}{3}A_{1/2}^h\left(\tau_b\right) + A_1^h\left(\tau_W\right)\ \right|^2} . \qquad (6.56)$$

Next we consider the Higgs triplet model in the *Wrong Sign Limit*(WSL) where either of the fermionic couplings to up-type or down-type quarks of the SM like Higgs bears a relative negative sign as compared to the gauge couplings [31,39–41]. The Higgs diphoton decay width is written below along with the relative decay width in this limit.

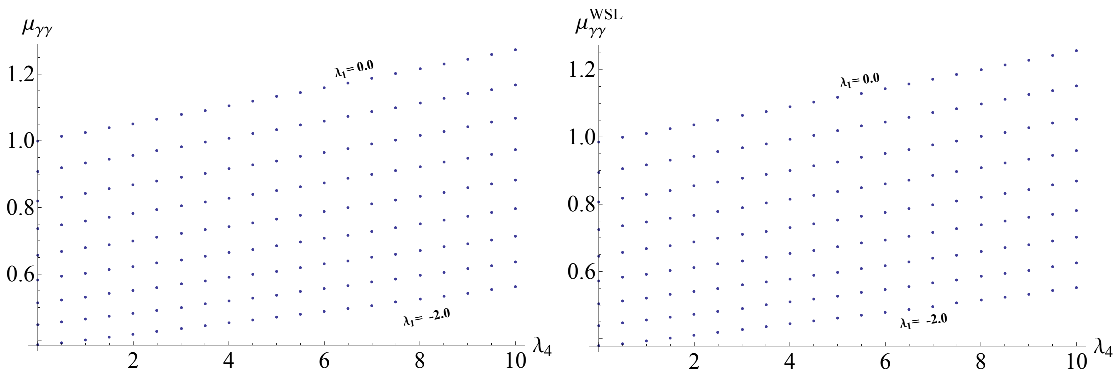
$$
\begin{aligned}
\Gamma\left(h\to\gamma\gamma\right)^{HTM}_{WSL} &= \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\Big|\sum_f N_c Q_f^2 g_{hff} A^h_{1/2}\left(\tau_f\right) + g_{hWW} A^h_1\left(\tau_W\right) \\
&\quad + \tilde{g}_{hH^\pm H^\mp} A^h_0\left(\tau_{H^\pm}\right) + 4\tilde{g}_{hH^{\pm\pm}H^{\mp\mp}} A^h_0\left(\tau_{H^{\pm\pm}}\right)\Big|^2 \qquad (6.57)\\
&= \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\Big|3\times\left(\frac{2}{3}\right)^2\times 1\times A^h_{1/2}\left(\tau_t\right) - 3\times\left(\frac{-1}{3}\right)^2\times 1\times A^h_{1/2}\left(\tau_b\right) \\
&\quad + 1\times A^h_1\left(\tau_W\right) + \frac{\left(\lambda_1+\frac{\lambda_4}{2}\right)}{2m^2_{H^\pm}} v^2 A^h_0\left(\tau_{H^\pm}\right) + 4\times\frac{\lambda_1}{2m^2_{H^{\pm\pm}}} v^2 A^h_0\left(\tau_{H^{\pm\pm}}\right)\Big|^2 \\
&= \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\Big|\frac{4}{3}A^h_{1/2}\left(\tau_t\right) - \frac{1}{3}A^h_{1/2}\left(\tau_b\right) + A^h_1\left(\tau_W\right) \\
&\quad + \frac{\left(\lambda_1+\frac{\lambda_4}{2}\right)}{2m^2_{H^\pm}} v^2 A^h_0\left(\tau_{H^\pm}\right) + 4\times\frac{\lambda_1}{2m^2_{H^{\pm\pm}}} v^2 A^h_0\left(\tau_{H^{\pm\pm}}\right)\Big|^2
\end{aligned}
$$

$$
\begin{aligned}
\text{and }\ \Gamma\left(h\to\gamma\gamma\right)^{SM} &= \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\Big|\sum_f N_c Q_f^2 g_{hff} A^h_{1/2}\left(\tau_f\right) + g_{hWW} A^h_1\left(\tau_W\right)\Big|^2 \qquad (6.58)\\
&= \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\ \Big|\ 3\times\left(\frac{2}{3}\right)^2\times 1\times A^h_{1/2}\left(\tau_t\right) + 3\times\left(\frac{-1}{3}\right)^2\times 1\times A^h_{1/2}\left(\tau_b\right) \\
&\quad + 1\times A^h_1\left(\tau_W\right)\Big|^2 \\
&= \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\Big|\frac{4}{3}A^h_{1/2}\left(\tau_t\right) + \frac{1}{3}A^h_{1/2}\left(\tau_b\right) + A^h_1\left(\tau_W\right)\Big|^2 .
\end{aligned}
$$

$$
\therefore \mu^{WSL}_{\gamma\gamma} = \frac{\Big|\ \frac{4}{3}A^h_{1/2}\left(\tau_t\right) - \frac{1}{3}A^h_{1/2}\left(\tau_b\right) + A^h_1\left(\tau_W\right) + \frac{\left(\lambda_1+\frac{\lambda_4}{2}\right)}{2m^2_{H^\pm}} v^2 A^h_0\left(\tau_{H^\pm}\right) + 4\times\frac{\lambda_1}{2m^2_{H^{\pm\pm}}} v^2 A^h_0\left(\tau_{H^{\pm\pm}}\right)\Big|^2}{\Big|\ \frac{4}{3}A^h_{1/2}\left(\tau_t\right) + \frac{1}{3}A^h_{1/2}\left(\tau_b\right) + A^h_1\left(\tau_W\right)\Big|^2} . \qquad (6.59)
$$

6.6 Results and Conclusion

In figure 6.1 $\mu_{\gamma\gamma}$ vs λ_4 and $\mu^{WSL}_{\gamma\gamma}$ vs λ_4 have been plotted for various values of λ_1. The quartic couplings λ_4 and λ_1 are responsible for $H^\pm$ to $H^{\pm\pm}$ decay and constructive contribution from $H^{\pm\pm}$ loop respectively and we wanted to study the variation of the relative diphoton decay width with these two quartic couplings. As we can see for any particular value of λ_1, $\mu_{\gamma\gamma}$ and $\mu^{WSL}_{\gamma\gamma}$ increases as λ_4 increases. This indicates that as the mass difference between $H^\pm$ and $H^{\pm\pm}$ increases ($m^2_{H^\pm} - m^2_{H^{\pm\pm}} = \frac{\lambda_4}{4}v^2$), relative diphoton decay width increases since the tendency for $H^\pm$ to decay to $H^{\pm\pm}$ increases. More the number of $H^{\pm\pm}$ produced more the increment since there is an enhancement factor of 4 which comes from the electric charge in case of $H^{\pm\pm}$'s contribution to $h\to\gamma\gamma$



(a) Same sign (b) Wrong sign

Figure 6.1: Diphoton decay width of the SM-like Higgs particle (normalized to SM) as a function of λ_4 for $m_{H^\pm} = 250$GeV, $m_{H^{\pm\pm}} = 200$GeV and various negative values of λ_1 , for (a) same sign and (b) wrong sign, of down-type Yukawa couplings in Higgs Triplet model.

as compared to that of $H^\pm$. Since the mass of the bottom quark is significantly smaller than the top quark mass therefore the the coupling of the bottom quark is also much smaller in magnitude compared to the top quark coupling with h. Thus the plots for the same sign and wrong sign of the Yukawa couplings are more or less same. As seen from the plots for both the same sign and the wrong sign of the Yukawa couplings there is an enhancement of $\sim 20\%$ for $\lambda_1 = 0$ and $\lambda_4 = 10$ where both the quartic couplings are well within the perturbative bounds.

The ratio of the diphoton decay width when the down type Yukawa coupling have *wrong sign* relative to the case with the *same sign* Yukawa coupling has been plotted in figure 6.2. We can easily point out that this ratio varies within a very narrow range and converges for higher values of λ_4.

In passing, we comment that although we have assumed $v_\Delta = 0.1$ MeV, the above conclusions do not crucially depend on the numerical value of v_Δ as long as it remains small.

Thus if tighter bounds are imposed on the discovery of $H^{\pm\pm}$ and $H^\pm$ via the four lepton($4l$) and three lepton($3l$) channels and their mass ranges be confined, then these non-standard scalars may account for the excess in the loop induced Higgs to diphoton decay. In that case the decay of $H^\pm$ to $H^{\pm\pm}$ must also be taken into account for the enhancement. This will in turn signal that the Higgs discovered at the LHC is a part of a richer scalar sector and not merely the Standard Model.

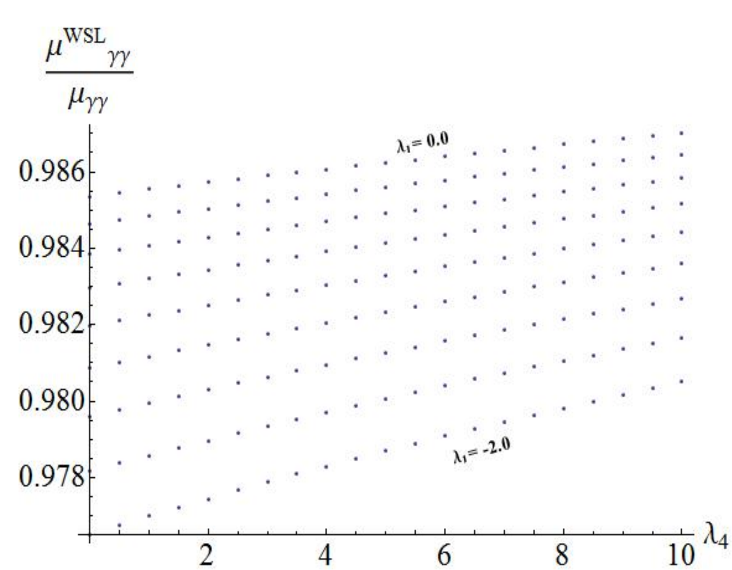


Figure 6.2: $h\gamma\gamma$ decay width for 'wrong sign' $h\bar{D}D$ coupling relative to the case with 'same sign' Yukawa couplings

References

[1] W. Konetschny and W. Kummer, Phys. Lett. B 70, 433 (1977).

[2] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[3] M. Magg and C. Wetterich, Phys. Lett. B 94, 61 (1980); G. Lazarides, Q. Shafi and C. Wet- terich, Nucl. Phys. B 181, 287 (1981).

[4] J. Schechter and J. W. F. Valle, Phys. Rev. D 22, 2227 (1980).

[5] T. P. Cheng and L. F. Li, Phys. Rev. D 22, 2860 (1980).

[6] **CMS** Collaboration, CMS PAS HIG-12-005 (March 2011).

[7] G. Aad et al. [**ATLAS** Collaboration], Phys. Rev. D 85, 032004 (2012).

[8] P. Dey, A. Kundu and B. Mukhopadhyaya, J. Phys. G 36, 025002 (2009).

[9] A. G. Akeroyd and C. W. Chiang, Phys. Rev. D 81, 115007 (2010).

[10] A. Arhrib, R. Benbrik, M. Chabab, G. Moultaka, M. C. Peyranere, L. Rahili and J. Ramadan, Phys. Rev. D 84, 095005 (2011).

[11] The Review of Particle Physics (2018), M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).

[12] The **ATLAS** and **CMS** Collaboration, *"Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at* $\sqrt{s}$ *= 7 and 8 TeV"*, ATLAS-CONF-2015-044 (2015).

[13] A. Arhrib, R. Benbrik, M. Chabab, G. Moultaka and L. Rahili, JHEP 1204, 136 (2012).

[14] A. G. Akeroyd and S. Moretti, *"Enhancement of* $H \to \gamma\gamma$ *from doubly charged scalars in the Higgs Triplet Model,"* arXiv:1206.0535v1[hep-ph].

[15] V. D. Barger, H. Baer, W. Y. Keung and R. J. N. Phillips, Phys. Rev. D 26, 218 (1982).

[16] J. F. Gunion, J. Grifols, A. Mendez, B. Kayser and F. I. Olness, Phys. Rev. D 40, 1546 (1989); J. F. Gunion, C. Loomis and K. T. Pitts, eConf C960625, LTH096 (1996) [arXiv:hep-ph/9610237].

[17] M. Muhlleitner and M. Spira, Phys. Rev. D 68, 117701 (2003).

[18] T. Han, B. Mukhopadhyaya, Z. Si and K. Wang, Phys. Rev. D 76, 075013 (2007).

[19] K. Huitu, J. Maalampi, A. Pietila and M. Raidal, Nucl. Phys. B 487, 27 (1997).

[20] B. Dion, T. Gregoire, D. London, L. Marleau and H. Nadeau, Phys. Rev. D 59, 075006 (1999).

[21] A. G. Akeroyd and M. Aoki, Phys. Rev. D 72, 035011 (2005).

[22] E. Ma, M. Raidal and U. Sarkar, Phys. Rev. Lett. 85, 3769 (2000); E. Ma, M. Raidal and U. Sarkar, Nucl. Phys. B 615, 313 (2001).

[23] E. J. Chun, K. Y. Lee and S. C. Park, Phys. Lett. B 566, 142 (2003).

[24] C. Bonilla, R. M. Fonseca, and J. W. F. Valle, *"Consistency of the triplet seesaw model revisited"*, Phys.Rev. D92 (2015), no. 7 075028, [arXiv:1508.02323].

[25] E. J. Chun, H. M. Lee, and P. Sharma, *"Vacuum Stability, Perturbativity, EWPD and Higgs-to-diphoton rate in Type II Seesaw Models"*, JHEP 11 (2012) 106, [arXiv:1209.1303].

[26] L. Lavoura and L. F. Li, *"Making the small oblique parameters large"*, Phys. Rev. D49 (1994) 1409–1416,[hep-ph/9309262].

[27] D. Das, A Santamaria, *"Updated scalar sector constraints in Higgs triplet model,"* [arXiv:1604.08099 [hep-ph]].

[28] A. Djouadi, V. Driesen, W. Hollik and J. I. Illana, Eur. Phys. J. C 1, 149 (1998); C. S. Huang and X. H. Wu, Phys. Rev. D 66, 075002 (2002); M. Carena, S. Gori, N. R. Shah and C. E. M. Wagner, JHEP 1203, 014 (2012).

[29] I. F. Ginzburg, M. Krawczyk and P. Osland, Nucl. Instrum. Meth. A 472, 149 (2001) [hep-ph/0101229]; L. Brucher and R. Santos, Eur. Phys. J. C 12, 87 (2000); A. G. Akeroyd, M. A. Diaz and M. A. Rivera, Phys. Rev. D 76, 115012 (2007); A. Arhrib, R. Benbrik, R. B. Guedes and R. Santos, Phys. Rev. D 78, 075002 (2008); N. Bernal, D. Lopez-Val and J. Sola, Phys. Lett. B 677, 39 (2009); P. Posch, Phys. Lett. B 696, 447 (2011).

[30] A. Arhrib, W. Hollik, S. Penaranda and M. Capdequi Peyranere, Phys. Lett. B 579, 361 (2004).

[31] A. Biswas and A. Lahiri *"Alignment, reverse alignment, and wrong sign Yukawa couplings in two Higgs doublet models,"* PHYSICAL REVIEW D 93, 115017 (2016), arXiv:1511.07159 [hep-ph]

[32] U. Ellwanger, JHEP 1203, 044 (2012); S. Moretti and S. Munir, Eur. Phys. J. C 47, 791 (2006).

[33] T. Han, H. E. Logan, B. McElrath and L. T. Wang, Phys. Lett. B 563, 191 (2003) [Erratum- ibid. B 603, 257 (2004)]; L. Wang and J. M. Yang, Phys. Rev. D 84, 075024 (2011).

[34] S. Kanemura and K. Yagyu, arXiv:1201.6287 [hep-ph].

[35] M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Sov. J. Nucl. Phys. 30, 711 (1979); J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *"The Higgs Hunter's Guide"*, Front. Phys. 80, 1 (2000); M. Spira, Fortsch. Phys. 46, 203 (1998); A. Djouadi, Phys. Rept. 459, 1 (2008).

[36] A. G. Akeroyd and S. Moretti, Phys. Rev. D 84,035028 (2011).

[37] A. Djouadi, J. Kalinowski and P. M. Zerwas, Z. Phys. C 70, 435 (1996).

[38] S. Moretti, W. J. Stirling, Phys. Lett. B347, 291-299 (1995).

[39] P. M. Ferreira, R. Guedes, J. F. Gunion, H. E. Haber, M. O. P. Sampaio, and R. Santos, *"The wrong sign limit in the 2HDM"*, arXiv:1410.1926.

[40] P. M. Ferreira, R. Guedes, M. O. P. Sampaio, and R. Santos, *"Wrong sign and symmetric limits and non-decoupling in 2HDMs"*, J. High Energy Phys. 12 (2014) 067.

[41] P. M. Ferreira, J. F. Gunion, H. E. Haber, and R. Santos, *"Probing wrong-sign Yukawa couplings at the LHC and a future linear collider"*, Phys. Rev. D 89, 115003 (2014).

Chapter 7

Cessation

> It is scientific only to say what's more likely or less likely and not to be proving all the time what's possible or impossible.
>
> *Richard Feynman*

Science has always been an interplay between theory and experiment. Building a theory and experimentally verifying it leads to unravelling the *Nature*. Only experiment can prove the validation of a theory. To this end scientists have been devising sophisticated experiments and manning its accuracy to higher levels in order to validate the theory precisely. In this process the success of a theory is never guaranteed. In science negative results particularly pave the way for a correct theory. Thus sometimes before the correct theory is formulated many theories have to bear the failed stigma. Nature is resting peacefully with all its phenomenon and man is devising new theories to be verified by experiments which could precisely explain the observables.

July, 2012 witnessed a remarkable discovery of the last building block of the celebrated Standard Model, the *Higgs* boson. This discovery seemed to complete the standard model but the precise measurements of its couplings with the other standard model particles are necessary to validate the *standardness* of the model. Thus aim of the future experiments were also set along with the discovery of this boson. As the experiments were run deviations from the SM prediction surfaced. As for instance preliminary data from ATLAS and CMS showed 2σ excess over the SM prediction for the Higgs diphoton decay rate. Thus as sophisticated experiments were devised, so were many new theories to explain the obtained set of data. Though the discovered Higgs turns out to be the SM Higgs particle still there are many unanswered questions that motivated us to embark on beyond SM physics. The major shortcomings of the SM are,

1. Neutrino mass: Neutrinos as predicted by the SM were massless. But neutrino oscillation experiments have predicted non-zero masses for them. Moreover neu-

trino mixing are very different from quark mixing and thus not explained by the SM.

2. Baryon asymmetry: The universe is supposed to contain equal quantities of matter and antimatter. But this isn't the case. There is way more matter than antimatter around us today. Where did all the antimatter go? Physicists are trying to find answer to this question believing that the universe was created with equal quantities of matter and antimatter however the laws of nature that subsequently came into effect were and are biased against antimatter for some reason. This problem did not find an appropriate answer within the framework of the standard model.

3. Dark matter: The existence of dark matter has been confirmed by many cosmological and astrophysical experiments but we cannot see it. This suggests that dark matter doesn't have any strong and electromagnetic interactions. Whereas nearly 27% of dark matter in the universe is suggestive of the fact that dark matter must have some other kind of interaction that is not predicted by the SM. We thus have to look beyond it to find an explanation for this unknown matter.

Speaking of BSM scenarios we have extended the horizon beyond the SM by adding one extra Higgs doublet to the SM theory. The resulting theory was consequently named as the two Higgs doublet model and have been discussed elaborately in Chapter 1. Different basis can be chosen to parametrize the Higgs doublets and one such basis is the Higgs-basis which potentially simplifies our calculations. The first chapter introduces this basis. Though we have worked with a CP-conserving model but we need to study the general 2HDM potential which allows possible sources of CP violation. If the vacuum expectation values and the parameters of the theory are so chosen that there is no mixing between the CP-odd and the CP-even states then CP violation can be avoided. In SM things are pretty simple since it has only one Higgs doublet. Flavour changing neutral currents are easily avoided. But with two Higgs doublets the scenario is a bit difficult since there are two independent Yukawa structures in each quark sector. But Glashow-Weinberg- Paschos (GWP) theorem came to the rescue by restricting the sources for the mass matrices of the fermions. Fermions with same charge and helicity couple to one of the doublets only. Thus as FCNCs were done away with we landed up classifying 2HDMs into four types. Nature advocates of many symmetries which make our lives easier. Following this trend 2HDM has also been imposed with many symmetries. Many authors impose the discrete Z_2 symmetry but we have built the model with the continuous $U(1)$ symmetry. Chapter 1 lastly recapitulates the previous work done in the framework of the 2HDMs.

A theory cannot flow on its own. It needs to be restricted by physical phenomenon and observations. Similarly the 2HDM scalar potential needs to be bounded from below. Moreover for a theory to be perturbative upto higher orders there are some upper bounds on the combinations of the quartic couplings of the 2HDM potential. New physics corrections impose further restrictions on the allowed parameter space.

Veltman had long ago gave a solution to ameliorate the fine tuning problem by setting the quadratically divergent Higgs self-energy terms to zero by some symmetry of the model. This naturally keeps the Higgs mass at the electroweak scale and is denoted as *Naturalness* criteria. Thus naturalness further restricts the choice of the parameter space. These restrictions have been discussed elaborately in Chapter 2 and Chapter 3. All these constraints combined were imposed on the 2HDMs in various limits of it to restrict the mass ranges for the non-standard Higgs bosons. Among the various limits are the well known alignment limit and the reverse alignment limit where the heavier CP even Higgs is considered to be the SM-like Higgs unlike the alignment limit where the lighter one is considered to play the role of the Higgs discovered at the LHC. Apart from the regular signs of the Yukawa couplings there is yet another scenario where one of the up-type or down-type Yukawa couplings is of opposite sign. 2HDMs were considered under this reversed sign of the Yukawa couplings in both the alignment and the reverse alignment limits. The obtained mass ranges were elaborately calculated and plotted in Chapter 4. In the process it was found that the reverse alignment limit was not compatible with Naturalness criteria. It is to be mentioned here that the quadratic divergences were calculated upto one loop and for higher loops there will be corrections to Veltman conditions though small. Thus the mass ranges predicted for one loop can be commented on as they are within certain percentage of tolerance.

The excess in the diphoton decay width can be tried to reason out for by considering the effect of the charged Higgs particle of the model in the loop induced decay. Theory advocates of a $\sim 6\%$ excess above the SM prediction.

The possibility of the formation of the $H-H$ bound state was studied in Chapter 5. A non-relativistic version of Higgs effective field theory was formulated and the relative strengths of the attractive and repulsive contact interactions were compared to see if a bound state was possible or not. It was found that for values of m_H on the higher end of the mass scale the attractive coupling was stronger than the repulsive coupling thus indicating a bound state. Again from the consideration of the formation and decay times of the bound state it was found that $H-H$ bound state will not form in the Type II and flipped 2HDMs, but may form in Type I and lepton specific models. But even that conclusion is not a strong one, as the range of parameters for bound state formation are at the edge of the allowed values.

Lastly another model the Higgs triplet model was discussed in the last chapter Chapter 6. In this model an $SU(2)$ triplet of scalars is present in addition to the SM particles. Thus there is a doubly charged Higgs particle in addition to the singly charged one that can also contribute to the loop induced diphoton decay width. Moreover the singly charged particle may decay to the doubly charged one when $m_{H^+} > m_{H^{++}}$ which occurs when $\lambda_1 < 0$ and $\lambda_4 > 0$, λ's being the quartic coupling constants of the theory. The variation of the relative diphoton decay width with λ_1 and λ_4 was studied in the *same* sign and *wrong* sign of the Yukawa couplings. For certain allowed range of the quartic couplings there was a substantial increase in the relative diphoton decay width in both cases.

Thus models are constructed beyond the SM with a motivation to suffice for the shortcomings of the SM. But still now not a single model has been formulated which has been able to provide a complete theory consistent with the observables. The models explain some of the experimental data obtained but fail to explain others. Sometimes it may appear that a particular model has explained all the experimental data but with the devising of higher and higher energy accelerators the pitfalls of these models surface. But does that mean that we theoreticians should discard our models in despair? The answer is no. In fact these innumerable failed or incomplete attempts are the stepping stones for formulating a universal theory of the Universe.

Science is not only a disciple of reason but also one of romance and passion.

- Stephen Hawking

Appendix A

Simultaneous diagonalisation

The necessary and sufficient conditions for the simultaneous diagonalisation of a set of matrices $P_1, \cdots, P_n$ were first discussed in [1–4].

Theorem 1 *For a set of $P_1, \cdots, P_n$ complex $m \times m$ matrices, unitary matrices L and R such that $L^\dagger P_i R$ is diagonal for all $i = 1, \cdots, n$ exist if and only if both sets*

$$\{P_i P_j^\dagger\}_{i,j=1,\cdots,n} \quad \text{and} \quad \{P_i^\dagger P_j\}_{i,j=1,\cdots,n} \tag{A.1}$$

are abelian, that is

$$\left[P_i P_j^\dagger, P_k P_l^\dagger\right] = 0 \quad \text{and} \quad \left[P_i^\dagger P_j, P_k^\dagger P_l\right] = 0, \;\; i,j,k,l = 1, \cdots, n\,. \tag{A.2}$$

It is now shown that a weaker requirement on the sets $\{P_i P_j^\dagger\}_{i,j=1,\cdots,n}$ and $\{P_i^\dagger P_j\}_{i,j=1,\cdots,n}$ is already necessary and sufficient.
Consider instead the sets,

$$\{P_1 P_1^\dagger, \cdots, P_n P_n^\dagger\}, \quad \{P_1^\dagger P_1, \cdots, P_n^\dagger P_n\}\,, \tag{A.3}$$

which is the subset of Eq. (A.1) with $i = j$. Each set in Eq. (A.3) has n hermitian elements rather than the n^2 elements in Eq. (A.1). If both sets in Eq. (A.3) are abelian then complete sets of orthonormal eigenvectors $\{\overrightarrow{u}_j\}$, $\{\overrightarrow{v}_j\}$, $j = 1, \cdots, m$ exist [5]. This is mathematically shown below,

$$\begin{aligned} & P_i P_i^\dagger \overrightarrow{u}_j = \lambda_{(i)j} \overrightarrow{u}_j, \quad P_i^\dagger P_i \overrightarrow{v}_j = \lambda_{(i)j} \overrightarrow{v}_j \\ \text{where,} \quad & \lambda_{(i)j} \in \mathbb{R}, \quad \lambda_{(i)j} \geq 0, \quad \overrightarrow{u}_i \cdot \overrightarrow{u}_j = \overrightarrow{v}_i \cdot \overrightarrow{v}_j = \delta_{ij}\,. \end{aligned} \tag{A.4}$$

The matrices in Eq. (A.3) are simultaneously diagonalised for $i = 1, \cdots, n$ as follows.

$$\begin{aligned} & U^\dagger P_i P_i^\dagger U = \text{diag}\left(\lambda_{(i)1}, \cdots, \lambda_{(i)m}\right) \\ \text{where,} \quad & U = \begin{pmatrix} \uparrow & \cdot & \uparrow \\ \overrightarrow{u}_1 & \cdot & \overrightarrow{u}_m \\ \downarrow & \cdot & \downarrow \end{pmatrix}, \end{aligned} \tag{A.5}$$

$$V^\dagger P_i^\dagger P_i V = \mathrm{diag}\left(\lambda_{(i)1}, \cdots, \lambda_{(i)m}\right) \tag{A.6}$$

$$\text{where,} \quad V = \begin{pmatrix} \uparrow & \cdot & \uparrow \\ \overrightarrow{v}_1 & \cdot & \overrightarrow{v}_m \\ \downarrow & \cdot & \downarrow \end{pmatrix},$$

and

$$U^\dagger P_i V = \mathrm{diag}\left(\sqrt{\lambda_{(i)1}}, \cdots, \sqrt{\lambda_{(i)m}}\right) . \tag{A.7}$$

With Eq. (A.7), it follows trivially that Eq. (A.2) is fulfilled. Thus the requirement that the sets in Eq. (A.3) are abelian implies that the larger sets in Eq. (A.1) are abelian too and is sufficient for the simultaneous bi-diagonalisability of $P_1, \cdots, P_n$.

References

[1] R. Gatto, G. Morchio, and F. Strocchi, *"Natural Flavor Conservation in the Neutral Currents and the Determination of the Cabibbo Angle",* Phys. Lett. 80B (1979) 265–268.

[2] R. Gatto, G. Morchio, G. Sartori, and F. Strocchi, *"Natural Flavor Conservation in Higgs Induced Neutral Currents and the Quark Mixing Angles",* Nucl. Phys. B163 (1980) 221–253.

[3] G. Sartori, *"Discrete Symmetries, Natural Flavor Conservation and Weak Mixing Angles",* Phys. Lett. 82B (1979) 255–259.

[4] W. Grimus and G. Ecker, *"On the Simultaneous Diagonalizability of Matrices",* J. Phys. A19 (1986) 3917.

[5] F. Gantmacher, *"The Theory of Matrices (Vol. I)",* American Mathematical Society (1977).

Appendix B

RGEs for 2HDMs

The two Higgs doublet model scalar potential is,

$$\begin{aligned} V\left(\Phi_1,\Phi_2\right) &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right) + \frac{1}{2}\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{1}{2}\lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 \\ & +\lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \frac{1}{2}\lambda_5\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right]. \quad \text{(B.1)} \end{aligned}$$

Haber and Hempfling in their paper [1] had calculated the one loop renormalization group equations for the quadratic parameters m_{ij}^2 and Branco et al. in their review of the two Higgs doublet models [2] had calculated the one loop RGEs for the gauge couplings, the Yukawa couplings and the quartic couplings, λ_i's. We define the beta function as,

$$\beta_x = 16\pi^2\frac{\partial x}{\partial \ln\mu}\,. \qquad \text{(B.2)}$$

The one loop RGEs for the various couplings are written below.

The beta functions for the $U(1)_Y$, $SU(2)_L$ and $SU(3)$ gauge couplings g, g' and g_s are,

$$\begin{aligned} \beta_g &= -3g^3\,, & \text{(B.3)} \\ \beta_{g'} &= 7g'^3\,, & \text{(B.4)} \\ \beta_{g_s} &= -7g_s^3\,. & \text{(B.5)} \end{aligned}$$

The starting values of the gauge couplings are,

$$\begin{aligned} g &= \frac{2m_{\text{W}}}{v}\,, & \text{(B.6)} \\ g' &= 2\frac{\sqrt{m_{\text{Z}}^2 - m_{\text{W}}^2}}{v}\,, & \text{(B.7)} \\ g_s &= \sqrt{4\pi\alpha_s}\,, & \text{(B.8)} \end{aligned}$$

where, $\alpha_s = g_s^2/(4\pi)$ is the strong coupling constant and v is the vacuum expectation value. The value for the strong coupling is $\alpha_s = 0.119$ [3–6].

With

$$\begin{aligned}
Y_u &= \frac{\sqrt{2}}{v_2}\begin{pmatrix} m_{\rm u} & 0 & 0 \\ 0 & m_{\rm c} & 0 \\ 0 & 0 & m_{\rm t} \end{pmatrix}, && \text{(B.9)}\\
Y_d &= \frac{\sqrt{2}}{v_d}V_{CKM}\begin{pmatrix} m_{\rm d} & 0 & 0 \\ 0 & m_{\rm s} & 0 \\ 0 & 0 & m_{\rm b} \end{pmatrix}V_{CKM}^\dagger, && \text{(B.10)}\\
Y_e &= \frac{\sqrt{2}}{v_e}\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, && \text{(B.11)}\\
V_{CKM} &= \mathbf{1}_{3\times 3}, && \text{(B.12)}
\end{aligned}$$

as the starting values of the Yukawa matrices we write below their beta functions for type I and type II 2HDMs.

For type I 2HDM,

$$\begin{aligned}
\beta_{Y_u} &= a_u Y_u + T_{22} Y_u - \frac{3}{2}\left(Y_d Y_d^\dagger - Y_u Y_u^\dagger\right) Y_u, && \text{(B.13)}\\
\beta_{Y_d} &= a_d Y_d + T_{22} Y_d + \frac{3}{2}\left(Y_d Y_d^\dagger - Y_u Y_u^\dagger\right) Y_d, && \text{(B.14)}\\
\beta_{Y_e} &= a_e Y_e + T_{22} Y_e + \frac{3}{2} Y_e Y_e^\dagger Y_e. && \text{(B.15)}
\end{aligned}$$

For type II 2HDM,

$$\begin{aligned}
\beta_{Y_u} &= a_u Y_u + T_{22} Y_u + \frac{1}{2}\left(Y_d Y_d^\dagger + 3 Y_u Y_u^\dagger\right) Y_u, && \text{(B.16)}\\
\beta_{Y_d} &= a_d Y_d + T_{11} Y_d + \frac{1}{2}\left(Y_u Y_u^\dagger + 3 Y_d Y_d^\dagger\right) Y_d, && \text{(B.17)}\\
\beta_{Y_e} &= a_e Y_e + T_{11} Y_e + \frac{3}{2} Y_e Y_e^\dagger Y_e, && \text{(B.18)}
\end{aligned}$$

with,

$$\begin{aligned}
a_d &= -8 g_s^2 - \frac{9}{4} g^2 - \frac{5}{12} g'^2, && \text{(B.19)}\\
a_u &= -8 g_s^2 - \frac{9}{4} g^2 - \frac{17}{12} g'^2, && \text{(B.20)}\\
a_e &= -\frac{9}{4} g^2 - \frac{15}{4} g'^2. && \text{(B.21)}
\end{aligned}$$

Now we define T_{11} and T_{22} for type I and type II 2HDMs.

For type I,

$$\begin{aligned} T_{11} &= 0\,, && \text{(B.22)}\\ T_{22} &= 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e\,, && \text{(B.23)} \end{aligned}$$

and for type II,

$$\begin{aligned} T_{11} &= 3Y_d^\dagger Y_d + Y_e^\dagger Y_e\,, && \text{(B.24)}\\ T_{22} &= 3Y_u^\dagger Y_u\,. && \text{(B.25)} \end{aligned}$$

For type I,

$$\begin{aligned} v_e &= v_2\,, && \text{(B.26)}\\ v_d &= v_2\,, && \text{(B.27)} \end{aligned}$$

and for type II,

$$\begin{aligned} v_e &= v_1\,, && \text{(B.28)}\\ v_d &= v_1\,. && \text{(B.29)} \end{aligned}$$

Now we proceed towards the RGEs for the quartic couplings of the 2HDM potential.

For type I 2HDM,

$$\begin{aligned} \beta_{\lambda_1} &= 12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + \frac{9}{4}g^4 + \frac{3}{2}g^2g'^2 + \frac{3}{4}g'^4 - 4\gamma_1\lambda_1\,, && \text{(B.30)}\\ \beta_{\lambda_2} &= 12\lambda_2^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + \frac{9}{4}g^4 + \frac{3}{2}g^2g'^2 + \frac{3}{4}g'^4 - 4\gamma_2\lambda_2\\ &\quad -12\,\mathrm{Tr}\left[Y_d^\dagger Y_d Y_d^\dagger Y_d + Y_u^\dagger Y_u Y_u^\dagger Y_u\right] - 4\,\mathrm{Tr}\left[Y_e^\dagger Y_e Y_e^\dagger Y_e\right]\,, && \text{(B.31)}\\ \beta_{\lambda_3} &= (\lambda_1+\lambda_2)(6\lambda_3+2\lambda_4) + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 + \frac{9}{4}g^4 - \frac{3}{2}g^2g'^2 + \frac{3}{4}g'^4\\ &\quad -2(\gamma_1+\gamma_2)\lambda_3\,, && \text{(B.32)}\\ \beta_{\lambda_4} &= 2(\lambda_1+\lambda_2)\lambda_4 + 8\lambda_3\lambda_4 + 4\lambda_4^2 + 8\lambda_5^2 + 3g^2g'^2 - 2(\gamma_1+\gamma_2)\lambda_4\,, && \text{(B.33)}\\ \beta_{\lambda_5} &= 2(\lambda_1+\lambda_2+4\lambda_3+6\lambda_4)\lambda_5 - 2(\gamma_1+\gamma_2)\lambda_5\,, && \text{(B.34)} \end{aligned}$$

and for type II,

$$\beta_{\lambda_1} = 12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + \frac{9}{4}g^4 + \frac{3}{2}g^2 g'^2 + \frac{3}{4}g'^4 - 4\gamma_1\lambda_1 - 12\,\mathrm{Tr}\left[Y_d^\dagger Y_d Y_d^\dagger Y_d\right] - 4\,\mathrm{Tr}\left[Y_e^\dagger Y_e Y_e^\dagger Y_e\right], \quad (B.35)$$

$$\beta_{\lambda_2} = 12\lambda_2^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + \frac{9}{4}g^4 + \frac{3}{2}g^2 g'^2 + \frac{3}{4}g'^4 - 4\gamma_2\lambda_2 - 12\,\mathrm{Tr}\left[Y_u^\dagger Y_u Y_u^\dagger Y_u\right], \quad (B.36)$$

$$\beta_{\lambda_3} = (\lambda_1 + \lambda_2)(6\lambda_3 + 2\lambda_4) + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 + \frac{9}{4}g^4 - \frac{3}{2}g^2 g'^2 + \frac{3}{4}g'^4 - 2(\gamma_1 + \gamma_2)\lambda_3 - 12\,\mathrm{Tr}\left[Y_d^\dagger Y_d Y_u^\dagger Y_u\right], \quad (B.37)$$

$$\beta_{\lambda_4} = 2(\lambda_1 + \lambda_2)\lambda_4 + 8\lambda_3\lambda_4 + 4\lambda_4^2 + 8\lambda_5^2 + 3g^2 g'^2 - 2(\gamma_1 + \gamma_2)\lambda_4 + 12\,\mathrm{Tr}\left[Y_d^\dagger Y_d Y_u^\dagger Y_u\right], \quad (B.38)$$

$$\beta_{\lambda_5} = 2(\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4)\lambda_5 - 2(\gamma_1 + \gamma_2)\lambda_5 . \quad (B.39)$$

Here γ_1 and γ_2 are defined as,

$$\gamma_1 = \frac{9}{4}g^2 + \frac{3}{4}g'^2 - T_{11}, \quad (B.40)$$

$$\gamma_2 = \frac{9}{4}g^2 + \frac{3}{4}g'^2 - T_{22}. \quad (B.41)$$

For the quadratic couplings, the RGEs are,

$$\beta_{m_{11}^2} = 6\lambda_1 m_{11}^2 + (4\lambda_3 + 2\lambda_4) m_{22}^2 - 2\gamma_1 m_{11}^2, \quad (B.42)$$

$$\beta_{m_{22}^2} = 6\lambda_2 m_{22}^2 + (4\lambda_3 + 2\lambda_4) m_{11}^2 - 2\gamma_2 m_{22}^2, \quad (B.43)$$

$$\beta_{m_{12}^2} = (2\lambda_3 + 4\lambda_4 + 6\lambda_5) m_{12}^2 - (\gamma_1 + \gamma_2) m_{12}^2. \quad (B.44)$$

The RGEs for the vacuum expectation values were calculated in [7, 8]. We write them below.

$$\beta_{v_1} = \gamma_1 v_1, \quad (B.45)$$

$$\beta_{v_2} = \gamma_2 v_2. \quad (B.46)$$

The vacuum expectation value is given by,

$$v = \frac{1}{\sqrt{\sqrt{2}G_F}}, \quad (B.47)$$

where,

$$G_F = 1.1663787 \times 10^{-5}\mathrm{GeV}^{-2}. \quad (B.48)$$

The fermion and the gauge boson masses as calculated in references [3–6] are,

$$m_u = 0.1\,\mathrm{GeV}\,, \tag{B.49}$$
$$m_c = 1.51\,\mathrm{GeV}\,, \tag{B.50}$$
$$m_t = 172.5\,\mathrm{GeV}\,, \tag{B.51}$$
$$m_d = 0.1\,\mathrm{GeV}\,, \tag{B.52}$$
$$m_s = 0.1\,\mathrm{GeV}\,, \tag{B.53}$$
$$m_b = 4.92\,\mathrm{GeV}\,, \tag{B.54}$$
$$m_e = 0.510998928 \times 10^{-3}\,\mathrm{GeV}\,, \tag{B.55}$$
$$m_\mu = 0.1056583715\,\mathrm{GeV}\,, \tag{B.56}$$
$$m_\tau = 1.77682\,\mathrm{GeV}\,, \tag{B.57}$$
$$m_{\mathrm{W}} = 80.385\,\mathrm{GeV}\,, \tag{B.58}$$
$$m_{\mathrm{Z}} = 91.1876\,\mathrm{GeV}\,. \tag{B.59}$$

Useful calculations on the evaluation of the beta functions of the various coupling constants were done by authors of [9, 10].

References

[1] H. E. Haber and R. Hempfling, Phys. Rev. D48, 4280 (1993), hep-ph/9307201.

[2] G. C. Branco et al., Phys. Rept. 516, 1 (2012), 1106.0034.

[3] Particle Data Group, K. A. Olive et al., Chin. Phys. C38, 090001 (2014).

[4] A. Denner et al., Lhchxswg-int-2015-006, https://cds.cern.ch/record/2047636.

[5] L. H. C. S. Group, https://twiki.cern.ch/twiki/bin/view/LHCPhysics/LHCHXSWG.

[6] LHC Higgs Cross Section Working Group, S. Dittmaier et al., (2011), 1101.0593.

[7] M. Sperling, D. Stöckinger, and A. Voigt, 1305.1548v3.

[8] M. Sperling, D. Stöckinger, and A. Voigt, 1310.7629v1.

[9] P Basler et al., *" High scale impact in alignment and decoupling in two-Higgs doublet models",* arXiv: 1710.10410v2 [hep-ph].

[10] F. Herren, L. Mihaila and M. Steinhauser, *"Gauge and Yukawa coupling beta functions of two Higgs doublet models of three-loop order",* arXiv: 1712.06614[hep-ph].

Appendix C

Contribution of Higgs towards the oblique electroweak parameters

h, H, A, $\xi^{\pm}$ are the physical Higgs bosons of the general two Higgs doublet models with masses denoted respectively by m_h, m_H, m_A and m_ξ. We consider h to be the SM-like Higgs boson and thus after subtracting the SM Higgs corrections to S, T and U with m_h as the reference point, the one loop Higgs contributions to S, T and U read as [1],

$$\begin{aligned} S' &= \frac{1}{\pi m_Z^2}\Big\{ \sin^2(\beta-\alpha)\,\mathcal{B}_{22}\left(m_Z^2; m_H^2, m_A^2\right) - \mathcal{B}_{22}\left(m_Z^2; m_\xi^2, m_\xi^2\right) \\ &\quad + \cos^2(\beta-\alpha)\Big[\mathcal{B}_{22}\left(m_Z^2; m_h^2, m_A^2\right) + \mathcal{B}_{22}\left(m_Z^2; m_Z^2, m_H^2\right) - \mathcal{B}_{22}\left(m_Z^2; m_Z^2, m_h^2\right) \\ &\quad - m_Z^2\mathcal{B}_0\left(m_Z^2; m_Z^2, m_H^2\right) + m_Z^2\mathcal{B}_0\left(m_Z^2; m_Z^2, m_h^2\right)\Big]\Big\}, \end{aligned} \tag{C.1}$$

$$\begin{aligned} T' &= \frac{1}{16\pi m_W^2 s_{\theta_W}^2}\Big\{ F\left(m_\xi^2, m_A^2\right) + \sin^2(\beta-\alpha)\left[F\left(m_\xi^2, m_H^2\right) - F\left(m_A^2, m_H^2\right)\right] \\ &\quad + \cos^2(\beta-\alpha)\Big[F\left(m_\xi^2, m_h^2\right) - F\left(m_A^2, m_h^2\right) + F\left(m_W^2, m_H^2\right) - F\left(m_W^2, m_h^2\right) \\ &\quad - F\left(m_Z^2, m_H^2\right) + F\left(m_Z^2, m_h^2\right) + 4m_Z^2\overline{B}_0\left(m_Z^2, m_H^2, m_h^2\right) \\ &\quad - 4m_W^2\overline{B}_0\left(m_W^2, m_H^2, m_h^2\right)\Big]\Big\}, \end{aligned} \tag{C.2}$$

$$\begin{aligned} U' &= -S' + \frac{1}{\pi m_Z^2}\Big\{\mathcal{B}_{22}\left(m_W^2; m_A^2, m_\xi^2\right) - 2\mathcal{B}_{22}\left(m_W^2; m_\xi^2, m_\xi^2\right) \\ &\quad + \sin^2(\beta-\alpha)\,\mathcal{B}_{22}\left(m_W^2; m_H^2, m_\xi^2\right) + \cos^2(\beta-\alpha)\Big[\mathcal{B}_{22}\left(m_W^2; m_h^2, m_\xi^2\right) \\ &\quad - \mathcal{B}_{22}\left(m_W^2; m_W^2, m_H^2\right) - \mathcal{B}_{22}\left(m_W^2; m_W^2, m_h^2\right) \\ &\quad - m_W^2\mathcal{B}_0\left(m_W^2; m_W^2, m_H^2\right) + m_W^2\mathcal{B}_0\left(m_W^2; m_W^2, m_h^2\right)\Big]\Big\}. \end{aligned} \tag{C.3}$$

He, Polonsky and Su have explicitly calculated the finite part of the $\mathcal{B}$- functions in their paper [2]. The results are quoted below. Before proceeding, two parameters x_1 and x_2 need to be defined as $x_1 \equiv \frac{m_1^2}{q^2}$ and $x_2 \equiv \frac{m_2^2}{q^2}$.

$$\mathcal{B}_0\left(q^2; m_1^2, m_2^2\right) = 1 + \frac{1}{2}\left[\frac{x_1+x_2}{x_1-x_2} - (x_1-x_2)\right]\ln\frac{x_1}{x_2} + \frac{1}{2}f\left(x_1,x_2\right)\,, \tag{C.4}$$

$$= 2 - 2\sqrt{4x_1-1}\arctan\frac{1}{\sqrt{4x_1-1}} \quad \text{when } m_1 = m_2\,, \tag{C.5}$$

$$\overline{B}_0\left(m_1^2, m_2^2, m_3^2\right) \equiv B_0\left(0, m_1^2, m_2^2\right) - B_0\left(0, m_1^2, m_3^2\right)$$

$$= \frac{m_1^2\ln m_1^2 - m_3^2\ln m_3^2}{m_1^2 - m_3^2} - \frac{m_1^2\ln m_1^2 - m_2^2\ln m_2^2}{m_1^2 - m_2^2}\,, \tag{C.6}$$

$$\begin{aligned}\mathcal{B}_{22}\left(q^2; m_1^2, m_2^2\right) &\equiv B_{22}\left(q^2, m_1^2, m_2^2\right) - B_{22}\left(0, m_1^2, m_2^2\right)\\ &= \frac{q^2}{24}\Big\{2\ln q^2 + \ln\left(x_1,x_2\right) + \Big[\left(x_1-x_2\right)^3 - 3\left(x_1^2 - x_2^2\right)\\ &+3\left(x_1-x_2\right)\Big]\ln\frac{x_1}{x_2} - \left[2\left(x_1-x_2\right)^2 - 8\left(x_1+x_2\right) + \frac{10}{3}\right]\\ &- \left[\left(x_1-x_2\right)^2 - 2\left(x_1+x_2\right) + 1\right] f\left(x_1,x_2\right) - 6F\left(x_1,x_2\right)\Big\}\end{aligned} \tag{C.7}$$

$$\mathcal{B}_{22}\left(q^2; m_1^2, m_1^2\right) \equiv \frac{q^2}{24}\left[2\ln q^2 + 2\ln x_1 + \left(16x_1 - \frac{10}{3}\right) + \left(4x_1 - 1\right)G\left(x_1\right)\right] \quad \text{when } m_1 = m_2\,. \tag{C.8}$$

The functions $F\left(x_1,x_2\right)$, $G\left(x\right)$ and $f\left(x_1,x_2\right)$ are defined below.

$$F\left(x_1,x_2\right) = \begin{cases} \frac{x_1+x_2}{2} - \frac{x_1x_2}{x_1-x_2}\ln\frac{x_1}{x_2}\,, & x_1 \neq x_2 \\ 0 & x_1 = x_2 \end{cases} \tag{C.9}$$

$$G\left(x\right) = -4\sqrt{4x-1}\arcsin\frac{1}{\sqrt{4x-1}}\,, \tag{C.10}$$

$$f\left(x_1,x_2\right) = \begin{cases} -2\sqrt{\Delta}\left[\arctan\frac{x_1-x_2+1}{\sqrt{\Delta}} - \arctan\frac{x_1-x_2-1}{\sqrt{\Delta}}\right]\,, & (\Delta > 0) \\ 0\,, & (\Delta = 0) \\ \sqrt{-\Delta}\ln\frac{x_1+x_2-1+\sqrt{-\Delta}}{x_1+x_2-1-\sqrt{-\Delta}}\,, & (\Delta < 0)\,, \end{cases} \tag{C.11}$$

where,

$$\Delta = 2\left(x_1+x_2\right) - \left(x_1-x_2\right)^2 - 1\,. \tag{C.12}$$

The expressions for S', T' and U' are simplified in the limit assuming $m_{\text{Higgs}}^2 \gg m_Z^2$ and are written below,

$$S' = \frac{1}{12\pi}\Bigg\{\cos^2(\beta-\alpha)\left[\ln\frac{m_H^2}{m_h^2} + g\left(m_h^2, m_A^2\right) - \ln\frac{m_\xi^2}{m_h m_A}\right] + \sin^2(\beta-\alpha)\left[g\left(m_H^2, m_A^2\right) - \ln\frac{m_\xi^2}{m_H m_A}\right]\Bigg\}, \quad \text{(C.13)}$$

$$T' = \frac{1}{16\pi s_{\theta_W}^2 m_W^2}\Big\{\cos^2(\beta-\alpha)\left[F\left(m_\xi^2, m_h^2\right) + F\left(m_\xi^2, m_A^2\right) - F\left(m_A^2, m_h^2\right)\right] + \sin^2(\beta-\alpha)\left[F\left(m_\xi^2, m_H^2\right) + F\left(m_\xi^2, m_A^2\right) - F\left(m_A^2, m_H^2\right)\right]\Big\}, \quad \text{(C.14)}$$

$$U' = \frac{1}{12\pi}\Big\{\cos^2(\beta-\alpha)\left[g\left(m_h^2, m_\xi^2\right) + g\left(m_A^2, m_\xi^2\right) - g\left(m_h^2, m_A^2\right)\right] + \sin^2(\beta-\alpha)\left[g\left(m_H^2, m_\xi^2\right) + g\left(m_A^2, m_\xi^2\right) - g\left(m_H^2, m_A^2\right)\right]\Big\}, \quad \text{(C.15)}$$

where,

$$g(x_1, x_2) = -\frac{5}{6} + \frac{2x_1x_2}{(x_1-x_2)^2} + \frac{(x_1+x_2)\left(x_1^2 - 4x_1x_2 + x_2^2\right)}{2(x_1-x_2)^3}\ln\frac{x_1}{x_2}\,. \quad \text{(C.16)}$$

In alignment limit when $\sin(\beta-\alpha) \sim 1$, S', T' and U' takes the form,

$$S' = \frac{1}{12\pi}\left(g\left(m_H^2, m_A^2\right) - \ln\frac{m_\xi^2}{m_H m_A}\right), \quad \text{(C.17)}$$

$$T = \frac{1}{16\pi\sin^2\theta_W M_W^2}\left[F(m_\xi^2, m_H^2) + F(m_\xi^2, m_A^2) - F(m_H^2, m_A^2)\right], \quad \text{(C.18)}$$

$$U' = \frac{1}{12\pi}\left(g\left(m_H^2, m_\xi^2\right) + g\left(m_A^2, m_\xi^2\right) - g\left(m_H^2, m_A^2\right)\right). \quad \text{(C.19)}$$

References

[1] H. E. Haber, hep-ph/9306207, presented at the Theoretical Advanced Study Institute(TASI 92), Boulder, CO, June, 1992; H. E. Haber and H. E. Logan, Phys. Rev. D 62, 015011 (2000), [hep-ph/9909335].

[2] H.-J. He, N. Polonsky and S. Su *"Extra families, Higgs Spectrum and Oblique Corrections"*, Phys.Rev.D64:053004,2001, hep-ph/9306207.

Appendix D

$h \to Z\gamma$ decay width

If we proceed analogously for the decay width of $h \to Z\gamma$ as we have done for $h \to \gamma\gamma$ in Section 4.4.1 we have,

$$\Gamma\left(h \to Z\gamma\right) = \frac{\alpha^2 g^2}{2^9 \pi^3} \frac{m_h^3}{M_W^2} \left|G_W + G_t + \chi G_\xi\right|^2 \left(1 - \frac{M_Z^2}{m_h^2}\right)^3 , \tag{D.1}$$

where again a new notation has been adopted as stated below.

$$\rho_x \equiv \left(2 m_x / M_Z\right)^2 . \tag{D.2}$$

The functions G_W, G_t and G_ξ are given by [1],

$$\begin{aligned} G_W &= \cot\theta_w \Bigg[4\left(\tan^2\theta_w - 3\right) I_2\left(\tau_W, \rho_W\right) \\ &+ \left(\left(5 + \frac{2}{\tau_W}\right) - \left(1 + \frac{2}{\tau_W}\right)\tan^2\theta_w\right) I_1\left(\tau_w, \rho_W\right)\Bigg] , \end{aligned} \tag{D.3}$$

$$G_t = \frac{4\left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_w\right)}{\sin\theta_w \cos\theta_w}\left[I_2\left(\tau_t, \rho_t\right) - I_1\left(\tau_t, \rho_t\right)\right] , \tag{D.4}$$

$$G_\xi = \frac{\left(2\sin^2\theta_w - 1\right)}{\sin\theta_w \cos\theta_w} I_1\left(\tau_\xi, \rho_\xi\right) . \tag{D.5}$$

The functions I_1 and I_2 are given by

$$I_1\left(\tau, \rho\right) = \frac{\tau\rho}{2\left(\tau - \rho\right)} + \frac{\tau^2\rho^2}{2\left(\tau - \rho\right)^2}\left[f\left(\tau\right) - f\left(\rho\right)\right] + \frac{\tau^2\rho}{\left(\tau - \rho\right)^2}\left[g\left(\tau\right) - g\left(\rho\right)\right] , \tag{D.6}$$

$$I_2\left(\tau, \rho\right) = -\frac{\tau\rho}{2\left(\tau - \rho\right)}\left[f\left(\tau\right) - f\left(\rho\right)\right] . \tag{D.7}$$

The function f is defined as,

$$f(\tau) = \begin{cases} \arcsin^2\sqrt{1/\tau} , & \tau \geq 1 \\ -\dfrac{1}{4}\left[\log\dfrac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} - i\pi\right]^2 , & \tau < 1 . \end{cases} \tag{D.8}$$

Since $\tau_x > 1$ and $\rho_x > 1$ for $x = W$, t and ξ the function g assumes the following form:

$$g\,(a) = \sqrt{a-1}\,\sin^{-1}(\sqrt{1/a})\,. \tag{D.9}$$

The parameter χ which carries the significance of the new physics contribution to the $h \to Z\gamma$ in Eq. (D.1) takes the below form in the alignment limit.

$$\chi = \frac{1}{m_\xi^2}\left(m_A^2 - m_\xi^2 - \frac{1}{2}m_h^2\right)\,. \tag{D.10}$$

References

[1] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *"The Higgs Hunter's Guide,"* Front. Phys. **80**, 1 (2000).

Appendix E

Custodial Symmetry

An approximate symmetry of the Standard model is the custodial symmetry that protect the value of the well known ρ parameter from large radiative corrections.

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W}\,, \tag{E.1}$$

m_W and m_Z being the masses of the W boson and Z boson respectively and θ_W being the Weinberg angle.

$\rho = 1$ for first order perturbation theory and is true for all orders of perturbation theory when custodial symmetry is exact.

If we write the SM Higgs doublet as,

$$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \tag{E.2}$$

then the SM scalar potential will solely depend on the scalar invariant $\Phi^\dagger\Phi = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2$. This automatically imposes the $SO(4)$ symmetry on the SM potential. $SO(4)$ is isomorphic to $SU(2) \times SU(2)$ which is larger than the SM gauge group $SU(2)_L \times U(1)_Y$. The scalar gauge kinetic terms specifically those involving the weak-hypercharge coupling g' and and the Yukawa terms linear in Φ do not abide by the $SO(4)$ symmetry. Thus $SO(4)$ is an approximate symmetry of the SM potential only and not the full Lagrangian and is also called the *custodial symmetry* [1, 2].

In general the 2HDM potential does not bear any $SO(4)$ symmetry and thus large radiative corrections to ρ is inevitable. If one wants to avoid them, one may impose custodial symmetry. Pomarol and Vega did considerable amount of work in this regard [3].

We write the two Higgs doublet fields as given in [4]

$$\phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix}, \qquad i = 1, 2. \tag{E.3}$$

Then $\epsilon\phi_i^\star$ are also two $SU(2)_L$ doublets with components

$$\epsilon\phi_i^\star = \begin{pmatrix} \phi_i^{0\star} \\ -\phi_i^- \end{pmatrix} , \tag{E.4}$$

where $\phi_i^- = \phi_i^{+\star}$. The Higgs bi-doublet fields are given by

$$\begin{aligned} \Phi_i &= \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon\phi_i^\star & \phi_i \end{pmatrix} , \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_i^{0\star} & \phi_i^+ \\ -\phi_i^- & \phi_i^0 \end{pmatrix} . \end{aligned} \tag{E.5}$$

The $SU(2)_L \times U(1)_Y$ gauge symmetry acts on the Higgs bi-doublets as

$$SU(2)_L \quad : \quad \Phi_i \to L\Phi_i \tag{E.6}$$

$$U(1)_Y \quad : \quad \Phi_i \to \Phi_i e^{-i\sigma_3\theta_i/2} . \tag{E.7}$$

In the limit that hyper charge vanishes, the Lagrangian also has the following global symmetry

$$SU(2)_R : \Phi_i \to \Phi_i R^\dagger \tag{E.8}$$

When the Higgs fields acquire their respective vacuum expectation values, both $SU(2)_L$ and $SU(2)_R$ are broken, however the subgroup $SU(2)_{L=R}$ is unbroken, i.e at $\langle \phi_i^0 \rangle = v_i$, one has

$$\langle \Phi_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_i^\star & 0 \\ 0 & v_i \end{pmatrix} . \tag{E.9}$$

The vacuum expectation values are chosen to be real, $v_i^\star = v_i$, then $\langle \Phi_i \rangle$ is proportional to the 2×2 identity matrix and the vacuum preserves a group $SU(2)_V$ (the V stands for "vectorial") corresponding to the identical matrices $SU(2)_{L=R}$ i.e,

$$L \langle \Phi_i \rangle L^\dagger = \langle \Phi_i \rangle \ . \tag{E.10}$$

This remaining group preserved by the vacuum is the custodial-symmetry group and the corresponding transformation of the Higgs bi-doublet under this group.

References

[1] P. Sikivie, L. Susskind, M. B. Voloshin, and V. I. Zakharov, Nucl. Phys. B 173 (1980) 189.

[2] M. S. Chanowitz and M. Golden, Phys. Lett. B 165 (1985) 105.

[3] A. Pomarol and R. Vega, Nucl. Phys. B 413 (1994) 3 [hep-ph/9305272].

[4] S. Willenbrock, (2004), hep-ph/0410370.

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Masses of physical scalars in two Higgs doublet models

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We find bounds on scalar masses resulting from a criterion of naturalness, in a broad class of two Higgs doublet models. Specifically, we assume the cancellation of quadratic divergences in what are called the type I, type II, lepton-specific, and flipped two Higgs doublet models, with an additional U(1) symmetry. This results in a set of relations among masses of the physical scalars and coupling constants, a generalization of the Veltman conditions of the standard model. Assuming that the lighter CP-even neutral Higgs particle is the observed scalar particle of mass ∼125 GeV, and imposing further the constraints from the electroweak T-parameter, stability, and perturbative unitarity, we calculate the range of the mass of each of the remaining physical scalars.

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I. INTRODUCTION

With the discovery of a 125 GeV neutral scalar boson [1,2], the menagerie of fundamental particles in the standard model appears to be complete. Some questions still remain unanswered, including the origins of neutrino mass and dark matter, keeping the door open for physics beyond the standard model. Among the simplest extensions of the standard model are two Higgs doublet models (2HDMs) (for a recent review see [3]). Originally motivated by supersymmetry, where a second Higgs doublet is essential, 2HDMs have also been studied in several other contexts. Peccei-Quinn symmetry [4,5] solves the strong CP problem, but must be spontaneously broken. The corresponding Goldstone boson is the axion, which can be a combination of the phases of two Higgs doublets. Models of baryogenesis often involve 2HDMs [6] because their mass spectrum can be adjusted to produce CP violation, both explicit and spontaneous. Another motivation, one that is important to us, is their use in models of dark matter [7–9]. These models are the inert doublet models, so called because one of the Higgs doublets does not couple to the fermions. Of the 2HDMs we will consider, the Yukawa couplings of one model (type I) approach the inert doublet model for large values of the ratio of the vacuum expectation values (VEVs) of the two Higgs fields. The other models also have small couplings to one or more types of fermions in that limit.

In this paper we consider 2HDMs with a softly broken global U(1) symmetry [4,10], with the parameters chosen so as to make the 2HDM "SM-like." We choose the fermion transformations under this U(1) symmetry, and impose a naturalness condition of vanishing quadratic divergences on the scalar sector of the models. Using additional restrictions coming from partial wave unitarity, vacuum stability, and the T parameter measuring "new physics," and assuming that the lighter CP-even Higgs particle in the 2HDMs is the one observed at the Large Hadron Collider (LHC), we find bounds on the masses of the additional scalar particles for each of the 2HDMs.

We will work with the scalar potential [11,12]

$$\begin{aligned} V = {}& \lambda_1\left(|\Phi_1|^2 - \frac{v_1^2}{2}\right)^2 + \lambda_2\left(|\Phi_2|^2 - \frac{v_2^2}{2}\right)^2 \\ & + \lambda_3\left(|\Phi_1|^2 + |\Phi_2|^2 - \frac{v_1^2+v_2^2}{2}\right)^2 \\ & + \lambda_4(|\Phi_1|^2|\Phi_2|^2 - |\Phi_1^\dagger\Phi_2|^2) \\ & + \lambda_5\left|\Phi_1^\dagger\Phi_2 - \frac{v_1 v_2}{2}\right|^2, \end{aligned} \tag{1.1}$$

with real λ_i. This potential is invariant under the symmetry $\Phi_1 \to e^{i\theta}\Phi_1$, $\Phi_2 \to \Phi_2$, except for a soft breaking term $\lambda_5 v_1 v_2 \Re(\Phi_1^\dagger\Phi_2)$. Additional dimension-4 terms, including one allowed by a softly broken Z_2 symmetry [13] are also set to zero by this U(1) symmetry.

The scalar doublets are parametrized as

$$\Phi_i = \begin{pmatrix} w_i^+(x) \\ \frac{v_i + h_i(x) + i z_i(x)}{\sqrt{2}} \end{pmatrix}, \qquad i = 1, 2 \tag{1.2}$$

where the VEVs v_i may be taken to be real and positive without any loss of generality. Three of these fields get "eaten" by the $W^\pm$ and Z^0 gauge bosons; the remaining five are physical scalar (Higgs) fields. There is a pair of charged scalars denoted by $\xi^\pm$, two neutral CP even scalars H and h, and one CP odd pseudoscalar denoted by A. With

$$\tan\beta = \frac{v_2}{v_1}, \tag{1.3}$$

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these fields are given by the combinations

$$\begin{pmatrix} \omega^\pm \\ \xi^\pm \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} w_1^\pm \\ w_2^\pm \end{pmatrix}, \qquad (1.4)$$

$$\begin{pmatrix} \zeta \\ A \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \qquad (1.5)$$

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \qquad (1.6)$$

where $c_\alpha \equiv \cos\alpha$, etc.

If we rotated $h_1 - h_2$ fields by the angle β,

$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \qquad (1.7)$$

we would find that H^0 has exactly the standard model Higgs couplings with the fermions and gauge bosons [14,15]. The physical scalar h is related to H^0 and R via

$$h = \sin(\beta - \alpha) H^0 + \cos(\beta - \alpha) R. \qquad (1.8)$$

Thus in order for h to be the Higgs boson of the standard model, we require $\sin(\beta - \alpha) \approx 1$, which has been called the SM-like or alignment limit [16]. Accordingly, we will assume $\beta - \alpha = \frac{\pi}{2}$ in the rest of this paper.

II. VELTMAN CONDITIONS

The scalar masses get quadratically divergent contributions which require a fine-tuning of parameters. We thus impose naturalness conditions, a generalization of the Veltman conditions for the standard model, that these contributions cancel [17]. The resulting masses and couplings should not then require fine-tuning.

The Yukawa potential for the 2HDMs is of the form

$$\mathcal{L}_Y = \sum_{i=1,2} [-\bar{l}_L \Phi_i G_e^i e_R - \bar{Q}_L \tilde{\Phi}_i G_u^i u_R - \bar{Q}_L \Phi_i G_d^i d_R + \text{H.c.}], \qquad (2.1)$$

where l_L, Q_L are 3-vectors of isodoublets in the space of generations, e_R, u_R, d_R are 3-vectors of singlets, G_e^1, etc. are complex 3×3 matrices in generation space containing the Yukawa coupling constants, and $\tilde{\Phi}_i = i\tau_2 \Phi_i^*$.

Cancellation of quadratic divergences in the scalar masses gives rise to four mass relations, which we may call the Veltman conditions for the 2HDMs being considered [18],

$$2\text{Tr}G_e^1 G_e^{1\dagger} + 6\text{Tr}G_u^{1\dagger} G_u^1 + 6\text{Tr}G_d^1 G_d^{1\dagger} = \frac{9}{4} g^2 + \frac{3}{4} g'^2 + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5, \qquad (2.2)$$

$$2\text{Tr}G_e^2 G_e^{2\dagger} + 6\text{Tr}G_u^{2\dagger} G_u^2 + 6\text{Tr}G_d^2 G_d^{2\dagger} = \frac{9}{4} g^2 + \frac{3}{4} g'^2 + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5, \qquad (2.3)$$

$$2\text{Tr}G_e^1 G_e^{2\dagger} + 6\text{Tr}G_u^{1\dagger} G_u^2 + 6\text{Tr}G_d^1 G_d^{2\dagger} = 0, \qquad (2.4)$$

where g, g' are the $SU(2)$ and $U(1)_Y$ coupling constants. A fourth equation is the complex conjugate of the third one. As we will see below, the last equation vanishes identically for all the 2HDMs we consider. The mass relations come from the first two equations above.

When the fermions are in mass eigenstates, the Yukawa matrices are automatically diagonal if there is only one Higgs doublet as in the standard model, so there is no flavor changing neutral current (FCNC) at the tree level. But in the presence of a second scalar doublet, the two Yukawa matrices will not be simultaneously diagonalizable in general, and thus the Yukawa couplings will not be flavor diagonal. Neutral Higgs scalars will mediate FCNCs. The necessary and sufficient condition for the absence of FCNC at tree level is that all fermions of a given charge and helicity transform according to the same irreducible representation of SU(2), corresponding to the same eigenvalue of T_3, and that a basis exists in which they receive their contributions in the mass matrix from a single source [19,20].

For the fermions of the standard model, this theorem implies that all right-handed singlets of a given charge must couple to the same Higgs doublet. We will ensure this using the global U(1) symmetry mentioned earlier, which generalizes a Z_2 symmetry more commonly employed for this purpose. The left-handed fermion doublets remain unchanged under this symmetry, $Q_L \to Q_L$, $l_L \to l_L$. The transformations of right-handed fermion singlets determine the type of 2HDM. There are four such possibilities, which may be identified by the right-handed fields which transform under the U(1): type I (none), type II ($d_R \to e^{-i\theta} d_R, e_R \to e^{-i\theta} e_R$), lepton specific ($e_R \to e^{-i\theta} e_R$), flipped ($d_R \to e^{-i\theta} d_R$). We note in passing that another way of avoiding FCNCs at tree level is by aligning the Yukawa and mass matrices in flavor space

TABLE I. Veltman conditions for the different 2HDMs.

Model	Zero Yukawa	VC1	VC2
		$6M_W^2+3M_Z^2+5m_h^2+2m_\xi^2$ $+m_H^2(3\tan^2\beta-2)-\frac{3v^2}{2}\lambda_5\tan^2\beta=$	$6M_W^2+3M_Z^2+5m_h^2+2m_\xi^2$ $+m_H^2(3\cot^2\beta-2)-\frac{3v^2}{2}\lambda_5\cot^2\beta=$
Type I	G_{1e}, G_{1d}, G_{1u}	0	$4[\sum m_e^2+3\sum m_u^2+3\sum m_d^2]\csc^2\beta$
Type II	G_{2e}, G_{2d}, G_{1u}	$4[\sum m_e^2+3\sum m_d^2]\sec^2\beta$	$12\sum m_u^2\csc^2\beta$
LS	G_{2e}, G_{1d}, G_{1u}	$4\sum m_e^2\sec^2\beta$	$12[\sum m_u^2+\sum m_d^2]\csc^2\beta$
Flipped	G_{1e}, G_{2d}, G_{1u}	$12\sum m_d^2\sec^2\beta$	$4[\sum m_e^2+3\sum m_u^2]\csc^2\beta$

[21]. However, only these four 2HDMs admit symmetries such as the U(1) [22].

The fermion mass matrix is diagonalized by independent unitary transformations on the left and right-handed fermion fields. In any of the 2HDMs, either G_{1f} or G_{2f} vanish for each fermion type f. For example, in the Type II model Φ_1 couples to down-type quarks and charged leptons, while Φ_2 couples to up-type quarks, so $G_{2e}=G_{2d}=G_{1u}=0$. Thus Eq. (2.4) is automatically satisfied in each 2HDM. The nonvanishing Yukawa matrices are related to the fermion masses by [18]

$$\mathrm{Tr}[G_{1f}^\dagger G_{1f}]=\frac{2}{v^2\cos^2\beta}\sum m_f^2, \tag{2.5}$$

$$\mathrm{Tr}[G_{2f}^\dagger G_{2f}]=\frac{2}{v^2\sin^2\beta}\sum m_f^2, \tag{2.6}$$

where f stands for charged leptons, up-type quarks, or down-type quarks, and the sum is taken over generations.

In order to rewrite the Veltman conditions in terms of the known masses, we first note that in the alignment limit and with the global U(1) symmetry, the independent parameters in the scalar potential may be taken to be the masses m_h, m_H, m_ξ, the angle β, the electroweak VEV $v=\sqrt{v_1^2+v_2^2}$, and the constant λ_5. The λ_i are related to these parameters by [23]

$$\lambda_1=\frac{1}{2v^2c_\beta^2}m_H^2-\frac{\lambda_5}{4}(\tan^2\beta-1), \tag{2.7}$$

$$\lambda_2=\frac{1}{2v^2s_\beta^2}m_H^2-\frac{\lambda_5}{4}\left(\frac{1}{\tan^2\beta}-1\right), \tag{2.8}$$

$$\lambda_3=-\frac{1}{2v^2}(m_H^2-m_h^2)-\frac{\lambda_5}{4}, \tag{2.9}$$

$$\lambda_4=\frac{2}{v^2}m_\xi^2,\qquad \lambda_5=\frac{2}{v^2}m_A^2. \tag{2.10}$$

Inserting Eq. (2.5)–Eq. (2.10) into Eq. (2.2) and Eq. (2.3), we get the Veltman conditions in terms of the physical particle masses. These are shown in Table I. The Yukawa matrices which vanish in each model are listed in the second column. We note here that although naturalness conditions in specific 2HDMs have been studied earlier on a few occasions [24,25], they were not done in the SM-like scenario, nor expressed in terms of the physical masses for the different types as in here.

III. BOUNDS ON THE MASSES OF HEAVY AND CHARGED SCALARS

We now display our main results, the bounds we have obtained for the masses of the heavy and charged Higgs particles. We will assume that the h particle is the one that has been observed at the LHC, so that $m_h=125$ GeV, and $v=246$ GeV. Let us consider the example of the type II model to explain our derivation of the bounds.

Since we want the bounds on m_H and m_ξ, let us rewrite VC1 and VC2 for the type II model in a convenient form,

$$m_H^2(3\tan^2\beta-2)+2m_\xi^2=4\left[\sum m_e^2+3\sum m_d^2\right]\sec^2\beta-6M_W^2-3M_Z^2-5m_h^2+\lambda_5\frac{3v^2}{2}\tan^2\beta, \tag{3.1}$$

$$m_H^2(3\cot^2\beta-2)+2m_\xi^2=12\sum m_u^2\csc^2\beta-6M_W^2-3M_Z^2-5m_h^2+\lambda_5\frac{3v^2}{2}\cot^2\beta. \tag{3.2}$$

On the right-hand side of either equation, all but the last term are experimentally known. The $U(1)$ symmetry implies that $\lambda_5>0$, and we impose the restriction of $|\lambda_i|\le 4\pi$ based on the validity of perturbativity. Comparing with Eq. (2.10), we see that this last puts a restriction $m_A\lesssim 617$ GeV.

For a fixed value of $\tan\beta$, we plot both equations on the m_H-m_ξ plane for various values of λ_5. The point where the two curves cross for a given value of λ_5, is an allowed value of the pair (m_H, m_ξ).

We can restrict the allowed range of the masses even further by imposing constraints coming from stability, perturbative unitarity, and the oblique electroweak T-parameter. Conditions for stability, i.e., for the scalar potential being bounded from below, were examined in [3,15,26], and found to provide lower bounds on certain combinations of the quartic couplings λ_i. On the other hand, the requirement of perturbative unitarity translates into upper limits on combinations of the λ_i, which for two-Higgs models have been derived by many authors [23,27–29]. One condition coming from perturbative unitarity is

$$|3(\lambda_1+\lambda_2+2\lambda_3) \pm \sqrt{9(\lambda_1-\lambda_2)^2+(4\lambda_3+\lambda_4+\lambda_5)^2}| \leq 16\pi. \tag{3.3}$$

Stability provides the inequalities

$$\lambda_1+\lambda_3 > 0, \qquad \lambda_2+\lambda_3 > 0, \tag{3.4}$$

so that we can write Eq. (3.3) as $|A \pm B| \leq 16\pi$, with $A, B \geq 0$. It then follows that

$$0 \leq \lambda_1+\lambda_2+2\lambda_3 \leq \frac{16\pi}{3}. \tag{3.5}$$

In terms of the scalar masses, this reads

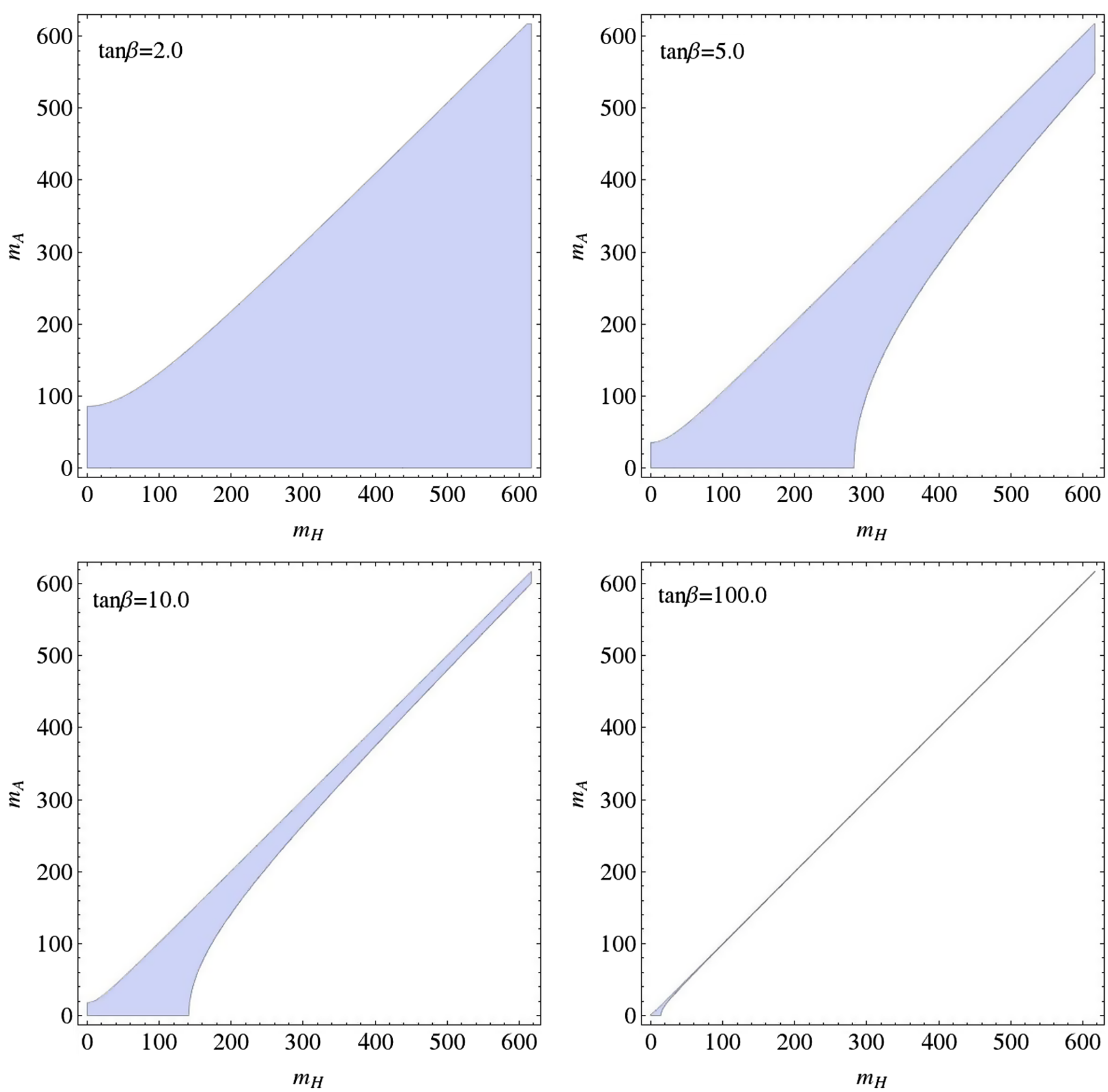

FIG. 1 (color online). Degeneracy of $m_H - m_A$ (in GeV) for progressively increasing $\tan\beta$. The condition $|\lambda_i| \leq 4\pi$ restricts $m_A \lesssim 617$ GeV.

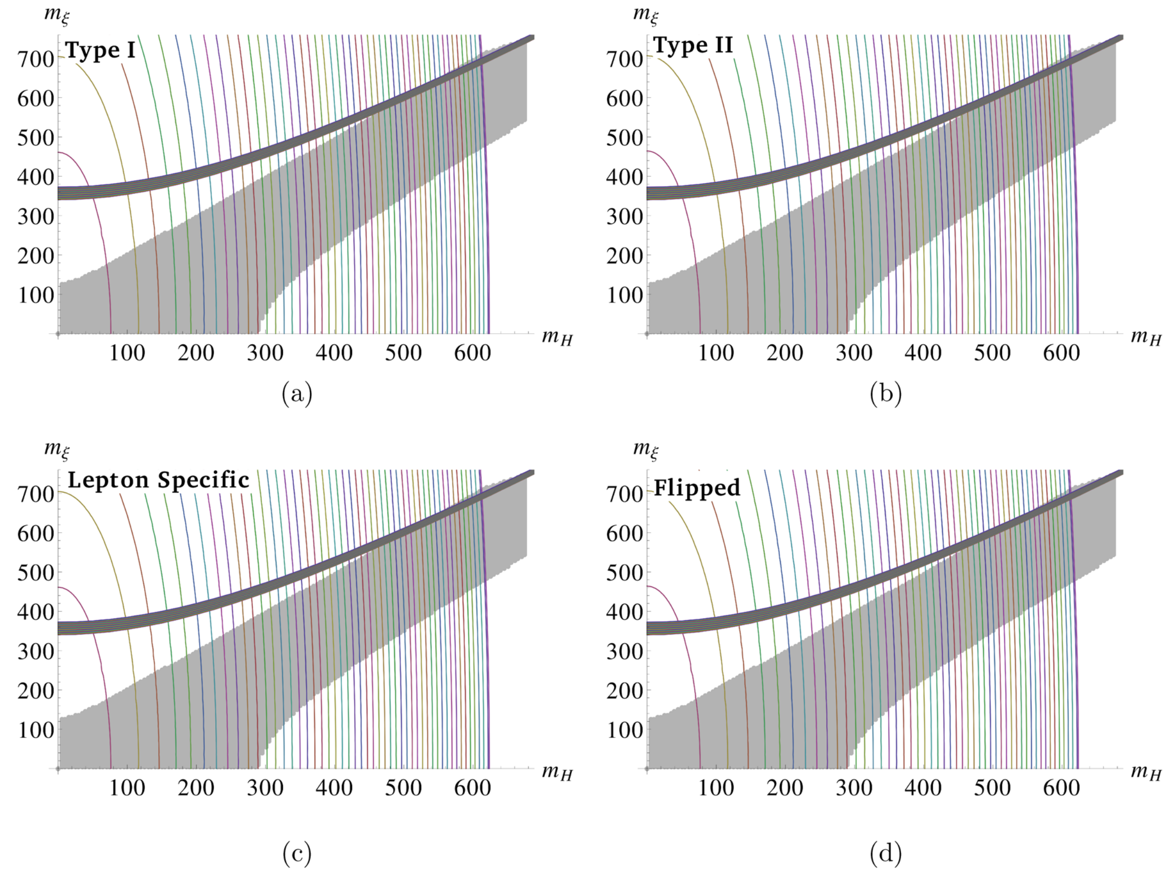

FIG. 2 (color online). Allowed mass range (in GeV) for the charged Higgs and the heavy *CP* even Higgs in (a) type I (b) type II (c) lepton specific and (d) flipped 2HDM for $|\lambda_5| \le 4\pi$ and $\tan\beta = 5$.

$$0 < (m_H^2 - m_A^2)(\tan^2\beta + \cot^2\beta) + 2m_h^2 < \frac{32\pi v^2}{3}. \tag{3.6}$$

For $\tan\beta \gg 1$, this inequality implies that m_H and m_A are almost degenerate, a result also found in [30]. In Fig. 1 we have shown this degeneracy by plotting m_A against m_H for different values of $\tan\beta$. It is easy to see from the plots that the degeneracy is more pronounced at higher values of m_A for any value of $\tan\beta$. For these plots we have used the perturbativity condition $|\lambda_i| \le 4\pi$, which restricts $m_A \lesssim 617$ GeV.

We will also need another inequality which follows from the condition

$$|2\lambda_3 + \lambda_4| \le 16\pi \tag{3.7}$$

required for perturbative unitarity. Substituting the mass relations Eq. (2.9) and (2.10) into this, we get

$$|2m_\xi^2 - m_H^2 - m_A^2 + m_h^2| \le 16\pi v^2. \tag{3.8}$$

Next we take into account the oblique parameter T for the 2HDMs, which has the expression [31,32]

$$T = \frac{1}{16\pi \sin^2\theta_W M_W^2}[F(m_\xi^2, m_H^2) + F(m_\xi^2, m_A^2) - F(m_H^2, m_A^2)], \tag{3.9}$$

with

$$F(x,y) = \begin{cases} \frac{x+y}{2} - \frac{xy}{x-y}\ln\frac{x}{y}, & x \ne y \\ 0 & x = y. \end{cases} \tag{3.10}$$

The T parameter is constrained by the global fit to precision electroweak data to be [33]

$$T = 0.05 \pm 0.12. \tag{3.11}$$

Our results consist of the pairs (m_H, m_ξ) for each type of 2HDM, satisfying the two Veltman conditions, and consistent with the constraints from stability, tree-level unitarity and the T parameter. For $\tan\beta = 5$, we have plotted the $m_H - m_\xi$ curves corresponding to VC1 and VC2 for several values of λ_5. These have been superimposed on the bound determined by (3.6), (3.8), and (3.11). The resulting plot is shown in Fig. 2. VC1 produces ellipses, and VC2 gives a narrow band of hyperbolas. Their crossings which fall inside the band representing the bound from the inequalities are the allowed masses. From the plot we can estimate the individual bounds: for all four models, we find approximately $550\ \mathrm{GeV} \lesssim m_\xi \lesssim 700\ \mathrm{GeV}$, and about $450\ \mathrm{GeV} \lesssim m_H \lesssim 620\ \mathrm{GeV}$, with a higher m_H implying a higher m_ξ. As mentioned earlier, m_A is close to m_H as a result of (3.6). We also note that direct searches have put a rough lower bound of $m_\xi > 100\ \mathrm{GeV}$ [34].

IV. DISCUSSION

Some comments are in order for the values of some parameters that we have used in this analysis. We chose $\beta - \alpha = \frac{\pi}{2}$ so that the 2HDMs are in the alignment limit, in which the lighter *CP*-even scalar h has the couplings of the Higgs particle of the standard model. We note that in the decoupling limit [15] defined by $m_A^2 \gg |\lambda_i| v^2$ subject to a condition of perturbativity $|\lambda_i| \lesssim 4\pi$, we also find $\sin(\beta - \alpha) \approx 1$. (The relation between these λ_i and ours may be found in [15].) Although we find from our computations in this paper that m_A must be large, we do not require it *a priori*, so our results are valid for the SM-like alignment limit of the 2HDMs, without going to the decoupling limit. It is worth pointing out that the issue of distinguishing between the decoupling limit and the SM-like scenario was first explored in [35].

Perturbativity requires that the quartic couplings of the physical Higgs fields are small. Our choice of $|\lambda_i| \leq 4\pi$ keeps the models inside the perturbative regime, and this requirement also keeps $m_A \lesssim 617\ \mathrm{GeV}$. Allowing for larger values of λ_i would also allow higher values of m_A as well as of m_H and m_ξ. In that sense, what we have found in this paper are the lower bounds on the masses of H, A, and $\xi^\pm$, in the SM-like limit of 2HDMs.

The most important parameter in the 2HDMs is $\tan\beta$. There is no consensus on the value of $\tan\beta$, except that it should be larger than unity, based on constraints coming from $Z \to b\bar{b}$ and $B_q \bar{B}_q$ mixing [36]. Several arguments have been proffered for a large $\tan\beta$ in 2HDMs of different types, using muon $g-2$ in lepton specific 2HDM [37], or using $b \to s\gamma$ in type I and flipped models [38], which also suppresses the $t \to bH^+$ branching ratio to a rough agreement with 95% CL limits from the light charged Higgs searches at the LHC [39,40]. A large value of $\tan\beta$ also makes the heavy Higgs particle difficult to detect [41]. We have used a conservative $\tan\beta = 5$ to estimate the scalar masses m_H and $m_{\xi^\pm}$—note that m_A is not very far from m_H because of the degeneracy relation (3.6). A larger $\tan\beta$ makes the $m_H - m_A$ degeneracy more pronounced, so the inequality band becomes narrower. This narrows the ranges of m_H and m_ξ, also pushing the region of overlap upwards, making the heavy and charged Higgses more difficult to detect. Recent analyses of LHC data at $\sqrt{s} = 8$ TeV, as a search for the pseudoscalar Higgs particle, also appear to favor a value of 5 or larger for $\tan\beta$ near the alignment limit [42,43].

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[1] G. Aad *et al.* (ATLAS Collaboration), Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B **716**, 1 (2012).
[2] S. Chatrchyan *et al.* (CMS Collaboration), Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys. Lett. B **716**, 30 (2012).
[3] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, Theory and phenomenology of two-Higgs-doublet models, Phys. Rep. **516**, 1 (2012).
[4] R. D. Peccei and H. R. Quinn, *CP* Conservation in the Presence of Instantons, Phys. Rev. Lett. **38**, 1440 (1977).
[5] R. D. Peccei and H. R. Quinn, Constraints imposed by *CP* conservation in the presence of instantons, Phys. Rev. D **16**, 1791 (1977).
[6] N. Turok and J. Zadrozny, Electroweak baryogenesis in the two doublet model, Nucl. Phys. **B358**, 471 (1991).
[7] E. Ma, Verifiable radiative seesaw mechanism of neutrino mass and dark matter, Phys. Rev. D **73**, 077301 (2006).
[8] E. Ma, Utility of a special second scalar doublet, Mod. Phys. Lett. A **23**, 647 (2008).
[9] R. Barbieri, L. J. Hall, and V. S. Rychkov, Improved naturalness with a heavy Higgs: An alternative road to LHC physics, Phys. Rev. D **74**, 015007 (2006).
[10] P. M. Ferreira and D. R. T. Jones, Bounds on scalar masses in two Higgs doublet models, J. High Energy Phys. 08 (2009) 069.
[11] T. D. Lee, A theory of spontaneous T violation, Phys. Rev. D **8**, 1226 (1973).
[12] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, *The Higgs Hunter's Guide*, Frontiers in Physics Vol. 80 (Perseus Publishing, Cambridge, 2000).
[13] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, The Higgs hunter's guide, arXiv:hep-ph/9302272.

[14] G. C. Branco, W. Grimus, and L. Lavoura, Relating the scalar flavor changing neutral couplings to the CKM matrix, Phys. Lett. B **380**, 119 (1996).
[15] J. F. Gunion and H. E. Haber, The *CP* conserving two Higgs doublet model: The approach to the decoupling limit, Phys. Rev. D **67**, 075019 (2003).
[16] P. M. Ferreira, J. F. Gunion, H. E. Haber, and R. Santos, Probing wrong-sign Yukawa couplings at the LHC and a future linear collider, Phys. Rev. D **89**, 115003 (2014).
[17] M. J. G. Veltman, The infrared—ultraviolet connection, Acta Phys. Pol. B **12**, 437 (1981).
[18] C. Newton and T. T. Wu, Mass relations in the two Higgs doublet model from the absence of quadratic divergences, Z. Phys. C **62**, 253 (1994).
[19] S. L. Glashow and S. Weinberg, Natural conservation laws for neutral currents, Phys. Rev. D **15**, 1958 (1977).
[20] E. A. Paschos, Diagonal neutral currents, Phys. Rev. D **15**, 1966 (1977).
[21] A. Pich and P. Tuzon, Yukawa alignment in the two-Higgs-doublet model, Phys. Rev. D **80**, 091702 (2009).
[22] P. M. Ferreira, L. Lavoura, and J. P. Silva, Renormalization-group constraints on Yukawa alignment in multi-Higgs-doublet models, Phys. Lett. B **688**, 341 (2010).
[23] A. G. Akeroyd, A. Arhrib, and E. M. Naimi, Note on tree level unitarity in the general two Higgs doublet model, Phys. Lett. B **490**, 119 (2000).
[24] B. Grzadkowski and P. Osland, Tempered two-Higgs-doublet model, Phys. Rev. D **82**, 125026 (2010).
[25] R. Jora, S. Nasri, and J. Schechter, Naturalness in a simple two Higgs doublet model, Int. J. Mod. Phys. A **28**, 1350036 (2013).
[26] M. Sher, Electroweak Higgs potentials and vacuum stability, Phys. Rep. **179**, 273 (1989).
[27] J. Maalampi, J. Sirkka, and I. Vilja, Tree level unitarity and triviality bounds for two Higgs models, Phys. Lett. B **265**, 371 (1991).
[28] S. Kanemura, T. Kubota, and E. Takasugi, Lee-Quigg-Thacker bounds for Higgs boson masses in a two doublet model, Phys. Lett. B **313**, 155 (1993).
[29] J. Horejsi and M. Kladiva, Tree-unitarity bounds for THDM Higgs masses revisited, Eur. Phys. J. C **46**, 81 (2006).
[30] G. Bhattacharyya, D. Das, P. B. Pal, and M. N. Rebelo, Scalar sector properties of two-Higgs-doublet models with a global U(1) symmetry, J. High Energy Phys. 10 (2013) 081.
[31] H. J. He, N. Polonsky, and S. f. Su, Extra families, Higgs spectrum and oblique corrections, Phys. Rev. D **64**, 053004 (2001).
[32] W. Grimus, L. Lavoura, O. M. Ogreid, and P. Osland, A precision constraint on multi-Higgs-doublet models, J. Phys. G **35**, 075001 (2008).
[33] M. Baak and R. Kogler, The global electroweak Standard Model fit after the Higgs discovery, arXiv:1306.0571.
[34] J. Beringer *et al.* (Particle Data Group Collaboration), Review of particle physics (RPP), Phys. Rev. D **86**, 010001 (2012) and 2013 partial update for the 2014 edition.
[35] I. F. Ginzburg and M. Krawczyk, Symmetries of two Higgs doublet model and *CP* violation, Phys. Rev. D **72**, 115013 (2005).
[36] A. Arhrib, R. Benbrik, C. H. Chen, R. Guedes, and R. Santos, Double neutral Higgs production in the two-Higgs doublet model at the LHC, J. High Energy Phys. 08 (2009) 035.
[37] J. Cao, P. Wan, L. Wu, and J. M. Yang, Lepton-specific two-Higgs doublet model: Experimental constraints and implication on Higgs phenomenology, Phys. Rev. D **80**, 071701 (2009).
[38] J. h. Park, Lepton non-universality at LEP and charged Higgs, J. High Energy Phys. 10 (2006) 077.
[39] G. Aad *et al.* (ATLAS Collaboration), Search for charged Higgs bosons decaying via $H^+ \to \tau\nu$ in top quark pair events using pp collision data at $\sqrt{s} = 7$ TeV with the ATLAS detector, J. High Energy Phys. 06 (2012) 039.
[40] S. Chatrchyan *et al.* (CMS Collaboration), Search for a light charged Higgs boson in top quark decays in pp collisions at $\sqrt{s} = 7$ TeV, J. High Energy Phys. 07 (2012) 143.
[41] L. Randall, Two Higgs models for large tan beta and heavy second Higgs, J. High Energy Phys. 02 (2008) 084.
[42] G. Aad *et al.* (ATLAS Collaboration), Search for a *CP*-odd Higgs boson decaying to Zh in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector, Phys. Lett. B **744**, 163 (2015).
[43] V. Khachatryan *et al.* (CMS Collaboration), Search for a pseudoscalar boson decaying into a Z boson and the 125 GeV Higgs boson in llbb final states, arXiv:1504.04710.

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Alignment, reverse alignment, and wrong sign Yukawa couplings in two Higgs doublet models

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We consider two Higgs doublet models with a softly broken U(1) symmetry, for various limiting values of the scalar mixing angles α and β. These correspond to the Standard Model Higgs particle being the lighter *CP*-even scalar (alignment) or the heavier *CP*-even scalar (reverse alignment), and also the limit in which some of the Yukawa couplings of this particle are the opposite sign of the vector boson couplings (wrong sign). In these limits we impose a criterion for naturalness by demanding that quadratic divergences cancel at one loop. We plot the allowed masses of the remaining physical scalars based on naturalness, stability, perturbative unitarity, and constraints coming from the ρ parameter. We also calculate the $h \to \gamma\gamma$ decay width in the wrong sign limit.

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I. INTRODUCTION

The discovery of a new boson in July 2012 by the ATLAS [1] and CMS collaborations [2] at the Large Hadron Collider (LHC) is a landmark in the history of particle physics. This scalar is most likely *the* Higgs boson, which is the last missing block in the Standard Model (SM). Although it answers most of the questions concerning fundamental particles, the SM has a few shortcomings, thus encouraging a search for theories beyond the Standard Model. Among the inadequacies are the lack of clear answers on the questions of the origins of neutrino mass and dark matter. It also cannot provide the observed matter-antimatter asymmetry of the Universe.

One of the simplest ways to go beyond the SM is by extending the scalar sector. This of course affects the ρ parameter, whose deviation from the tree-level value of unity is a measure of new physics. The general expression for the tree-level ρ parameter for an $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge theory with N scalar multiplets is [3]

$$\rho \equiv \frac{m_W^2}{\cos^2\theta_W m_Z^2} = \frac{\sum_{i=1}^N [T_i(T_i+1) - \frac{1}{4}Y_i^2]v_i^2}{\frac{1}{2}\sum_{i=1}^N Y_i^2 v_i^2}, \quad (1)$$

where T_i and Y_i denote the weak isospin and the hypercharge of the ith scalar multiplet respectively, and v_i is the vacuum expectation value (VEV) of the neutral component of that multiplet. If the scalar sector contains only SU(2) singlets with $Y = 0$ and doublets with $Y = \pm 1$, then $\rho = 1$ is automatically satisfied without requiring any fine-tuning among the VEVs. This conforms with the experimental value of ρ, which is very close to unity [4]. We therefore confine our discussions to the doublet extensions, specifically the two Higgs doublet models (2HDMs) [5], which have received a lot of attention mainly because the type II 2HDM arises as part of minimal supersymmetry.

In this paper we consider the restrictions imposed on the scalar masses by a criterion of naturalness, embodied in the Veltman conditions, in various limits of 2HDMs of all types. The alignment limit and the reverse alignment limit are two scenarios in which the lighter and the heavier *CP*-even neutral scalar, respectively, correspond to the observed Higgs particle. We also consider the cases where these occur in conjunction with the wrong sign limit, in which the Yukawa coupling of at least one type of fermion is of the opposite sign as the vector coupling. Using the naturalness conditions we analyze the parameter space of masses of scalars in 2HDMs of different types. The parameter space is further restricted by constraints arising from the ρ parameter, global stability of the scalar potential, and requirement of perturbative unitarity. Section II gives a brief review of the 2HDM. Sections III and IV deal with various limits of two Higgs doublet models and their permutations. In Sec. V we calculate the Higgs-diphoton decay width for one of the scenarios and Sec. VI concludes with a discussion of the results.

II. BRIEF REVIEW OF 2HDMs

We will work with the scalar potential [6,7] considered under the imposition of a U(1) symmetry which forbids flavor-changing neutral currents (FCNCs),

$$\begin{aligned} V = {} & \lambda_1\left(|\Phi_1|^2 - \frac{v_1^2}{2}\right)^2 + \lambda_2\left(|\Phi_2|^2 - \frac{v_2^2}{2}\right)^2 \\ & + \lambda_3\left(|\Phi_1|^2 + |\Phi_2|^2 - \frac{v_1^2+v_2^2}{2}\right)^2 \\ & + \lambda_4(|\Phi_1|^2|\Phi_2|^2 - |\Phi_1^\dagger\Phi_2|^2) \\ & + \lambda_5\left|\Phi_1^\dagger\Phi_2 - \frac{v_1 v_2}{2}\right|^2, \end{aligned} \quad (2)$$

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with real λ_i's. This potential is invariant under the symmetry $\Phi_1 \to e^{i\theta}\Phi_1$, $\Phi_2 \to \Phi_2$, except for a soft breaking term $\lambda_5 v_1 v_2 \Re(\Phi_1^\dagger \Phi_2)$. Additional dimension-4 terms, including one allowed by a softly broken Z_2 symmetry [8], are also set to zero by this U(1) symmetry. This is the same U(1) symmetry which prevents FCNC by having left- and right-handed fermions transform differently under it, leading to the four types of 2HDMs.

The scalar doublets are parametrized as

$$\Phi_i = \begin{pmatrix} w_i^+(x) \\ \frac{v_i + h_i(x) + iz_i(x)}{\sqrt{2}} \end{pmatrix}, \qquad i = 1, 2 \tag{3}$$

where the VEVs v_i may be taken to be real and positive without any loss of generality. Three of these fields get "eaten" by the $W^\pm$ and Z^0 gauge bosons; the remaining five are physical scalar fields. There is a pair of charged scalars denoted by $\xi^\pm$, two neutral CP-even scalars H and h, and one CP-odd pseudoscalar denoted by A. The two CP-even scalars have distinct masses, and $m_h < m_H$. With

$$\tan\beta = \frac{v_2}{v_1}, \tag{4}$$

the scalar fields are given by the combinations

$$\begin{pmatrix} \omega^\pm \\ \xi^\pm \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} w_1^\pm \\ w_2^\pm \end{pmatrix}, \tag{5}$$

$$\begin{pmatrix} \zeta \\ A \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \tag{6}$$

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \tag{7}$$

where $c_\alpha \equiv \cos\alpha$, etc. We will assume, without loss of generality, that $0 \le \beta \le \frac{\pi}{2}$, and $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$.

The quartic couplings are related to the physical Higgs masses by [9,10]

$$\lambda_1 = \frac{1}{2v^2 c_\beta^2}\left[c_\alpha^2 m_H^2 + s_\alpha^2 m_h^2 - \frac{s_\alpha c_\alpha}{\tan\beta}(m_H^2 - m_h^2)\right] - \frac{\lambda_5}{4}(\tan^2\beta - 1), \tag{8}$$

$$\lambda_2 = \frac{1}{2v^2 s_\beta^2}[s_\alpha^2 m_H^2 + c_\alpha^2 m_h^2 - s_\alpha c_\alpha \tan\beta (m_H^2 - m_h^2)] - \frac{\lambda_5}{4}\left(\frac{1}{\tan^2\beta} - 1\right), \tag{9}$$

$$\lambda_3 = \frac{1}{2v^2}\frac{s_\alpha c_\alpha}{s_\beta c_\beta}(m_H^2 - m_h^2) - \frac{\lambda_5}{4}, \tag{10}$$

$$\lambda_4 = \frac{2}{v^2} m_\xi^2, \tag{11}$$

$$\lambda_5 = \frac{2}{v^2} m_A^2. \tag{12}$$

Let us now turn our attention to the fermion couplings. The scalar doublets couple to the fermions in the theory via the Yukawa Lagrangian

$$\mathcal{L}_Y = \sum_{i=1,2} [-\bar{l}_L \Phi_i G_e^i e_R - \bar{Q}_L \tilde{\Phi}_i G_u^i u_R - \bar{Q}_L \Phi_i G_d^i d_R + \text{H.c.}]. \tag{13}$$

Here l_L, Q_L are three-vectors of isodoublets in the space of generations, e_R, u_R, d_R are three-vectors of singlets, G_e^1 etc. are complex 3×3 matrices in generation space containing the Yukawa coupling constants, and $\tilde{\Phi}_i = i\tau_2 \Phi_i^*$.

When the fermions are in mass eigenstates, the Yukawa matrices are automatically diagonal if there is only one Higgs doublet, as in the Standard Model. But in the presence of a second scalar doublet, the two Yukawa matrices will not be simultaneously diagonalizable in general. Thus the Yukawa couplings will not be flavor diagonal, and neutral Higgs scalars will mediate FCNCs [11–13]. The necessary and sufficient condition for the absence of FCNCs at tree level is that all fermions of a given charge and helicity transform according to the same irreducible representation of SU(2), corresponding to the same eigenvalue of T_3, and that a basis exists in which they receive their contributions in the mass matrix from a single source [14,15].

For the fermions of the Standard Model, this theorem implies that all right-handed singlets of a given charge must couple to the same Higgs doublet. This can be ensured by using the global U(1) symmetry mentioned earlier, which generalizes a Z_2 symmetry more commonly employed for this purpose. The left-handed fermion doublets remain unchanged under this symmetry, $Q_L \to Q_L$, $l_L \to l_L$. The transformations of right-handed fermion singlets determine the type of 2HDM. There are four such possibilities, which may be identified by the right-handed fields which transform under the U(1): type I (none), type II ($d_R \to e^{-i\theta} d_R$, $e_R \to e^{-i\theta} e_R$), lepton specific ($e_R \to e^{-i\theta} e_R$), and flipped ($d_R \to e^{-i\theta} d_R$).

The scalar masses get quadratically divergent contributions which require very large fine-tuning of parameters. We will impose a criterion of naturalness on the scalar masses, viz., the cancellation of these quadratic divergences. This gives rise to four mass relations, which we may call the Veltman conditions for the 2HDMs being considered [16],

$$2\mathrm{Tr}G_e^1 G_e^{1\dagger} + 6\mathrm{Tr}G_u^{1\dagger} G_u^1 + 6\mathrm{Tr}G_d^1 G_d^{1\dagger} = \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_1 + 10\lambda_3 + \lambda_4 + \lambda_5, \tag{14}$$

$$2\mathrm{Tr}G_e^2 G_e^{2\dagger} + 6\mathrm{Tr}G_u^{2\dagger} G_u^2 + 6\mathrm{Tr}G_d^2 G_d^{2\dagger} = \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_2 + 10\lambda_3 + \lambda_4 + \lambda_5, \tag{15}$$

$$2\mathrm{Tr}G_e^1 G_e^{2\dagger} + 6\mathrm{Tr}G_u^{1\dagger} G_u^2 + 6\mathrm{Tr}G_d^1 G_d^{2\dagger} = 0, \tag{16}$$

and another one which is the complex conjugate of the third equation. Here g, g' are the $SU(2)$ and $U(1)_Y$ coupling constants, respectively.

The fermion mass matrix is diagonalized by independent unitary transformations on the left- and right-handed fermion fields. In any of the 2HDMs, the U(1) symmetry implies that either G_{1f} or G_{2f} must vanish for each fermion type f. For example, in the type II model Φ_1 couples to down-type quarks and charged leptons, while Φ_2 couples to up-type quarks, so $G_{2e} = G_{2d} = G_{1u} = 0$. Thus Eq. (16) is automatically satisfied in each 2HDM, and the relevant mass relations come from the first two equations above. The nonvanishing Yukawa matrices are related to the fermion masses by

$$\mathrm{Tr}[G_{1f}^\dagger G_{1f}] = \frac{2}{v^2\cos^2\beta}\sum m_f^2, \tag{17}$$

$$\mathrm{Tr}[G_{2f}^\dagger G_{2f}] = \frac{2}{v^2\sin^2\beta}\sum m_f^2, \tag{18}$$

where f stands for charged leptons, up-type quarks, or down-type quarks, and the sum is taken over generations. These and the scalar mass relations of Eqs. (8)–(12) allow us to write the Veltman conditions in terms of the physical masses of particles.

There are some additional conditions on the parameters which further constrain the scalar masses. One is the pertubativity condition, which puts a constraint on the quartic coupling constants, $\lambda_i \leq 4\pi$ [17]. Another set comes from the condition that the potential is bounded from below. This was examined for more general potentials in 2HDM under U(1) symmetry in [18,19], and for the potential given in Eq. (2) these conditions become

$$\lambda_1 + \lambda_3 > 0, \tag{19}$$

$$\lambda_2 + \lambda_3 > 0, \tag{20}$$

$$2\lambda_3 + \lambda_4 + 2\sqrt{(\lambda_1+\lambda_3)(\lambda_2+\lambda_3)} > 0, \tag{21}$$

$$2\lambda_3 + \lambda_5 + 2\sqrt{(\lambda_1+\lambda_3)(\lambda_2+\lambda_3)} > 0. \tag{22}$$

These conditions put lower bounds on the above combinations of quartic couplings, but there are also upper bounds on these couplings arising from the considerations of perturbative unitarity [20]. These conditions are

$$|2\lambda_3 - \lambda_4 + 2\lambda_5| \leq 16\pi, \tag{23}$$

$$|2\lambda_3 + \lambda_4| \leq 16\pi, \tag{24}$$

$$|2\lambda_3 + \lambda_5| \leq 16\pi, \tag{25}$$

$$|2\lambda_3 + 2\lambda_4 - \lambda_5| \leq 16\pi, \tag{26}$$

$$|3(\lambda_1+\lambda_2+2\lambda_3) \pm \sqrt{9(\lambda_1-\lambda_2)^2 + (4\lambda_3+\lambda_4+\lambda_5)^2}| \leq 16\pi, \tag{27}$$

$$|(\lambda_1+\lambda_2+2\lambda_3) \pm \sqrt{(\lambda_1-\lambda_2)^2 + (\lambda_4-\lambda_5)^2}| \leq 16\pi, \tag{28}$$

$$|(\lambda_1+\lambda_2+2\lambda_3) \pm (\lambda_1-\lambda_2)| \leq 16\pi. \tag{29}$$

There is another condition that we need to take into account when we calculate bounds on the scalar masses. The oblique electroweak correction T, which measures deviations from the Standard Model due to new physics, is related to the deviation of the ρ parameter from its SM value of unity by

$$\delta\rho \equiv \rho - 1 = \alpha T, \tag{30}$$

where $\alpha = e^2/4\pi$ is the fine structure constant. The effect of the general 2HDM on the ρ parameter is known to be [21,22]

$$\begin{aligned}\delta\rho = \frac{g^2}{64\pi^2 m_w^2}&(F(m_\xi^2, m_A^2) + \sin^2(\beta-\alpha)F(m_\xi^2, m_H^2) \\ &+ \cos^2(\beta-\alpha)F(m_\xi^2, m_h^2) \\ &- \sin^2(\beta-\alpha)F(m_A^2, m_H^2) - \cos^2(\beta-\alpha)F(m_A^2, m_h^2) \\ &+ 3\cos^2(\beta-\alpha)[F(m_Z^2, m_H^2) - F(m_W^2, m_H^2)] \\ &+ 3\sin^2(\beta-\alpha)[F(m_Z^2, m_h^2) - F(m_W^2, m_h^2)] \\ &- 3[F(m_Z^2, m_{h_{SM}}^2) - F(m_W^2, m_{h_{SM}}^2)]),\end{aligned} \tag{31}$$

where $F(x,y)$ is a function of two non-negative arguments x and y, symmetrical under the exchange of the arguments, and vanishes only if $x = y$. The function has the property that it grows linearly with $\max(x,y)$, i.e., quadratically with the heaviest scalar mass when that mass becomes very large. The current experimental bound on the total new physics contribution to ρ is given by $\delta\rho = -0.00011$ [4].

III. LIMITS OF 2HDMs

In order to relate a 2HDM to the Higgs sector of the Standard Model, we need to identify some combination of the neutral scalar particles in the theory as the observed Higgs particle. This can be done in several ways, by considering different combinations of the angles α and β. Is this section we will consider the different limits for which part of the 2HDM matches the Standard Model, and calculate the allowed range of masses for the additional scalars.

A crucial parameter of the 2HDMs is $\tan\beta$. Its value is larger than one, based on constraints coming from $Z \to b\bar{b}$ and $B_q\bar{B}_q$ mixing [23]. A large $\tan\beta$ is suggested by muon $g-2$ in a lepton specific 2HDM [24], by using $b \to s\gamma$ in type I and flipped models [25], which also suppresses the $t \to bH^+$ branching ratio to a rough agreement with 95% C.L. limits from the light charged Higgs searches at the LHC [26,27]. We will assume that $\tan\beta$ is large, and certainly larger than unity, specific values will be considered for the plots as needed.

A. Alignment limit

If we rotated the neutral (h_1, h_2) doublet by the angle β,

$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \tag{32}$$

we would find that H^0 has exactly the Standard Model Higgs couplings with the fermions and gauge bosons [11,18]. The physical scalar h is related to H^0 and R via

$$h = \sin(\beta - \alpha)H^0 + \cos(\beta - \alpha)R. \tag{33}$$

Thus in order for h to be the Higgs boson of the Standard Model, we require $\sin(\beta - \alpha) \approx 1$, which has been called the SM-like or alignment limit [28].

There remain three unknown mass parameters, namely m_H, m_ξ, and m_A, which span the parameter space. By fixing $\tan\beta$ at some specific value, we can use the Veltman conditions to plot the accessible region of the $m_H - m_\xi$ plane corresponding to the allowed range of values for m_A. On the other hand, constraints from perturbative unitarity and the oblique correction T also restrict the accessible region on this plane. The intersection of all these regions provides the allowed ranges for m_H and m_ξ.

The mass ranges were studied for the alignment limit in [29], where it was found that if we set $m_h = 125$ GeV, and allowed m_A to run over its entire range of $0 < m_A \lesssim 617$ GeV as determined by the condition of perturbativity, the two unknown masses m_H and m_ξ became restricted to ranges of $550\ \text{GeV} \lesssim m_\xi \lesssim 700$ GeV, $450\ \text{GeV} \lesssim m_H \lesssim 620$ GeV. The value of $\tan\beta$ used in these calculations was $\tan\beta = 5$, and it was also found that a higher value of $\tan\beta$ pushed the ranges to higher values and also made them narrower. These mass ranges are in agreement with bounds found by analyzing experimental data [30].

B. Reverse alignment limit

Let us rearrange the equations described in the previous section. Using Eqs. (7) and (32) we obtain H in terms of H^0 and R,

$$H = H^0 \cos(\beta - \alpha) - R \sin(\beta - \alpha) \tag{34}$$

Had H been the SM-like Higgs boson, it would have to resemble the properties of H^0, and for that β would have to approximately equal α or $\pi + \alpha$. The ultimate results with $\beta \approx \alpha$ and $\beta \approx \pi + \alpha$ are identical, so in what follows we will work with $\beta \approx \alpha$ and call it the *reverse alignment limit*.

Equations (8)–(12) become, in the reverse alignment limit,

$$\lambda_1 = \frac{m_h^2}{2v^2}(\tan^2\beta + 1) - \frac{\lambda_5}{4}(\tan^2\beta - 1), \tag{35}$$

$$\lambda_2 = \frac{m_h^2}{2v^2}(\cot^2\beta + 1) - \frac{\lambda_5}{4}(\cot^2\beta - 1), \tag{36}$$

$$\lambda_3 = \frac{1}{2v^2}(m_H^2 - m_h^2) - \frac{\lambda_5}{4}, \tag{37}$$

$$\lambda_4 = \frac{2}{v^2} m_\xi^2, \tag{38}$$

$$\lambda_5 = \frac{2}{v^2} m_A^2. \tag{39}$$

Let us write the Veltman conditions defined in Eqs. (14) and (15) using the above equations. We will write the equations explicitly for one case, that of the type II 2HDM, for which the two Veltman conditions read, in the reverse alignment limit,

$$m_h^2(3\tan^2\beta - 2) + 2m_\xi^2 = 4\left[\sum m_e^2 + 3\sum m_d^2\right]\sec^2\beta - 6M_W^2 - 3M_Z^2 - 5m_H^2 + \lambda_5\frac{3v^2}{2}\tan^2\beta, \tag{40}$$

$$m_h^2(3\cot^2\beta - 2) + 2m_\xi^2 = 12\sum m_u^2 \csc^2\beta - 6M_W^2 - 3M_Z^2 - 5m_H^2 + \lambda_5\frac{3v^2}{2}\cot^2\beta. \tag{41}$$

We have plotted the above equalities on the $m_h - m_\xi$ plane for several values of λ_5 for a fixed value of $\tan\beta$ and with $m_H = 125$ GeV, with $m_h \leq m_H$. On the same plane, we have also plotted the region allowed by stability, perturbative unitarity, and constraints from $\delta\rho$. The conditions of stability and perturbative unitarity, Eqs. (19)–(29), produce the following two inequalities in the reverse alignment limit relevant to this plot:

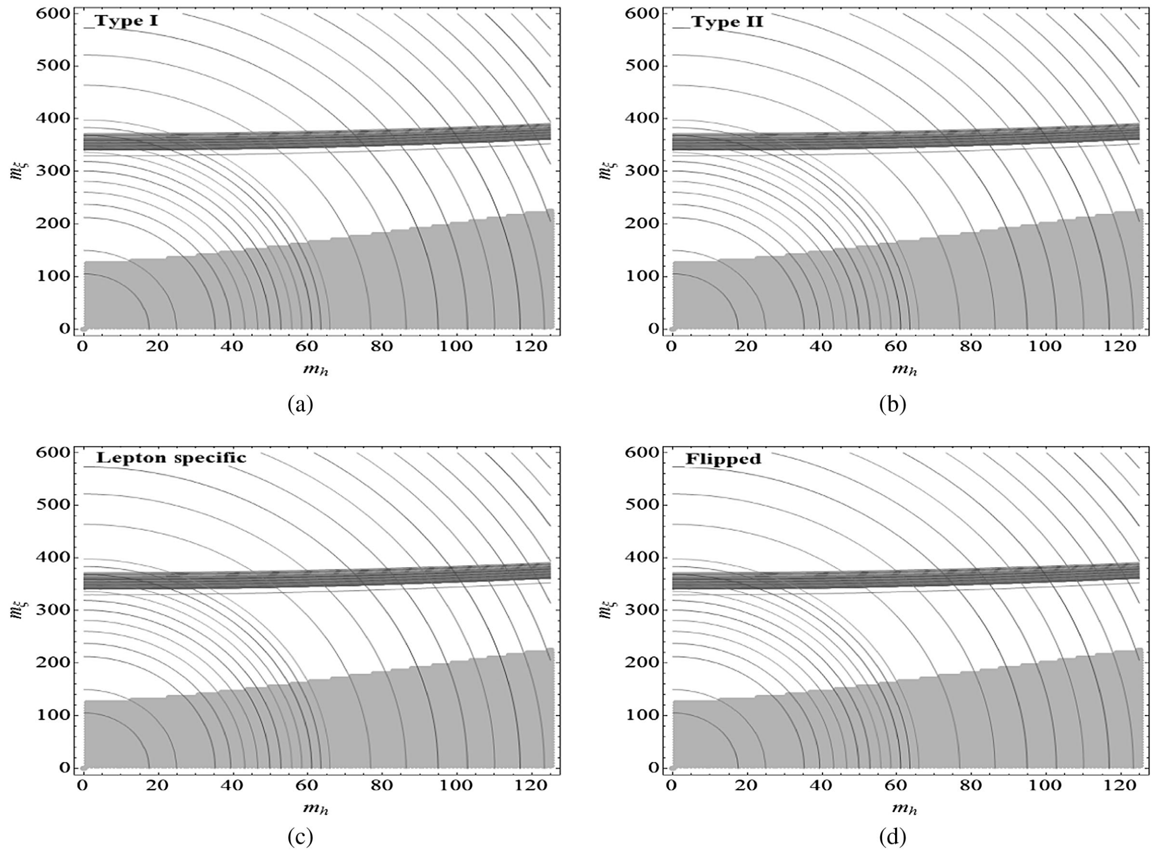

FIG. 1. Allowed mass range (in GeV) for the charged Higgs and the light CP-even Higgs in reverse alignment limit for (a) type I, (b) type II, (c) lepton specific, and (d) flipped 2HDMs for $|\lambda_5| \le 4\pi$ and $\tan\beta = 5$.

$$0 \le (m_h^2 - m_A^2)(\tan^2\beta + \cot^2\beta) + 2m_H^2 \le \frac{32\pi v^2}{3}, \quad (42)$$

$$|2m_\xi^2 - m_h^2 - m_A^2 + m_H^2| \le 16\pi v^2. \quad (43)$$

These are analogous to similar inequalities found in [29] in the alignment limit.

For $\tan\beta = 5$, the plots for all four types of 2HDM are shown in Fig. 1. The gray region covers the points which satisfy the inequalities (42) and (43) in addition to the constraints from $\delta\rho$, the first Veltman condition provides the curves (ellipses) which cross this region, and the second Veltman condition provides the nearly flat hyperbolas above the gray region.

As we can see from the plots in Fig. 1, there is no region on the $m_h - m_\xi$ plane where all the constraints are obeyed. In other words, if we insist on naturalness, as embodied by the Veltman conditions, the reverse alignment limit is not a valid limit for any of the 2HDMs, i.e., the observed Higgs particle cannot be the heavier CP-even neutral scalar in any of the 2HDMs.

It should be mentioned here that allowed mass ranges of scalars in both the alignment limit and the reverse alignment limit were studied in [31]. However, that paper considered an unbroken Z_2 symmetry, not a softly broken symmetry as we have considered. As a result the mass ranges of scalars, as well as the allowed range of $\tan\beta$ found in that paper, are different from the ones we have found.

IV. WRONG SIGN YUKAWA COUPLINGS

The wrong sign Yukawa coupling regime [28,32,33] is defined as the region of 2HDM parameter space in which at least one of the couplings of the SM-like Higgs to up-type and down-type quarks is opposite in sign to the corresponding coupling of SM-like Higgs to vectors bosons. This is to be contrasted with the Standard Model, where the couplings of h_{SM} to $\bar{f}f$ and vector bosons are of the same sign. The *wrong sign limit* needs to be considered in conjunction with either the alignment limit or the reverse alignment limit. We will now calculate the regions of parameter space when each of these two limits is combined with the wrong sign limit.

The CP-even neutral scalars couple to the up-type and down-type quarks in the various 2HDMs as shown in Table I, with the SM couplings of the quarks to the SM Higgs field normalized to unity.

TABLE I. Yukawa couplings for the different 2HDMs.

2HDMs	$h\bar{U}U$	$h\bar{D}D$	$H\bar{U}U$	$H\bar{D}D$
Type I	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$
Type II	$\frac{\cos\alpha}{\sin\beta}$	$-\frac{\sin\alpha}{\cos\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\cos\beta}$
Lepton specific	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$
Flipped	$\frac{\cos\alpha}{\sin\beta}$	$-\frac{\sin\alpha}{\cos\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\cos\beta}$

A. Wrong sign and reverse alignment limit

Let us first consider the case of wrong sign Yukawa couplings in the reverse alignment limit. The heavier CP-even neutral scalar H corresponds to the SM Higgs in the reverse alignment limit, with a coupling to vector bosons which is $\cos(\beta-\alpha)$ times the corresponding SM value. In the convention where $\cos(\beta-\alpha)\geq 0$, the HVV couplings in the 2HDM are always non-negative. To analyze the wrong sign coupling regime, we write the Yukawa couplings in the type II and Flipped 2HDMs in the following form:

$$H\bar{D}D\colon\ \frac{\cos\alpha}{\cos\beta} = \cos(\beta+\alpha)+\sin(\beta+\alpha)\tan\beta, \tag{44}$$

$$H\bar{U}U\colon\ \frac{\sin\alpha}{\sin\beta} = -\cos(\beta+\alpha)+\sin(\beta+\alpha)\cot\beta. \tag{45}$$

In the case when $\cos(\beta+\alpha)=-1$, the $H\bar{D}D$ coupling normalized to its SM value is equal to -1, whereas the normalized $H\bar{U}U$ coupling is $+1$. Thus in this case, when the reverse alignment limit is taken in conjunction with the wrong sign limit, we have $\alpha\approx\beta\approx\frac{\pi}{2}$. It turns out there is no point on the m_h-m_ξ plane which satisfies the Veltman conditions as well as the bounds coming from unitarity, stability and the ρ parameter. In Fig. 2 only the first Veltman condition has been plotted, and it does not cross the gray region corresponding to the bounds. The other Veltman condition does not show up in this picture at all, it is not satisfied for any point in this plot.

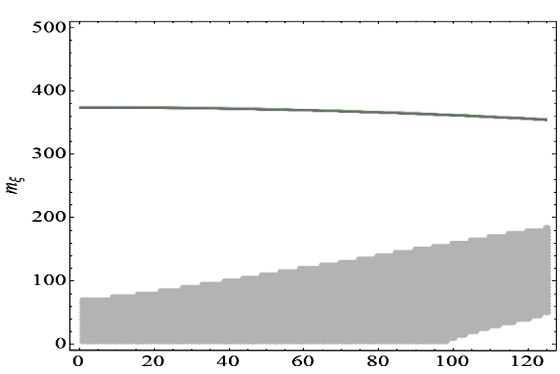

FIG. 2. Veltman conditions are not satisfied for any (m_h, m_ξ) satisfying unitarity and other bounds, in the reverse alignment limit with wrong sign Yukawa couplings.

On the other hand, in the case when $\cos(\beta+\alpha)=1$, the $H\bar{U}U$ coupling normalized to its SM value is equal to -1, while the normalized $H\bar{D}D$ coupling is $+1$. In this limiting case, $\cos(\beta-\alpha)=\cos 2\beta$, which implies that the wrong sign $H\bar{U}U$ couplings can only be achieved for $\tan\beta<1$ for the type II and flipped 2HDMs.

In the type I and lepton specific 2HDMs, both the $H\bar{D}D$ and $H\bar{U}U$ couplings are given by Eq. (45). Thus, for $\cos(\beta+\alpha)=1$, the normalized $H\bar{D}D$ and $H\bar{U}U$ couplings are both equal to -1, which is only possible if $\tan\beta<1$.

Since $\tan\beta>1$, we see that the wrong sign Yukawa coupling is incompatible with the reverse alignment limit in all of the four types of 2HDMs.

B. Wrong sign in the alignment limit

Let us now look at what happens if some Yukawa couplings are of the wrong sign, in the alignment limit. In this case h is the SM Higgs, and its coupling to the vector bosons is $\sin(\beta-\alpha)$ times the corresponding SM value. Then in the convention where $\sin(\beta-\alpha)\geq 0$, the hVV couplings in the 2HDM are always non-negative. As in the previous case, we write the type II and flipped Higgs-fermion Yukawa couplings, normalized with respect to the Standard Model couplings, in the following form:

$$h\bar{D}D\colon -\frac{\sin\alpha}{\cos\beta} = -\sin(\beta+\alpha)+\cos(\beta+\alpha)\tan\beta, \tag{46}$$

$$h\bar{U}U\colon \frac{\cos\alpha}{\sin\beta} = \sin(\beta+\alpha)+\cos(\beta+\alpha)\cot\beta. \tag{47}$$

In the case when $\sin(\beta+\alpha)=1$, the $h\bar{D}D$ coupling normalized to its SM value is equal to -1, while the normalized $h\bar{U}U$ coupling is $+1$. Note that in this limiting case, $\sin(\beta-\alpha)=-\cos 2\beta$, which implies that the wrong sign $h\bar{D}D$ Yukawa coupling can only be achieved for values of $\tan\beta>1$.

Likewise, in the case of $\sin(\beta+\alpha)=-1$, the $h\bar{U}U$ coupling normalized to its SM value is equal to -1, whereas the normalized $h\bar{D}D$ coupling is $+1$. Then $\sin(\beta-\alpha)=\cos 2\beta$, which implies that the wrong sign $h\bar{U}U$ couplings can occur only if $\tan\beta<1$. In the type I and lepton specific 2HDMs, the $h\bar{D}D$ and $h\bar{U}U$ couplings are both given by Eq. (47). Thus for $\sin(\beta+\alpha)=-1$, the normalized $h\bar{D}D$ and $h\bar{U}U$ couplings are both equal to -1, which is only possible if $\tan\beta<1$. Thus realistically only the $h\bar{D}D$ coupling of the type II and flipped 2HDM can be of the wrong sign, since $\tan\beta>1$.

Let us therefore consider a type II model with a wrong sign $h\bar{D}D$ coupling. The wrong sign limit approaches the alignment limit for $\tan\beta\approx 17$ as was displayed in [32,33] for the allowed parameter space of the type II CP-conserving 2HDM, based on the 8 TeV run of the LHC.

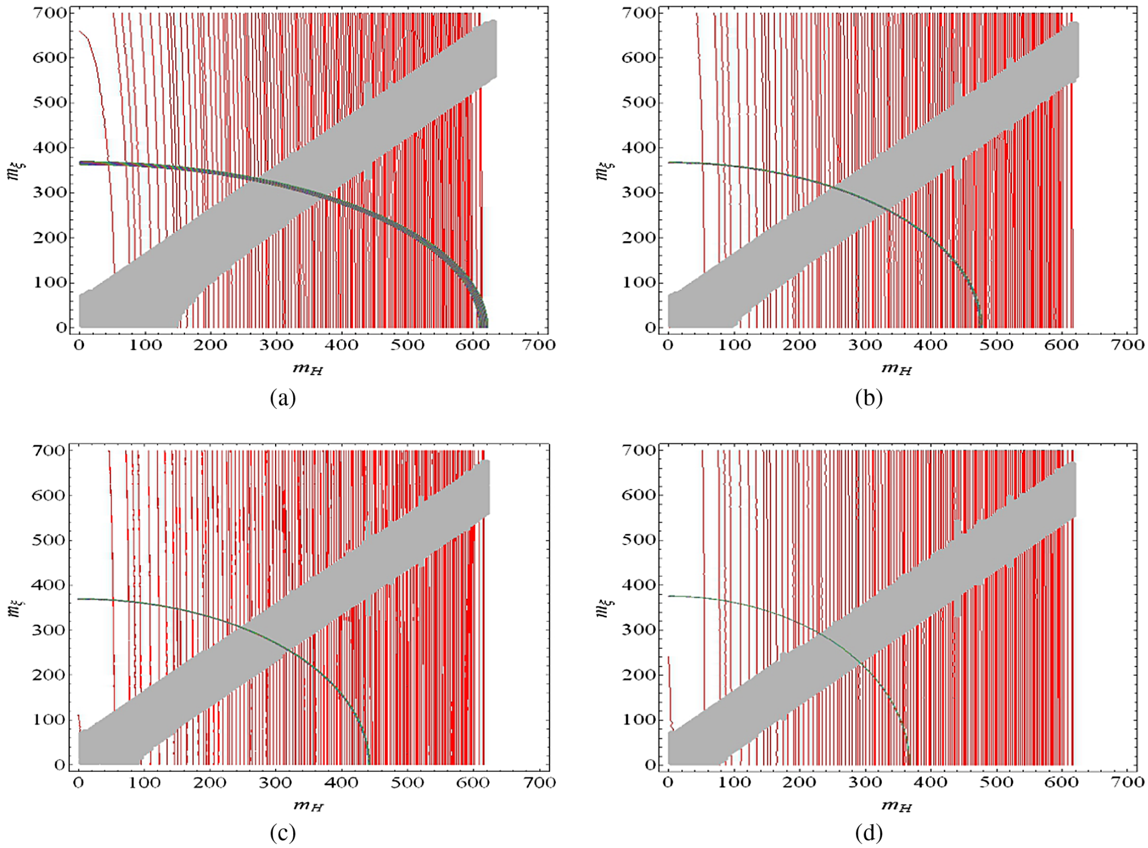

FIG. 3. Allowed mass range in GeV for the charged Higgs and the heavy CP-even Higgs when approaching wrong sign and alignment limits simultaneously for (a) $\tan\beta = 10$, (b) $\tan\beta = 17$, (c) $\tan\beta = 20$, and (d) $\tan\beta = 30$ for $|\lambda_5| \leq 4\pi$ and a type II 2HDM.

For this model, we will plot the values of the pair (m_H, m_ξ) allowed by the naturalness conditions as well as the constraints imposed by perturbativity, stability, tree-level unitarity, and the ρ parameter. We will do this for four different values of $\tan\beta$ around the "critical" value of 17. By choosing a small enough α we can ensure that for all these choices, both $\sin(\beta - \alpha) \approx 1$ and $\sin(\beta + \alpha) \approx 1$, as needed for the alignment limit and the wrong sign coupling.

In Fig. 3 we have plotted the Veltman conditions on the $m_H - m_\xi$ plane for type II 2HDM for the four choices of $\tan\beta$, for different values of m_A constrained by $|\lambda_5| \leq 4\pi$. These plots are further constrained by conditions coming from stability of the potential, perturbative unitarity, and experimental bounds on $\delta\rho$. We have also taken $m_h = 125$ GeV. One can estimate from the plots that for $\tan\beta = 17$ that the range of m_H is approximately (250, 330) GeV, and that of m_ξ is approximately (260, 310) GeV. At higher values of $\tan\beta$, both ranges become narrower and move down on the mass scale.

V. MODIFICATION OF HIGGS-DIPHOTON DECAY WIDTH

The $h \to \gamma\gamma$ decay channel is perhaps the most popular channel for Higgs and related searches. The decay width can be enhanced or reduced in the 2HDMs due to loop effects. In the alignment limit, the couplings of the lighter CP-even neutral scalar h to gauge bosons are identical to that for the SM Higgs. Then the tree-level decay widths of h will be the same as for the SM Higgs. For loop induced decays, such as $h \to \gamma\gamma$ and $h \to Z\gamma$, the contribution of the W boson loop and the top loop diagrams are the same as in the SM. But there will have to be some additional contributions due to the virtual charged scalars $\xi^\pm$ in the loop. Thus the decay widths will be different from the SM in general. Contributions from the fermion loops are the same in this case as for the SM.

On the other hand, suppose h has wrong sign Yukawa couplings to the down-type quarks. Then the bottom quarks will contribute with a relative negative sign in the loops, and the $h \to \gamma\gamma$ decay width will be different from the SM, as well as from 2HDMs in the usual alignment limit.

The Higgs-diphoton decay width is calculated using the formula [34]

$$\Gamma(h \to \gamma\gamma) = \frac{G_\mu \alpha^2 m_h^3}{128\sqrt{2}\pi^3} \left| \sum\nolimits_f N_c Q_f^2 g_{hff} A_{1/2}^h(\tau_f) + g_{hVV} A_1^h(\tau_W) + \frac{m_W^2 \lambda_{h\xi^+\xi^-}}{2c_W^2 M_{\xi^\pm}^2} A_0^h(\tau_{\xi^\pm}) \right|^2. \quad (48)$$

In this equation, N_c is the number color multiplicity, Q_f is the charge of the fermion f, G_μ is the Fermi constant, and the reduced couplings g_{hff} and g_{hVV} of the Higgs boson to fermions and W bosons are $g_{htt} = \frac{\cos\alpha}{\sin\beta}$, $g_{htt} = -\frac{\sin\alpha}{\cos\beta}$, and $g_{hWW} = \sin(\beta - \alpha)$, while the trilinear $\lambda_{h\xi^+\xi^-}$ couplings to charged Higgs bosons is given by

$$\lambda_{h\xi^+\xi^-} = \cos 2\beta \sin(\beta+\alpha) + 2c_W^2 \sin(\beta-\alpha) \quad (49)$$

$$= \lambda_{hAA} + 2c_W^2 g_{hVV}, \quad (50)$$

where $c_W = \cos\theta_W$, with θ_W being the Weinberg angle. The decay width does not depend on the type of the 2HDM.

The amplitudes A_i at lowest order for the spin-1, spin-$\frac{1}{2}$, and spin-0 particle contributions are given by [7]

$$A^h_{1/2} = -2\tau[1 + (1-\tau)f(\tau)], \quad (51)$$

$$A^h_1 = 2 + 3\tau + 3\tau(2-\tau)f(\tau), \quad (52)$$

$$A^h_0 = \tau[1 - \tau f(\tau)] \quad (53)$$

in the case of the CP-even Higgs boson h.

Here

$$\tau_x = 4m_x^2/m_h^2 \quad (54)$$

and

$$f(\tau) = \begin{cases} \arcsin^2\sqrt{1/\tau}, & \tau \geq 1 \\ -\frac{1}{4}\left[\log\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2, & \tau < 1 \end{cases}. \quad (55)$$

Using the above definitions in the decay width formula given in Eq. (48), we arrive at a much simplified expression for the decay width,

$$\Gamma(h\to\gamma\gamma) = \frac{G_\mu \alpha^2 m_h^3}{128\sqrt{2}\pi^3}\left| g_{hVV}A_W^h + \frac{4}{3}g_{htt}A_t^h \pm \frac{1}{3}g_{hbb}A_b^h + \kappa A_\xi^h \right|^2, \quad (56)$$

where the + before A_b^h is for when the $h\bar{b}b$ Yukawa coupling has the same sign as the hVV coupling and the − is for the wrong sign of the Yukawa coupling, and κ is defined as

$$\kappa = \frac{1}{m_\xi^2}\left(m_\xi^2 + \frac{1}{2}m_h^2 - m_A^2\right). \quad (57)$$

The appearance of m_A in Eq. (57) is merely an artifact of U(1) symmetry of the scalar potential. For a more general potential the expression for κ involves λ_5 [35]. In Fig. 4 we have plotted the $h\to\gamma\gamma$ decay width in 2HDMs in the alignment limit, normalized with respect to the SM value, against the mass of the charged Higgs particle, and for different values of the mass of the CP-odd scalar. Figure 4(a) shows the decay width for the case where the $h\bar{q}q$ Yukawa coupling has the same sign as the hVV coupling, whereas Fig. 4(b) is for the decay width corresponding to the case where the Yukawa coupling of h to the down-type quarks is the opposite sign of the hVV coupling. We note that the first case has been plotted, albeit for smaller values of $\tan\beta$ and without the use of the Veltman conditions (thus for a much larger range of m_ξ), in [36].

As we have seen in the previous section, simultaneously choosing the alignment limit and the wrong sign limit also sets $\tan\beta$ at a high value. The critical value $\tan\beta = 17$, and a small but nonzero value of α, namely $\alpha \simeq 0.035$, was chosen for both the plots. The plots are not noticeably different for other high values of $\tan\beta$ or other similar values of α. The decay width does not depend on the type of 2HDM once the masses of the charged Higgs particle and the CP-odd Higgs particle are fixed. However, the range of allowed masses depends on the type of 2HDM being considered. We have chosen the ranges

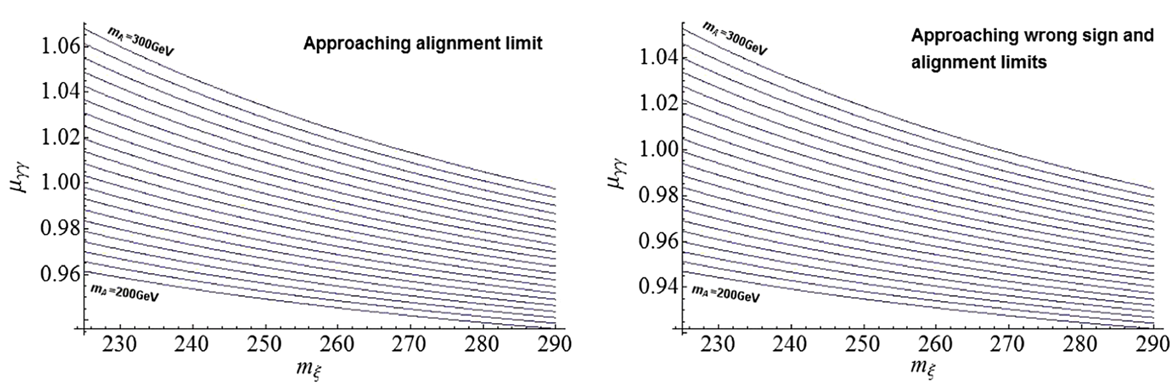

FIG. 4. Diphoton decay width of the SM-like Higgs particle (normalized to SM) as a function of the charged Higgs mass in GeV at $\tan\beta = 17$, for (a) the same sign and (b) the wrong sign, of down-type Yukawa couplings.

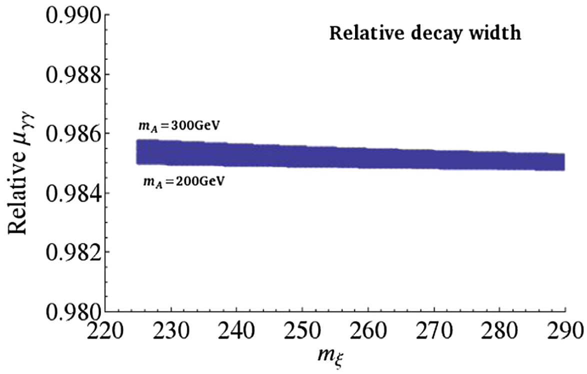

FIG. 5. $h\gamma\gamma$ decay width for a wrong sign $h\bar{D}D$ coupling relative to the case with same sign Yukawa couplings.

225 GeV $\leq m_\xi \leq$ 290 GeV and 200 GeV $\leq m_A \leq$ 300 GeV, which cover the allowed ranges for all four types for $\tan\beta = 17$. Although a picture is worth a thousand words, it is perhaps worth pointing out that when m_A is small, for example $m_A \simeq 200$ GeV, the diphoton decay width deviates from the SM value by 5%–7% for all values of m_ξ. The deviation is noticeable for many other values of m_A also, as can be easily seen from the plots. On the other hand, for specific choices of (m_A, m_ξ) the $h \to \gamma\gamma$ decay width is the same as for the SM, so the nonobservation of a deviation does not rule out 2HDMs.

The two plots are similar, but not identical. The decay width when the $h\bar{D}D$ Yukawa coupling is of the wrong sign is smaller than the decay width for the case when it is of the same sign (as hVV couplings) by about 1.5%, as can be seen from the ratio of the decay widths, displayed in Fig. 5.

VI. RESULTS AND CONCLUSION

In this paper we have looked at how a certain criterion of naturalness, namely the cancellation of quadratic divergences, affects the allowed ranges of masses of the additional scalars in 2HDMs in the alignment or SM-like limit with wrong sign Yukawa couplings, and also in the reverse alignment limit. A similar calculation was done in [29] for the alignment limit without the wrong sign assumption.

We found that reverse alignment, i.e., the scenario in which the heavier CP-even neutral scalar is the Standard Model Higgs particle, is clearly not a viable scenario for 2HDMs. Constraints arising from naturalness, stability, perturbative unitarity, and experimental bounds on the ρ parameter completely rule out this scenario. The naturalness criterion is crucial for this conclusion—reverse alignment is an allowed scenario if quadratic divergences are taken care of by some mechanism of fine-tuning, for example.

We have also considered a limit where the lighter CP-even neutral scalar corresponds to the SM-like Higgs but where the Yukawa couplings of this particle to D-type quarks are of the wrong sign relative to their gauge couplings. In this scenario we obtain mass ranges for the rest of the physical Higgs bosons for various benchmark values of $\tan\beta$. In this paper we have shown only the plot for the type II 2HDM, but the results are similar for the other 2HDMs with a small variation of a few GeV.

The Higgs-diphoton decay width in a 2HDM receives additional contributions from loops containing the charged scalar $\xi^\pm$, so the decay width in a 2HDM is different from the SM value. Furthermore, in the wrong sign limit, loops containing down-type quarks contribute with a different sign. We have plotted the $h \to 2\gamma$ decay width against the mass of the charged Higgs, and also for different values of the mass of the CP-odd neutral scalar, and found that the decay width can differ from its SM value by up to 6% for some values of the parameters.

ACKNOWLEDGMENTS

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Note added.—Recently, another paper which investigates what we call the reverse alignment limit appeared [37]. However, that paper uses fewer constraints, so limits on the masses of $\xi^\pm$ are less restrictive. Even more recently, the ATLAS and CMS collaborations at the LHC have reported an excess corresponding to a diphoton resonance at 750 GeV [38]. We note that according to the naturalness criterion we have used in this paper, this excess cannot be one of the scalar particles in any of the four types of 2HDMs, in agreement with the negative result found in [39] using several other lines of argument.

[1] G. Aad *et al.* (ATLAS Collaboration), Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B **716**, 1 (2012).

[2] S. Chatrchyan *et al.* (CMS Collaboration), Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys. Lett. B **716**, 30 (2012).

[3] P. Langacker, Grand unified theories and proton decay, Phys. Rep. **72**, 185 (1981).

[4] K. A. Olive *et al.* (Particle Data Group Collaboration), Review of particle physics, Chin. Phys. C **38**, 090001 (2014).

[5] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, Theory and phenomenology of two-Higgs-doublet models, Phys. Rep. **516**, 1 (2012).

[6] T. D. Lee, A theory of spontaneous T violation, Phys. Rev. D **8**, 1226 (1973).

[7] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, *The Higgs hunter's guide*, Frontiers in Physics Vol. 80 (Addison-Wesley, New York, 2000).

[8] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, Errata for The Higgs hunter's guide, arXiv:hep-ph/9302272.

[9] S. Kanemura, T. Kubota, and E. Takasugi, Lee-Quigg-Thacker bounds for Higgs boson masses in a two doublet model, Phys. Lett. B **313**, 155 (1993).

[10] A. G. Akeroyd, A. Arhrib, and E. M. Naimi, Note on tree level unitarity in the general two Higgs doublet model, Phys. Lett. B **490**, 119 (2000).

[11] G. C. Branco, W. Grimus, and L. Lavoura, Relating the scalar flavor changing neutral couplings to the CKM matrix, Phys. Lett. B **380**, 119 (1996).

[12] F. J. Botella, G. C. Branco, A. Carmona, M. Nebot, L. Pedro, and M. N. Rebelo, Physical constraints on a class of two-Higgs doublet models with FCNC at tree level, J. High Energy Phys. 07 (2014) 078.

[13] G. Bhattacharyya, D. Das, and A. Kundu, Feasibility of light scalars in a class of two-Higgs-doublet models and their decay signatures, Phys. Rev. D **89**, 095029 (2014).

[14] S. L. Glashow and S. Weinberg, Natural conservation laws for neutral currents, Phys. Rev. D **15**, 1958 (1977).

[15] E. A. Paschos, Diagonal neutral currents, Phys. Rev. D **15**, 1966 (1977).

[16] C. Newton and T. T. Wu, Mass relations in the two Higgs doublet model from the absence of quadratic divergences, Z. Phys. C **62**, 253 (1994).

[17] S. Kanemura, T. Kasai, and Y. Okada, Mass bounds of the lightest *CP* even Higgs boson in the two Higgs doublet model, Phys. Lett. B **471**, 182 (1999).

[18] J. F. Gunion and H. E. Haber, The *CP* conserving two Higgs doublet model: The approach to the decoupling limit, Phys. Rev. D **67**, 075019 (2003).

[19] M. Sher, Electroweak Higgs potentials and vacuum stability, Phys. Rep. **179**, 273 (1989).

[20] B. W. Lee, C. Quigg, and H. B. Thacker, Weak interactions at very high energies: The role of the Higgs-boson mass, Phys. Rev. D **16**, 1519 (1977).

[21] W. Grimus, L. Lavoura, O. M. Ogreid, and P. Osland, A precision constraint on multi-Higgs-doublet models, J. Phys. G **35**, 075001 (2008).

[22] S. Kanemura, Y. Okada, H. Taniguchi, and K. Tsumura, Indirect bounds on heavy scalar masses of the two-Higgs-doublet model in light of recent Higgs boson searches, Phys. Lett. B **704**, 303 (2011).

[23] A. Arhrib, R. Benbrik, C. H. Chen, R. Guedes, and R. Santos, Double neutral Higgs production in the two-Higgs doublet model at the LHC, J. High Energy Phys. 08 (2009) 035.

[24] J. Cao, P. Wan, L. Wu, and J. M. Yang, Lepton-specific two-Higgs doublet model: Experimental constraints and implication on Higgs phenomenology, Phys. Rev. D **80**, 071701 (2009).

[25] J. h. Park, Lepton non-universality at LEP and charged Higgs, J. High Energy Phys. 10 (2006) 077.

[26] G. Aad *et al.* (ATLAS Collaboration), Search for charged Higgs bosons decaying via $H^+ \to \tau\nu$ in top quark pair events using pp collision data at $\sqrt{s} = 7$ TeV with the ATLAS detector, J. High Energy Phys. 06 (2012) 039.

[27] S. Chatrchyan *et al.* (CMS Collaboration), Search for a light charged Higgs boson in top quark decays in pp collisions at $\sqrt{s} = 7$ TeV, J. High Energy Phys. 07 (2012) 143.

[28] P. M. Ferreira, J. F. Gunion, H. E. Haber, and R. Santos, Probing wrong-sign Yukawa couplings at the LHC and a future linear collider, Phys. Rev. D **89**, 115003 (2014).

[29] A. Biswas and A. Lahiri, Masses of physical scalars in two Higgs doublet models, Phys. Rev. D **91**, 115012 (2015).

[30] S. Kanemura, H. Yokoya, and Y. J. Zheng, Complementarity in direct searches for additional Higgs bosons at the LHC and the International Linear Collider, Nucl. Phys. **B886**, 524 (2014).

[31] B. Coleppa, F. Kling, and S. Su, Constraining type II 2HDM in light of LHC Higgs searches, J. High Energy Phys. 01 (2014) 161.

[32] P. M. Ferreira, R. Guedes, M. O. P. Sampaio, and R. Santos, Wrong sign and symmetric limits and non-decoupling in 2HDMs, J. High Energy Phys. 12 (2014) 067.

[33] P. M. Ferreira, R. Guedes, J. F. Gunion, H. E. Haber, M. O. P. Sampaio, and R. Santos, The wrong sign limit in the 2HDM, arXiv:1410.1926.

[34] A. Djouadi, The anatomy of electro-weak symmetry breaking. II. The Higgs bosons in the minimal supersymmetric model, Phys. Rep. **459**, 1 (2008).

[35] A. Arhrib, M. Capdequi Peyranere, W. Hollik, and S. Penaranda, Higgs decays in the two Higgs doublet model: Large quantum effects in the decoupling regime, Phys. Lett. B **579**, 361 (2004).

[36] G. Bhattacharyya, D. Das, P. B. Pal, and M. N. Rebelo, Scalar sector properties of two-Higgs-doublet models with a global U(1) symmetry, J. High Energy Phys. 10 (2013) 081.

[37] J. Bernon, J. F. Gunion, H. E. Haber, Y. Jiang, and S. Kraml, Scrutinizing the alignment limit in two-Higgs-doublet models. II. $m_H = 125$ GeV, Phys. Rev. D **93**, 035027 (2016).

[38] ATLAS Collaboration, Report No. ATLAS-CONF-2015-081, 2015; CMS Collaboration, Report No. CMS-PAS-EXO-15-004, 2015.

[39] A. Angelescu, A. Djouadi, and G. Moreau, Scenarii for interpretations of the LHC diphoton excess: Two Higgs doublets and vector-like quarks and leptons, Phys. Lett. B **756**, 126 (2016).